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INFLUENCE OF THE SOFTENING LENGTH ON STABILITY OF SPIRAL GALAXIES IN N-BODY SIMULATIONS

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Abstract. Softening length is a numerical parameter which is included in calculation of gravitational force between two bodies, in N-body simulations, in order to avoid divergence on small distances and ensure smooth function for gravitational potential. It depends on number of particles in simulations, mass of the particles, dimension of the system and timestep. In this paper we presented how specific choice of values for the softening length affects stability of spiral galaxies like Milky Way and M31. Galaxies in our simulations are consisted of three components: bulge, disk and dark matter halo. Components have different number of particles, dimension and particle mass. Differences between rotation curves, density profiles and energy conservation for different values of softening length were investigated.

1. INTRODUCTION

In order to explore Galaxy, Local group, clusters of galaxies and Universe, various types of numerical simulations were developed last almost half of the century. The main goal of these simulations is studying and describing galaxy formation and evolution. Part of these processes are interactions between galaxies. Unlike stars, interactions between galaxies are very often and very important for evolution if we assume bottom-up model for formation of large galaxies. Mergers, flybys, tidal stripping, etc. can be represented through numerical simulations. Values for some parameters for model constructing are known from observations and the other one are free parameters. Model which is numerical N-body representation of galaxy, or galaxy interaction is better if it differs from observed quantity as less as possible.

Galaxies have large number of stars, so these systems can be threaten through statistical mechanics (Binney & Tremaine, 2011). We can represent galaxy as N-body system (without gas) and only force in the system is gravitational force. Number of bodies, or "particles" is several orders of magnitude smaller then a number of stars in galaxy. Particles are used to represent a mass distribution of real system. As we assume collisionless system, we can use Boltcman's equation for distribution function

(Binney & Tremaine, 2011):

$$\frac{Df}{Dt} = 0 \tag{1}$$

Density of the system and gravitational potential are connected with Poisson equation:

$$\bigtriangledown^2 \Phi = 4\pi G\rho \tag{2}$$

The evolution of isolated self-gravitating collisionless system is described with two previous equations. In N-body simulation, particles of the system are moving due to gravitational attraction and dynamical description assume known coordinates and velocities of every particle at the end of each timestep. Equations that are integrated are equations of motion:

$$\frac{\overrightarrow{dr_i}}{dt} = \overrightarrow{v_i} \tag{3}$$

$$\frac{\overrightarrow{dv_i}}{dt} = \sum_{j\neq i}^{N} Gm_j \frac{\overrightarrow{r_j} - \overrightarrow{r_i}}{(|\overrightarrow{r_j} - \overrightarrow{r_i}|^2 + \epsilon^2)^{3/2}} \tag{4}$$

The quantity ϵ in equation (4) is called softening length. We assume that force is softened in order to get smooth gravitational potential and to avoid singularity as $r \rightarrow 0$. This modifies law of gravity at small distances. There is no optimal value for softening length, even if adaptive value is used(Iannuzzi & Dolag, 2011). In N-body codes value of softening length depends on number of particles in the system, mass of the particle (particle type), volume of the system. We need to adjust values for baryonic matter and dark matter if these particles have different masses in simulation and occupy different volumes. Also, timestep need to be adjusted. In practice there are many proposals for choosing a value for softening length: mean distance between particles, distance which is several orders of magnitude less then mean distance in most dense region of the galaxy, softening length for dark matter particle ten times larger then for baryonic particle, if this ratio is equal for masses (Sadoun et al. 2013),...

Stable model predicts conservation of energy and angular momentum (equations (5) and (6)) and virial theorem (equation (7)) should be satisfied.

$$E_{tot} = \frac{1}{2} m_{tot} v_{com}^2 - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{Gm_i m_j}{r_{ij}} + \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \tag{5}$$

$$L_{tot} = \sum_{i=1}^{N} m_i (\overrightarrow{r} \times \overrightarrow{v}) \tag{6}$$

$$\frac{2T}{|U|} = 1 \tag{7}$$

where T is kinetic and U is potential energy.

