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A SIMPLE FORMULA FOR CALENDARS

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Abstract. A simple formula which comprises and unifies the three calendars originating from the Roman calendar is proposed. Its significance is rather pedagogical than scientific because the purpose is to explain, in a way as simple as possible, the contributions of the Gregorian and Neojulian reforms.

1. INTRODUCTION

The calendar question is an old one and very well known. Calendars can be based in many ways, so that there exist, e. g. lunar, luni-solar and solar calendars. For instance, the Muslim calendar is lunar, that of the Jews luni-solar, etc. The calendar adopted by the Christians is solar, but it is due to the Romans. The version of the Roman calendar which was adopted in the Christianity is referred to as the Julian calendar. It was approved by Julius Caesar in 46 BC. An improved version of the Julian calendar was approved by Pope Gregory XIII in 1582. It is known as Gregorian calendar after him. From the time of its proclamation the use of the Gregorian calendar was increasing so that it has become, practically, a universal (world) calendar. On the other hand, it is also well known that some Christian churches still use the Julian calendar as the official one (for instance, Serbian Orthodox Church). There was an initiative among the Greek Orthodox Christians, almost a hundred years ago, aimed at introducing a new calendar which should be better than the Gregorian one. More precisely, the calendar question was considered at a Pan-Orthodox congress held in Constantinople in 1923. A new calendar, mainly referred to as Neojulian, was accepted and recommended. This calendar was the official proposal of the Serbian Church (except it only the Romanian Church had a proposal). Its first version is due to Maksim Trpković (1864-1924); later the second version, slightly changed in order to make 2000 a leap year, was proposed by Milutin Milanković (1879-1958). This is the version valid today; in more detail how this proposal was finally accepted see in, e. g. Milanković (1997).

The intention followed in the present contribution is to facilitate the explanation of the three calendars (say in teaching at levels of primary and secondary school). Here it is borne in mind that, in fact, they are three different versions of the same calendar. To this end a simple formula is proposed, which includes all the three calendars.

Table 1: Basic characteristics of calendars according to equation (1)

calendar	k	l
Julian	1	
Gregorian	100	3
Neojulian	225	7

2. THE APPROACH

As solar calendars the three calendars are based on the same time interval - tropical year. The tropical year expressed, as usually, in terms of the mean solar day is not an integer (or better a natural number). Its value lies between 365 and 366. Clearly, there can be no calendar wherein a year does not have an integral number of days. This was the reason why in the Julian calendar the concept of leap year was introduced. A leap year, unlike an ordinary one, has 366 days. The value by which the tropical year exceeds 365 is represented in a particular calendar by assuming a suitable frequency of leap years. The formula proposed here has namely this value as its subject.

3. RESULTS

Let the value by which the tropical year exceeds 365 be denoted as x. In principle x should be an irrational number. However, the value for x is established empirically, which implies a finite number of digits after the decimal point in accordance with the achieved accuracy. Therefore, x will be treated as a fraction. This fraction is the subject of the formula proposed here. Its form is

$$x = \frac{k-l}{4k},\ k \in \mathbb{N},\ l \in \mathbb{Z},\ l \geq 0,\ l < k\ . \tag{1}$$

The designations mean: $\mathbb{N}$ - set of natural numbers, $\mathbb{Z}$ - set of integers.

On the basis of equation (1) each of the three calendars can be described by means of its values for k and l. The corresponding values are given in Table 1.

The value for l in the case of the Julian calendar is not given because it is evident from the conditions of (1).

4. DISCUSSION AND CONCLUSIONS

The duration of the tropical year is $365 + x$ mean solar days, x ($0 < x < 1$) is a fraction given in (1). Its value obtained after substituting assumed values for k and l should yield a fit to the empirical values which is as good as possible. In the case of the three calendars a sufficiently good fit is achieved by introducing leap years. The circumstance that in the case of the Julian calendar (which preceded all others) x is equal to $1/4$ means that every fourth year, with no exception, is a leap one. Therefore, l is a correction aimed at improving the fit. The improvement is achieved by decreasing the number of leap years which would be obtained within an assumed cycle of $4k$ years if the Julian calendar were applied. This number is decreased exactly by l; for instance, in the case of the Gregorian calendar within its

400-year cycle there are 97 (100-3) leap years, instead of 100 as it is given in the Julian calendar. What years change the status in order to become ordinary is determined by means of a convention. In the Gregorian convention it is foreseen that secular years, in particular those wherein the factor multiplying 100 is not divisible by four without remainder (for instance in 1601-2000 these are 1700, 1800 and 1900) become ordinary. A similar principle is applied in the case of the Neojulian calendar, within 900 years seven secular years ($l = 7$) become ordinary, in practice the fourth and ninth in the sequence remain to be leap. In its first version (Trpković's one) the initial cycle was 1801-2700. As a consequence, out of the nine secular years only 2200 and 2700 would have been leap. As said above, in its current (Milanković's) version this is changed, the cycle is 2001-2900, 2000 remains to be leap, 2400 (in common with Gregorian calendar) is also leap, and finally 2900. This means that only in 2800 the discrepancy with the Gregorian calendar will occur, instead of 2000, which would have occured if Trpković's version had been accepted. In addition, out of the three calendars the Neojulian one yields the best fit, the value obtained for x (equation (1), Table 1) is $x = 0.2422$, as the evidence shows. On this matter there exist other articles (e. g. Simovljević 1996, Kečkić 2001).

Equation (1) has a pedagogical significance. By using (1) it becomes possible to give a simple explanation which includes all the three calendars. This is an advantage because the three calendars are in fact three versions of the same calendar, of that due to the Romans. Therefore, equation (1) is to be recommended in formation of the school curricula.

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