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# ILLUMINANCE AND VISIBILITY AT TWILIGHTS

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**Abstract.** Visibility is important in diverse human activities, among others in roadway traffic, where the good visibility is very important for safe driving. Decrease of visibility during the twilight-time is the cause of increased number of traffic accidents. Determination of the visibility under which the accident occurred is essential for accident analysis. Visibility of an object (target) depends on its illumination and on ability of human vision system to detect it. Here, the methods of determination of illuminance during twilight-time by sky brightness and some important factors of human visual system for visibility determination, like contrast sensitivity, are briefly presented.

## 1. INTRODUCTION

There is an increasing require of community for information about some astronomical events, since they have an important influence on human everyday activities. The most frequently needed information are the rise and set times of the Sun and Moon, the beginning and ending times of twilights, duration of daylight, azimuth and height above the horizon of the Sun or Moon, the amount of natural light at a designated time of day, at twilight or night etc. The determination of these quantities is the duty of astronomers. In this article I would like to concentrate on the determination of illumination of outdoor objects by solar scattered light in Earth atmosphere. It is very important information since it determines, along artificial light sources, the visibility of outdoor objects during twilight-time. Besides, the influence of human visual system on visibility will be discussed. Here the visibility is defined as the largest distance ($D_{\rm max}$) at which a target or object can be seen.

In every-day situations the visibility can be very important to detect, recognize or identify a given object (target). One of such every-day situation is the roadway traffic, where the good visibility is very important for safe driving conditions. Unfortunately, during the twilights this conditions decline very rapidly. Degradation of visual performance during these periods of the day is a leading contributor to increasing number of roadway accidents. Determination of the visibility under which the accident occurred is interesting for judicial system, insurance companies, low firms etc. Therefore it is often necessary to reconstruct the visibility at a given date, time and geographical position. The visibility is a very complex phenomenon, which depends on many factors, first of all on illuminance (total luminous flux incident on a surface, per unit area), luminance (photometric measure of the luminous intensity per unit area) and vision. Consequently, the estimation of visibility is affected by many physical (objective) and subjective factors.

In this article I would like to show some basic steps of illuminance and visibility determination during the twilight-time. The major determinant of the amount of natural light during moonless twilight is the atmosphere, which is a very complex physical (optical) medium. The illuminance of Earth surface by moon depends on its phase and can reach about 0.3 lux at full moon. It becomes a dominant source of illuminance and determines the outdoor visibility during nautical and astronomical twilights. The illuminance by starlight, zodiacal light and airglow is about 2 millilux. They are important light sources during a moonless night and determine the outdoor visibility. The detector is the human visual system (HVS), which is a very complex psychophysical system. I will present only the basics mathematical description of illuminance determination in implicit form. Similarly, the role of the human visual system in visibility determination will be described mainly by implicit mathematical equations and qualitatively by block diagram of processing of visual information.

## 2. OBJECTIVE FACTORS

### 2. 1. TWILIGHT ILLUMINANCE

The twilight-time of the day is occurring when the upper edge of apparent solar disk (uesd) is just graze the horizon (its elevation, $h_{\text{uesd}} = 0°$), but the elevation of solar disk center ($h_s$) is larger than $-18°$). Twilight-time is divided, to a certain extent artificially, into three periods: civil twilight ($-6° \leq h_s < 0°$) nautical twilight ($-12° \leq h_s < -6°$) and astronomical twilight ($-18° \leq h_s < -12°$). During the twilight-time the ambient is not illuminated by direct sunlight any longer, but by other light sources, like solar scattered light (sky), Moon and other celestial objects.

The target visibility during the twilight-time without moonlight (especially in civil and nautical twilight) depends on illumination by sky brightness. Therefore, I will deal with sky brightness determination caused by solar scattered light in Earth atmosphere. Let's, firstly, consider the geometry of solar light scattering (Figure 1). The position of an illuminated target or object ($N$) on the Earth is determined by its geographical coordinates: longitude, latitude and high above the sea level ($L$). The position of the Sun is determined by celestial horizontal coordinates: zenith distance ($z$) and azimuth ($A$). Let the light scattering occur on air particles in elementary volume ($dV$) at position $P$. The position of this elementary volume is determined by its zenith distance ($z_P$) and azimuth ($A_P$). The azimuth is measured from vertical of the Sun. In this case the azimuth of the Sun is equal to zero.

