**Publ. Astron. Obs. Belgrade No. 99 (2020), 213 – 216** *Contributed Paper*

# A NEW LOOK AT SURFACE-WAVE SUSTAINED PLASMA: MAGNETIC CURRENT MODEL TREATED BY A FIXED-POINT METHOD

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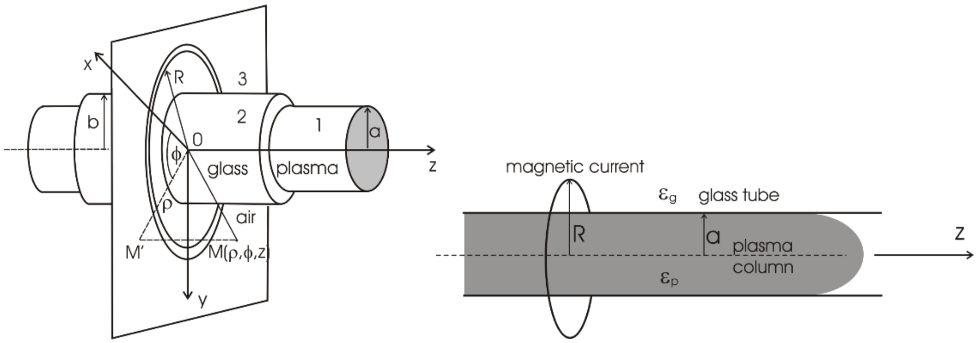
**Abstract.**An analytical description and numerical investigation of the cylindrical plasma columns produced and sustained by a weakly damped surface-wave is presented. This paper suggests using a simple magnetic current model in order to obtain a general expression for normalized power of surface-wave. This expression can be deduced theoretically from the full-wave theory already used for such plasmas. The diagrams of normalized power are computed by an iterative procedure known as the fixed-point method. The fixed-point method appears as the natural choice for the numerical treatment of formulae that could arise in a broad class of physical problems usually recognized as guided surface plasma waves.

## 1. INTRODUCTION

The problem of the propagation of surface-waves along plasma columns had been known for years and has been treated by many authors [Trivelpiece 1967, 1959, Moisan et al 1977, 1979]. Electron waves on a plasma guide attracted the attention of investigators for a long time. A comprehensive review on this topic can be found in [Babovic1999]. We have developed software packets which avoid any starting approximations and successfully calculate expressions in its full-electromagnetic formulations. In this connection, the fixed-point method was completely tested in this paper throughout the calculation of the normalized power of surface-wave in the plasma column.

## 2. STRUCTURE OF THE LINE

We consider the propagation of an azimuthally symmetric ($m = 0$ mode) surface-wave along cylindrical structure (Figure 1) consisting of a plasma column surrounded by air. Figure 1 shows a sketch of the adopted model. The central part of the line consists of a glass tube of radius $b$, whose interior is filled with a homogeneous plasma of radius $a$. Of course, one could imagine plasma is suspended in a glass tube, and we treat the glass wall as a very thin one. The glass tube is a dielectric characterized with permittivity $\varepsilon_g$ (in air $\varepsilon_r \approx 1$).




(a) (b)

Figure 1: (a) Geometric characteristic and applied cylindrical system, (b) plasma created by the magnetic current.

## 3. MAGNETIC CURRENT MODEL: FEATURES OF THE FIELD

The mathematical treatment we begin writing Maxwell's equations in a form adapted to our case

$$\nabla \times \vec{H} = \varepsilon_0 \varepsilon \, \partial \vec{E} / \partial t \, , \tag{1}$$

$$\nabla \times \vec{E} = -\mu_0 \partial \vec{H} / \partial t - \vec{J}_m \, . \tag{2}$$

where $\vec{J}_m$ is the magnetic current density which in the cylindrical coordinate system has only azimuthal component and in the form of Dirac delta function is given as