In this paper we investigate influence of softening length on stability of spiral galaxy in isolation. Particulary we investigate changing of density profiles for disk and dark matter halo with changing of softening length under condition that we have equal softening for different components of galaxy. We constructed stable N-body model of spiral galaxy and observe its evolution for several Gyrs in isolation in several runs of simulation with different values of softening length in each simulation.

2. METHOD

Model of spiral galaxy is consisted of disk, bulge and halo, as it is done for M31 in (Sadoun et al. 2013) and (Fardal et al. 2007). For density profiles of disk we used exponential profile in radial direction in the plane of the disk and $sech^2$ profile in orthogonal direction (z-axis). These profiles are given with equations (8) and (9).

$$\Sigma(R) = \Sigma_0 e^{\frac{-R}{R_d}} = \frac{M_d}{2\pi R_d^2} e^{\frac{-R}{R_d}} \tag{8}$$

where Σ_0 is central surface density of the disk and R_d is a scale length.

$$\rho_z \propto sech^2 \frac{z}{z_0} \tag{9}$$

where z_0 is scale length in direction orthogonal to disk.

Composit profile for the disk is given:

$$\rho(R, z) = \frac{\Sigma(R)}{2z_0} sech^2 \frac{z}{z_0} \tag{10}$$

For bulge, we used Hernquist profile (Hernquist, 1993) for mass distribution:

$$\rho_b = \frac{M_b r_b}{2\pi r (r + r_b)^3} \tag{11}$$

where ρ_b is central density of the bulge, and r_b is scale length for bulge.

Dark matter halo is a spherical structure which surrounds disk and bulge. Mass distribution is represented with NFW profile (Navaro, et al. 1996).

$$\rho_h = \frac{\rho_0}{\frac{r}{r_h}(1 + \frac{r}{r_h})^2} \tag{12}$$

where ρ_h is central density of the halo and r_h is scale length. NFW profile in theory ends to infinity, so it is necessary to cut off this function at some point. In many

Table 1: Table I: Parameters for disk, bulge and halo

N_d	108929	M_d	$3.71 \cdot 10^{10} M_\odot$	r_d	6.82 kpc
N_b	96347	M_b	$3.19 \cdot 10^{10} M_\odot$	r_b	1.23 kpc
N_h	261905	M_{halo}	$8.85 \cdot 10^{11} M_\odot$	r_h	122.5 kpc

Table 2: Table II: Different values for softening length in three cases

Model	ϵ_{bulge} (pc)	ϵ_{disk} (pc)	ϵ_{halo} (pc)
I	10	10	10
II	100	100	100
III	10	10	1000

papers (Fardal et al. 2007, Sadoun et al. 2013) it is done on the distance where density drop to $200\rho_{crit}$, where ρ_{crit} is given with:

$$\rho_{crit} = \frac{3H^2}{8\pi G} \tag{13}$$

Truncated density profile for dark matter halo is given with:

$$\rho(r) = \frac{2^{2-\alpha}\sigma_h^2}{4\pi r_s^2} \frac{\rho_0}{(r/r_s)^\alpha (1 + r/r_s)^{3-\alpha}} \frac{1}{2} erfc\left(\frac{r - r_h}{\sqrt{2}\delta r_h}\right) \tag{14}$$

Error function gives truncation after r_h. Some values for the parameters are summarised in Table 1.

We used GalactICs (Widrow, et al. 2005, Widrow, et al. 2008) for constructing initial conditions (distribution of particles at the beginning of the simulation). Content of the output file are masses, coordinates and velocities of the particles. Simulation were running with Gadget2 code (Springel, et al. 2005). Gadget2 code is Three-PM code. Timestep is adjusted to values of softening length:

$$\Delta t = min\left[\Delta t_{max}, \left(\frac{2\eta\epsilon}{|\overrightarrow{a}|}\right)^{1/2}\right] \tag{15}$$

Three different simulation were running, for different values of softening length (Table 2), for 3 Gyrs. This timescale is enough for several rotational periods and we used this model for further investigation of formation of streams during mergers.

3. DISCUSSION

We will discuss density profiles for disks and halos (bulges have drift that is not due to softening length, so it won't be discuss here) and energy conservation for working examples of stable spiral galaxies.