The most correct way of determination the sky brightness in one arbitrary direction is to solve the radiative transfer equation for each beam of light from Sun to the target trough scattering by the elementary volume. The observed brightness from a given direction (sight path) is the sum of scattered solar light by atmospheric particles along line-of-sight from twilight ray region toward target. The implicit mathematical solution of this problem will be described following the description given by Розенберг (1963).

Let the intensity of solar radiation, outside of the atmosphere,is $I_0(\lambda)$ and $\omega_0$ is the Solar disk surface measured in angular units from point P. The illuminance ($E_0$) at point $P$ without presence of Earth's atmosphere is:

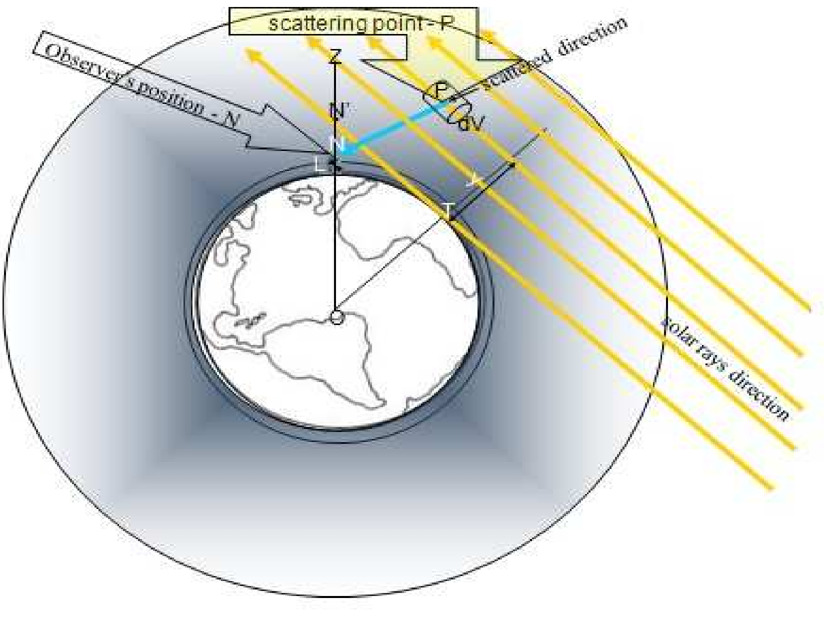


Figure 1: Geometry of scattered solar light in twilight-time (adopted from Розенберг, 1963).

$$E_0(\lambda, t) = I_0(\lambda) \cdot \omega_0(t). \tag{1}$$

For our purpose, we can consider that $I_0(\lambda) = \text{const.}$, but $\omega_0$ varies about 6% during the year due to the elliptical orbit of the Earth, achieving its maximum at the beginning of the year and its minimum at the beginning of July. The mean solar illuminance is around $128 \times 10^3$ lx.

Due to extinction of Earth atmosphere the illuminance of particles at point $P$ is:

$$E_P = \frac{T_P(z, z_P, A_P, h_P, \lambda) \cdot E_0(\lambda, t)}{\nu(z, y, r)} \tag{2}$$

where $T_P$ is the atmospheric transmission coefficient along solar light ray path to point $P$. It depends on optical properties of atmospheric particles along (refracted) path of solar ray trough atmosphere up to the point $P$. That is to say, it depends on air mass or optical thickness of atmosphere along solar light path. $\nu$ is the correction factor due to refraction of light.

On atmospheric particles in an elementary volume $dV$, formed around the point $P$, the light scatters in all direction. The characteristics of scattered radiation in a specific direction are defined by Stock's parameters $dS_i$ (Розенберг, 1963):

$$dS_i \cdot d\omega = \frac{1}{l^2} \sum_j D_{i,j}(\vec{l}, \vec{l_0}) \cdot S_j^0(\vec{l_0}) \cdot dV \cdot d\omega_0, \tag{3}$$

where $D_{i,j}$ is the scattering matrix, vectors $\vec{l}$ and $\vec{l_0}$ defined the scattered and the incident light directions, respectively. $S_j^0$ is the Stock's parameter of incident light. $l$

is the distance between the scattering point and target, $d\omega$ and $d\omega_0$ are the solid angle of $dV$ with vertex at target and the light source (Sun) with vertex in $P$, respectively.