$$J_{m\phi} = U\delta\left(\rho - R\right)\delta(z) \, . \tag{3}$$

The components of the electric field $\vec{E}$ are $(E_\rho, E_\phi, E_z)$ and magnetic field $\vec{H}$ $(H_\rho, H_\phi, H_z)$. The so-called H (or TE) field is composed of the group $E_\phi$, $H_\rho$, $H_z$ and the E (or TM) field has the components $H_\phi$, $E_\rho$, $E_z$. The typical method for finding the solution of wave equation connected to the mentioned problem of electromagnetic wave propagation along the plasma waveguide; (first, fairly general case wave equation is formed, and, secondly, the wave equation is solved in plasma, glass and air respectively; at the end, the boundary conditions were applied which is the working framework in [Kovačević 2000]and will be here omitted).

## 4. RESULTS

We begin with well-known expression for $Q$ [Aliev 1994] which will be the object of our attention:

$$Q = \frac{1}{2} \int_{S_\perp} \sigma \left| E \right|^2 dS_\perp \tag{4}$$

where $|E|^2 = |E_\rho|^2 + |E_z|^2$ is the amplitude of the electric field of axial symmetric surface-wave, and $\sigma = -\omega\varepsilon_0\varepsilon_i$ ( $\varepsilon_i$ is the imaginary part of complex plasma permittivity). The integration would be over the cross-section which is perpendicular to the wave propagation direction. Since we observe two mediums (plasma and the dielectric surrounding plasma), we can write $Q = Q_1 + Q_2$ where the marks 1 and 2 denote the plasma and glass tube respectively (the assumption of infinitely thick dielectric was made). In accordance with the cylindrical system, we have

$$Q_1 = \frac{\sigma_1}{2}\int_0^a |E_1|^2\, 2\pi\rho d\rho = \pi\sigma_1 \int_0^a \left(|E_{1z}|^2 + |E_{1\rho}|^2\right)\rho d\rho \tag{5}$$

$$Q_1 = \frac{\sigma_2}{2}\int_a^\infty |E_2|^2\, 2\pi\rho d\rho = \pi\sigma_2 \int_a^\infty \left(|E_{2z}|^2 + |E_{2\rho}|^2\right)\rho d\rho \tag{6}$$

where $\sigma_1 = -\omega\varepsilon_0\varepsilon_i = \omega\varepsilon_0(\omega_p^2/\omega)(\nu/\omega)$ and $\sigma_2 = -\omega\varepsilon_0\varepsilon_i = -\omega\varepsilon_0\varepsilon_g tg\delta$. Here $\nu$ is the collision frequency in the electron-neutral collision, and $tg\delta = \varepsilon_i/\varepsilon_g$ is tangent of loss. The equations (5) and (6) can be written in the form $Q_1 = Q_{1z} + Q_{1\rho}$ and $Q_2 = Q_{2z} + Q_{2\rho}$. Substituting the function $E_{1z}$, $E_\rho$, $E_{2z}$ and $E_{2\rho}$ [Kovačević 2000] the following expressions are obtained

$$\frac{Q_{1z}}{Q_0} = \frac{\omega_p^2}{\omega^2}\frac{\nu}{\omega}\frac{(u_1a)^4}{\varepsilon_p^2}\frac{\Delta_1^2}{\Delta'^2}\left[I_0^2(u_1a) - I_1^2(u_1a)\right], \tag{7}$$

$$\frac{Q_{1\rho}}{Q_0} = \frac{\omega_p^2}{\omega^2}\frac{\nu}{\omega}\frac{(\beta a)^2 (u_1a)^2}{\varepsilon_p^2}\frac{\Delta_1^2}{\Delta'^2}\left[I_1^2(u_1a) - I_0(u_1a)I_2(u_1a)\right], \tag{8}$$

