Publ. Astron. Obs. Belgrade No. 98 (2018), 313 - 316 *Poster*

LOCAL VELOCITY DISPERSION RATIO DESCRIBED BY MEANS OF A NEW FORMULA

S. NINKOVIĆ

Astronomical Observatory, Volgina 7, 11060 Belgrade 38, Serbia
E–mail: sninkovic@aob..rs

Abstract. The ratio of two mean random velocity squares (dispersions) in the Galactic plane is revisited. A new formula is obtained. It contains the classical one (known as epicyclic) as a special case. Like the classical formula the new one is also applicable to the thin disc (low orbital eccentricities). However, in the new formula the influences of the ratio of the Oort constants moduli and that of the distribution of orbital phases and eccentricities are clearly separated.

1. INTRODUCTION

Among the subsystems of the Milky Way the existence of which has been confirmed through kinematical studies for the solar neighbourhood the thin disc is, certainly, the most important. The Sun is also a thin disc star. Stars of the thin disc are known to orbit the centre of the Milky Way in nearly circular orbits. Their motion can be successfully decomposed into the motion in the Galactic plane and perpendicularly to it. The random velocities of thin disc stars are obtainable on the basis of kinematical studies, of importance are the second-order moments. They form a symmetric tensor. The subject of the present paper concerns the two diagonal elements in the plane, more precisely its ratio, for which a new formula is proposed. What is given here is a shortened presentation only. The full account is to be given elsewhere.

2. THE OBJECTIVE

A coordinate system centred on the projection of the Sun onto the Galactic plane x, y, z is introduced. The orientation of the axes is: x positive towards $l = 0°$, $b = 0°$; y positive towards $l = 90°$, $b = 0°$; z positive towards $b = 90°$. Due to such a choice of the coordinate axes it will be valid: $|v_R| = |v_x|$, $|v_\theta| = |v_y|$, $\vec{v}$ is random velocity, R is distance to the Z axis (parallel to z through the Galactic centre), θ is the angle in the Galactic plane. According to the formula well known from the literature (e. g. Angelov 2013, (29.14), p. 149) the mean squares taken at the projection of the Sun should satisfy the following relation

$$\frac{\overline{v_\theta^2}(R_\odot, 0)}{\overline{v_R^2}(R_\odot, 0)} = (1+\alpha)^{-1} \ , \quad \alpha = \frac{A}{|B|} \ ; \tag{1}$$

A and B are the Oort constants. Since the evidence usually suggests for this ratio to be about 0.4 (e. g. Dehnen & Binney 1998), on the basis of (1) one derives $\alpha = 1.5$. However, more recent evidence, especially the rotation curve, is in favour of smaller values, slightly exceeding 1.0 (e. g. Iocco et al. 2015). The new formula which is proposed here shows that the relationship between the ratio of the mean squares of random velocity and that of the moduli of the Oort constants is not as straightforward as it seems to be on the basis of (1).

3. RESULTS

The potential of the Milky Way is assumed to be stationary and axially symmetric. Since the motion of disc stars is the subject, the motion in R is assumed to be unaffected by that in Z. The consequence is a quasi integral of motion - specific (per unit mass) energy in the plane - $\frac{1}{2}V_p^2 - \Pi(0) \approx const$ (Π - potential, $V_p^2 = V_R^2 + V_\theta^2$). This integral together with the component J_Z of the specific angular momentum (exact integral) leads to approximately constant extremal distances, R_p and R_a which are replaced by the following quantities

$$R_m = \frac{R_p + R_a}{2} \ , \quad e = \frac{R_a - R_p}{R_p + R_a}. \tag{2}$$

Since all stars of a sample are at the Sun, the present distance is the same (equal to $R_\odot$). Because of this a dimensionless quantity χ, $\chi = R_\odot / R_m$, is introduced. If the eccentricity e (equation 2) is the same for all sample stars, then it will be: $1 - e \leq \chi \leq 1 + e$. Let the dependence of the potential on R for $Z = 0$ correspond to a power law for the circular speed u_c, $u_c \propto R^\delta$. Since $R_\odot$ lies within the interval $[R_{p\,\min}, R_{a\,\max}]$ (minimum and maximum for individual extremal distances in the sample), it will be $\delta = (1-\alpha)/(1+\alpha)$. Such a case for $-0.5 \leq \delta \leq 1$ has been studied by the present author (Ninkovich 1986). In this way expressions for V_p^2, V_θ^2 and $|V_\theta|$ as functions of χ with α and e as parameters and $u_c^2(R_\odot)$ or $u_c(R_\odot)$ as units can be obtained. Then the obtaining of all mean values, which are necessary, is reduced to the determination of the mean values of the corresponding functions of χ. Since stars of the thin disc are of interest, it will be: $e \approx 0$. This circumstance offers the possibility to obtain simplified expressions wherein χ is replaced by a new variable φ, $\varphi = (\chi - 1)e^{-1}$. It is clear that: $-1 \leq \varphi \leq 1$. It should be added that the mean value of v_R^2 is equal to the mean value of V_R^2 because $\overline{V_R} = 0$; $\overline{v_\theta^2} = \overline{V_\theta^2} - \overline{|V_\theta|}^2$. Finally it is obtained

$$\begin{aligned} \overline{v_R^2}(R_\odot, 0)_e &= 4(1+\alpha)^{-1} u_c^2(R_\odot)(1 - \overline{\varphi^2})e^2 \ , \\ \overline{v_\theta^2}(R_\odot, 0)_e &= 4(1+\alpha)^{-2} u_c^2(R_\odot)(\overline{\varphi^2} - \overline{\varphi}^2)e^2 \ . \end{aligned} \tag{3}$$