Stock's parameters of scattered light from elementary volume $dV$ toward target's position ($N$) are defined by

$$dS_i(z, z_P, A_P, h_P, \lambda) = \frac{D_{i,1} \cdot T_N(z_P, A_P, L, \lambda) \cdot T_P(z, z_P, A_P, h_P, \lambda) \cdot E_0 \cdot d\vec{l}}{\nu(z, y, r)}, \quad (4)$$

where $T_N$ is the coefficient of atmospheric transparency along the sight path toward the target $N$. Finally, Stock's parameters from line of sight direction $(A_P, h_P)$ are

$$S_i(z, z_P, A_P, h_P, \lambda) = \int_0^{l_{\max}} dS_i(z, z_P, A_P, h_P, \lambda), \quad (5)$$

The illuminance ($E$) of horizontal surface at position $N$ is defined as the light collected from the entire hemisphere:

$$E(z, \lambda) = \int_0^{2\pi} \int_0^1 S_1(z, z_P, A_P, \lambda) \cos z_P \cdot d(\cos z_P) dA; \qquad S_1 = I. \quad (6)$$

Since the illumination of scattered solar light depends on sun zenith distance and wavelength, the illumination of the horizontal surface undergoes both intensity and spectral changes during the twilight. At geographical position of our country, during twilight-time the illumination changes rather rapidly by a factor of about $10^7$: decreases at dusk and increases at dawn as a function of time. In the case of arbitrary oriented surface, the limits of integrals are different. For example, in the case of vertical target surface, which is regular situation in roadway traffic, the interval of azimuth limit decreases to $\pi$ (instead of $2\pi$).

### 2. 2. ANGULAR SIZE

The target size, or more exactly its angular size with vertex at eye lens, is also a very important factor in determining visibility. It has an influence on pupil diameter, too (see the next section). If the size is too small, then the object cannot be seen regardless how high the contrast is. Besides brightness of sky and target size, there are many other objective factors (clouds, raining, snowing, fog, windscreen, glasses etc.), which determine the visibility. Their influence on visibility will be discussed in separate paper (in preparation). It has to be note that some objective and subjective factors affect the visibility simultaneously.

## 3. SUBJECTIVE FACTORS

I will present only some of the most important subjective factors, following Barten's (1999) model of spatio-temporal contrast sensitivity function of human visual system.

### 3. 1. LUMINANCE ($L$)

Light from the twilight sky falls on a target surface, a fraction of which reflects from it and illuminate the observer's eye pupil. The amount of light falling on the pupil is the luminance of target ($L_t$). The luminance depends on target reflectance (objective

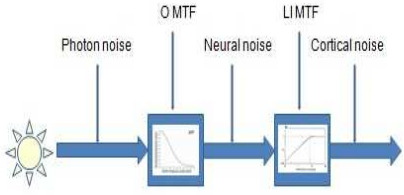


Figure 2: Block diagram of signal modulation and degradation in HVS.

factor), on target size (objective factor) and on eye pupil solid angle viewing from the target (subjective factor). If the luminance is lover than the sensitive threshold level of human visual system (HVS) the target will be invisible.

### 3. 2. CONTRAST ($C$)

Human eye can distinguish objects from each other or from their background (see the object), if the difference in their luminance or color is large enough. Here the luminance difference will be discussed, while the color difference will be considered in separate article. For an observer the ability to detect a target (object) mainly depends on target luminance ($L_t$), background luminance ($L_b$) and adaptation luminance ($L_a$). The visibility of a target at a given adaptation luminance level ($L_a = \text{const}$) is directly related to the contrast between the target and its surroundings. The quantification of this fact can be done by contrast coefficient ($C$), which is defined as

$$C = \frac{L_t - L_b}{L_b}, \tag{7}$$

$C$ can be determined by measuring the luminance of the target and the luminance of its background. If $C > 0$ (positive contrast) the observer detect the target in reflected light. If $C < 0$ (contrast is negative) the observer detect the target's silhouette. If $C = 0$ the target is invisible.