$$\frac{Q_{2z}}{Q_0} = tg\delta\frac{(u_2a)^4}{\varepsilon_g}\frac{\Delta_2^2}{\Delta'^2}\left[-K_0^2(u_2a) + K_1^2(u_2a)\right], \tag{9}$$

$$\frac{Q_{2\rho}}{Q_0} = tg\delta\frac{(\beta a)^2(u_2a)^2}{\varepsilon_g}\frac{\Delta_2^2}{\Delta'^2}\left[-K_1^2(u_2a) + K_0(u_2a)K_2(u_2a)\right], \tag{10}$$

where $Q_0 = (1/a^4)\left(J^2/8\pi\right)\left(1/\omega\varepsilon_0\right)$, $\Delta' = d\Delta/d(\beta a)$.To make the reading of the text easier, the meaning of individual abbreviations is:

$$\Delta = \begin{vmatrix} u_1aI_1(u_1a) & u_2aK_1(u_2a) \\ (u_1a)^2\varepsilon_g I_0(u_1a) & -(u_2a)^2\varepsilon_p K_0(u_2a) \end{vmatrix},\quad \Delta_1 = \begin{vmatrix} u_1aK_1(u_1a) & u_2aK_1(u_2a) \\ -(u_1a)^2\varepsilon_g K_0(u_1a) & -(u_2a)^2\varepsilon_p K_0(u_2a) \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} u_1aI_1(u_1a) & u_1aK_1(u_1a) \\ (u_1a)^2\varepsilon_g I_0(u_1a) & -(u_1a)^2\varepsilon_g K_0(u_1a) \end{vmatrix}, u_1 = \sqrt{\beta^2 - k_0^2\varepsilon_p},\ u_2 = \sqrt{\beta^2 - k_0^2\varepsilon_g}.$$

The total normalized power is

$$\tilde{Q} = \tilde{Q}_1 + \tilde{Q}_2 = \tilde{Q}_{1z} + \tilde{Q}_{1\rho} + \tilde{Q}_{2z} + \tilde{Q}_{2\rho}, \tag{11}$$

The equation (11) was treated by the fixed-point method using the first approximation $Y_1$ of the dispersion relation [Babović 2002].

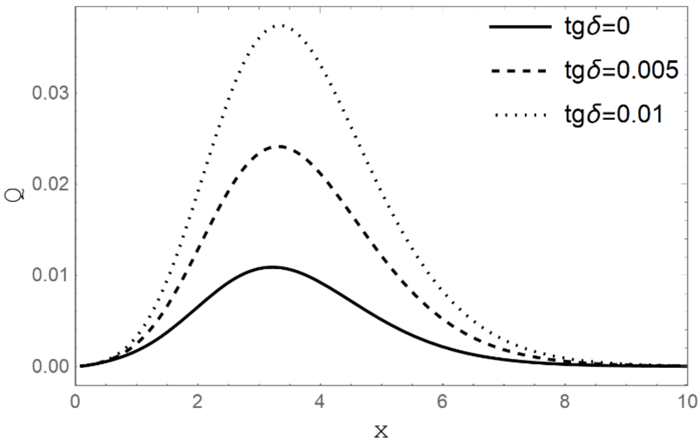



Figure 2: Normalized power $\tilde{Q}$ versus normalized wavenumber $X = \beta a$.

## 5. CONCLUSION

An analytical model, known as a magnetic current model, for the axialstructure of weakly collisional surface-wave sustained plasma was developed. In order to demonstrate the effectiveness of the fixed-point method, the normalized power vs. normalized wavenumber was calculated. The normalized power per unit length versus normalized phase coefficient for three values of tangent loss was presented. We notice that the absorbance of the power has a clearly expressed maximum around the point $X \sim 3$ for the given parameters; the magnitude of the maximum depends of dielectric losses such that $\tilde{Q}$ in generally increases as $tg\delta$ increases. This characteristic is very important for analyzing surfatron plasma as well as for calculation of the attenuation coefficient $\alpha$.

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