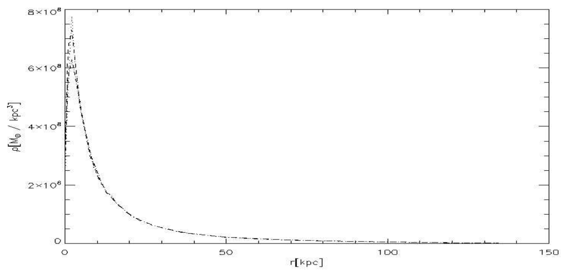

Figure 1: Density profile of dark matter halo for different values of softening length, models: I dot, II dash, III dash-dot.

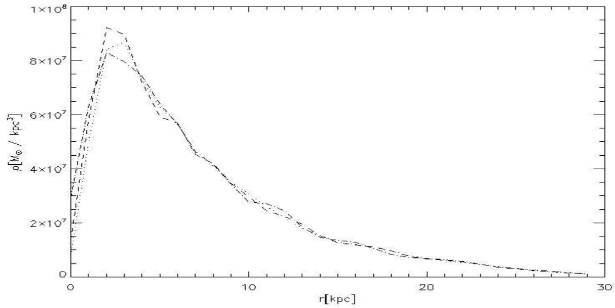

Figure 2: Density profile of disk for different values of softening length, models: I dot, II dash, III dash-dot.

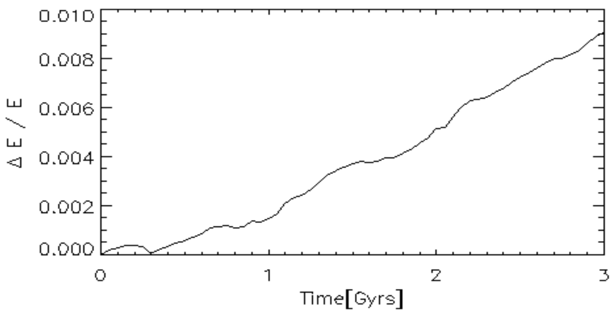

Figure 3: Changing of total energy for model I.

Softening length is a numerical parameter which is necessary for calculating gravitational force at small distances. We have modified law of gravity only at small distances and used classical Newtonian gravity above value of ϵ. From previous graphs we can conclude that softening length has no significant influence on stability of galaxy, in special case energy conservation and density profiles. Changes in density profiles are due to initial conditions and small dynamical instability after several Gyrs. We

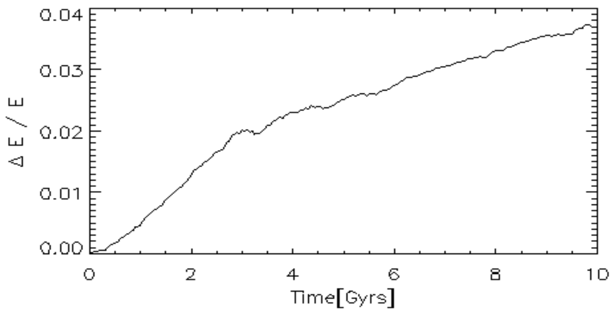

Figure 4: Changing of total energy for model II.

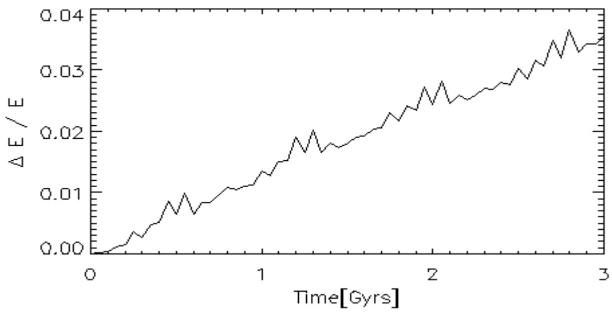

Figure 5: Changing of total energy for model III.

can see a very small differences between cases with equal and non equal values of softening length for different components of galaxy. In the case III, time duration for executing simulation is larger then in cases I and II, because at each step code is checking particle ID in order to choose value for the softening length. This results can be used for this specific type of simulation with Gadget2 code: N-body simulation with number of particles less then 10^6.

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