Whereas, due to the degradation of light signal while propagate trough HVS from optical to visual cortex, a minimal luminance difference between target and background ($\Delta L_{\text{min}}$) is needed to perceive the target with a certain probability level. Consequently the target becomes undetected even if the contrast differs from zero, but it is lover than a certain contrast value. This minimal contrast value is the threshold contrast ($C_{\text{th}}$). The threshold contrast is the ratio of minimal luminance difference ($\Delta L_{\text{min}}$) and the background luminance $L_b$. Therefore, the threshold contrast can be defined as (Adrian 1989):

$$C_{\text{th}} = \frac{L_{\text{th}} - L_b}{L_b}, \tag{8}$$

where $L_{\text{th}}$ is the object luminance at threshold contrast. The inverse of threshold contrast is the contrast sensitivity (CS). Threshold contrast is related to the fluctuation (noise) of luminosity (photon flux) and signal modulation, mathematically, describe by modulation transfer functions (MTFs), while propagate along human visual system path. The block diagram in Figure 2 illustrates the main degradation sources of target (objective) contrast by HVS.

As it noticed on block diagram, beside photon noise the neural and cortical noise is injected into the visual signal during the signal transfer from eye detecting system to visual center of brain. Therefore, threshold contrast depends not only on physical properties of observed object and its background but also from psychophysical properties of human visual system.

Now we can define a refined visibility criteria. If $C < C_{\rm th}$ the object is irresolvable from noise, which practically means that it is invisible. If an object's contrast is too low to be seen, then other visual factors are irrelevant. If $C > C_{\rm th}$ the object signal is resolved and might be visible.

In following sections the determination of factors designated in the block diagram will be shortly considered.

### 3. 3. PHOTON NOISE

The threshold contrast is directly related to the power spectral density of photon noise ($\Phi_{\rm ph}$). Namely, photon noise generates fluctuation in signal of photo-detectors in retina and increase the threshold contrast. At low luminance level photon noise can be the major contribution to threshold contrast level. The power spectral density of photon noise, as it can be proved, is

$$\Phi_{\rm ph} = \frac{1}{Q_e f_n E}, \qquad \text{where} \qquad E = \frac{\pi d^2}{4} L, \tag{9}$$

$Q_e$ is the quantum efficiency of the eye, $E$ is the retinal illuminance, $f_n$ is the conversion factor between light units and photon flux density, $d$ is the pupil diameter and $L$ is the target luminance.

### 3. 4. OMTF

Human visual system consists of optical, photoelectric conversion and information transmission part. The optical part of HVS focuses light on the retina and forms an object image. Optical modulation transfer function (OMTF) is the measure of eye's ability to transfer the contrast from the object to the image at a given resolution. The OMTF usually includes the modulation transfer functions (MTF) of the eye lens, the scattered light in eye media, light-sensitive detectors in retina (rods and cons) etc. The resulting MTF of optical part of HVS is a convolution of all these individual effects. Therefore, according to central limit theorem, the resulting MTF versus spatial frequency (u) can be describes by Gaussian function (Barten 1999):

$$\mathrm{OMTF} = e^{-2\pi^2\sigma^2 u^2}, \tag{10}$$

$\sigma$ is the standard deviation of the line-spread function resulting from the convolution of different elements of the convolution process. Its value depends on eye pupil diameter and the light-sensor density. Eye pupil diameter, otherwise, depends on luminance and angular field size of the object and viewer's age (decreases with age).

### 3. 5. NEURAL NOISE

Neurons in visual system consist of photoreceptor cells (rods and cons), cells for signal transmission and cells for processing the visual information. In modeling the neural noise we can assume that it is caused by statistical fluctuation of transported visual

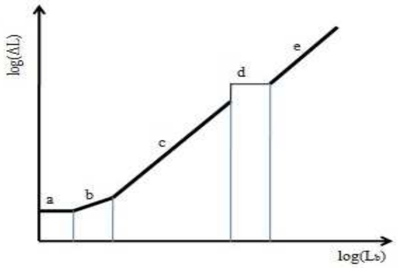


Figure 3: Schematic draw of luminance increment versus background luminance.