The subscript e in (3) means that the mean values are taken for a sample of thin-disc stars with the same orbital eccentricity. For a sample of thin disc stars of various eccentricities distributed following a function f(e) the corresponding mean values will be

$$\begin{aligned} \overline{v_R^2}(R_\odot,0) &= 4(1+\alpha)^{-1}u_c^2(R_\odot)\int_0^{e_l}(1-\overline{\varphi^2})e^2\mathrm{f}(e)de \ , \\ \overline{v_\theta^2}(R_\odot,0) &= 4(1+\alpha)^{-2}u_c^2(R_\odot)\int_0^{e_l}(\overline{\varphi^2}-\overline{\varphi}^2)e^2\mathrm{f}(e)de \ . \end{aligned} \tag{4}$$

4. DISCUSSION AND CONCLUSIONS

The ratio of the two mean velocity squares which are subject in (4) reduces to (1) provided that $\overline{\varphi}$ and $\overline{\varphi^2}$ are independent of eccentricity and have the following values $\overline{\varphi}=0$, $\overline{\varphi^2}=0.5$. In both cases we have the middles of the allowed intervals.

In the present paper only the behaviour of $\overline{\varphi}$ will be considered. It affects the asymmetric drift, i. e. the difference $\delta u = u_c - \overline{|V_\theta|}$. The quantity $\overline{|V_\theta|}$ is obtained from an integral, analogously to (4), wherein $f(e)de$ is multiplied by $\overline{|V_\theta|}(e)$, a quantity which depends on both $\overline{\varphi}$ and $\overline{\varphi^2}$, but the influence of the former one is much more significant.

On the other hand in view of the relations $\overline{V_p^2}=\overline{v_R^2}+\overline{|V_\theta|}^2+\overline{v_\theta^2}$ and $\overline{|V_\theta|}=u_c-\delta u$, mentioned above, and neglecting δu^2 one may write

$$\begin{aligned} \delta u &= \frac{\nu+1+\mu}{2u_c}\overline{v_R^2} \ , \\ \nu &= \frac{u_c^2-\overline{V_p^2}}{\overline{v_R^2}} \ , \\ \mu &= \frac{\overline{v_\theta^2}}{\overline{v_R^2}} \ . \end{aligned} \tag{5}$$

Relation (5) can be applied to a sequence of samples consisting of thin-disc stars. The sense of the word sequence is the condition that the quantities δu, $\overline{v_R^2}$ and $\overline{v_\theta^2}$ increase simultaneously. If the ratio μ were approximately the same for all sequence samples, as the literature shows (e. g. Dehnen & Binney 1998), then according to (5) δu would be proportional to $\overline{v_R^2}$, provided that the other ratio ν were also approximately the same. Examinations done by the present author show i) simultaneous increase of all the three δu, $\overline{v_R^2}$ and $\overline{v_\theta^2}$ is a consequence of $\overline{\varphi}$ as an increasing function of eccentricity; ii) such a $\overline{\varphi}$ is not consistent with $\nu \approx const$ in conditions when $\mu \approx const$.

A comparison of the samples studied in the paper by Dehnen and Binney (1998) with some of the samples from the paper by Cubarsi et al. (2017) for which the ratio μ is approximately the same (about 0.4) indicates that they also have approximately the same mean motion in θ with respect to the Sun and $\overline{v_R^2}$. This can be interpreted by assuming the same eccentricity distribution $f(e)$ for all these samples. A preliminary formula proposed by the present author is

$$f(e) = Ce^n\exp(-e/c) \tag{6}$$

The parameters of (6) are C, n and c. The value for n seems to be about 3, c is obtained from the condition for the maximum of $f(e)$ (most likely the maximum occurs at about $e = 0.1$). Samples within the thin disc would differ by the value for e_l (equation (4)), the smaller $\overline{v_R^2}$ and δu are, the lower is the corresponding e_l.

At the end it should be pointed out that $\overline{\varphi^2} = const = 0.5$, as required for the old formula (equation (1)), may be acceptable, but $\overline{\varphi} = const = 0$ is inconsistent with the asymmetric drift, i. e. it is not acceptable. Therefore, any clear correspondence between the ratios of the moduli of the Oort constants (α) and that of the mean velocity squares (μ - equation (5)), as follows from (1), does not exist.

Acknowledgment

This research has been supported by the Ministry of Education, Science and Technological Development (Project No 176011 "Dynamics and Kinematics of Celestial Bodies and Systems").

References

Angelov, T.: 2013, Zvezdana astronomija, Matematički fakultet, Beograd.
Cubarsi, R. Stojanović, M., Ninković, S.: 2017, *Serb. Astron. Journal*, **194**, 33.
Dehnen, W., Binney, J. J.: 1998, *Mon. Not. R. astr. Soc.*, **298**, 387.
Iocco, F. Pato, M. & Bertone, G.: 2015, *Nat. Phys. letters*, **11(3)**, 245.
Ninkovich, S.: 1986, *Astrofizika*, **24**, 411.