signal from retina to visual cortex in nerve fibers. This fluctuation can be described by standard deviation of signal in nerve fibers as (Barten, 1999):

$$\sigma = \sqrt{\frac{\Phi_0}{\Delta S \cdot \Delta t}}, \tag{11}$$

$\Phi_0$ is the relative power spectral density of neural noise, $\Delta S$ is the angular area covered by a nerve fiber and $\Delta t$ is the integration time of HVS. In equation (11) it was assumed that neural noise is independent on background luminosity. Whereas, in the case of increasing background luminosity, for successful target detection, the increase of minimal luminosity increment ($\Delta L_{\min}$) is needed. Figure 3 schematically shows the log–log dependence of target luminosity increment on background luminosity.[1] Short comments on properties of different segments (signed by letters) of luminance increment variation with background luminance level showed in Figure 3:

**a)** Very low dark background: neural noises define the luminance increment.

**b)** Photon noise is the main source of the luminance increment. The DeVries-Rose law is applicable, which predict that increment increases with the square root of background luminance.

**c)** The slope of linear function in log–log representation is equal to 1. The ratio of increment to background luminance is constant. Here the Weber's law is valid.

**d)** At this point the photosensitive detectors (cone type) are saturated by background luminosity ($\Delta L/L = \infty$).

**e)** At these background levels the rod type detectors in retina are activated. Again, the Weber's law is valid in this section.

### 3. 6. LATERAL INHIBITION MTF

In signal conduction process there exist an inhibition mechanism, which lowered the signal and noise in neighboring neural cells in lateral direction. Its efficiency increases with decreasing spatial frequency. This process can be modeled by the following modulation transfer function (Barten 1999):

[1] Adopted from Figure 1 in `https://www.yumpu.com/en/document/view/18209058/lecture-11-light-adaptation-webers-law-arapaho-nsuok`.

$$\mathrm{MTF} = \sqrt{1 - \mathrm{e}^{-(\frac{\mathrm{u}}{\mathrm{u_0}})^2}}, \tag{12}$$

where $u$ is the current spatial frequency and $u_0$ is the frequency at which the modulation practically stopped. According to measurements it happens at $u_0 \approx 7$.

## 4. VISIBILITY LEVEL

Visibility level (VL) is a measure of actual visibility in units of threshold contrast:

$$\mathrm{VL} = \frac{C}{C_{\mathrm{th}}}. \tag{13}$$

It can be used for mapping the visibility distribution on a target and its background. By comparing VL maps of different light sources it is possible to distinguish in which part of illuminated area the illumination of individual sources are dominant. Such situation can take place for example at twilight-time with moonlight, or twilight illumination combined with illumination by artificial light sources, like car headlamps or street-light.

## 5. DISCUSSION AND CONCLUSION

For our purpose of determination the maximal distance ($D_{\mathrm{max}}$) to which a given observer (a car driver, for instance), with a given acuity and a given contrast sensitivity may detect with a given probability a given object (a car or a pedestrian, for instance) illuminated by twilight sky can be calculated from condition VL = 1. Since $(D_{\mathrm{max}} = f(\mathrm{VL}((D_{\mathrm{max}}))$, for determination of ($D_{\mathrm{max}}$) we need to introduce iteration procedure. Moreover, as we can see from equations (6), (10), (11) and (13) the calculation of VL is a very tedious job since these equations depends on many parameters like air chemical composition and its density distribution, optical characteristics of target, observer's visual system parameters (acuity, sensitivity, and contrast sensitivity), which are not available in everyday situations, for instance at position and time of traffic accident. Consequently, we are forced to introduce in calculation some approximations, which can significantly lower the accuracy of visibility determination.

I have developed a computer program for visibility calculation, named VIDO (VIsibility Determination of Objects). VIDO takes into account the light sources characteristics (e.g. sky brightness distribution, Sun and Moon illuminance), the optical properties of Earth atmosphere (including also clouds, rain, fog, aerosols etc.), the physical properties of observed objects (size, shape, orientation, albedo, color), the optical and psychophysical properties of viewer's eye (observer's pupil diameter, its variation with observer's age, viewer adaptation state, viewer optical correction, light sensitivity, viewer eye disease).

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