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IMPACT OF YARKOVSKY EFFECT AND MEAN MOTION RESONANCES ON MAIN BELT ASTEROID'S TRANSPORT

I. MILIĆ ŽITNIK

Astronomical Observatory, Volgina 7, 11000 Belgrade, Serbia
E–mail: ivana@aob.rs

Abstract. We analyzed the effect of two body mean motion resonances with Jupiter on the mobility of an asteroid's semi-major axis induced by the Yarkovsky thermal mechanism. So far, the impact of the resonance on the semi-major axis drift speed has not been studied to that extent neither from that point of view. We established for the first time a functional relation that determine the connection between the average time spent inside the mean motion resonance, the strength of the resonance and the semi-major axis drift speed. Also, we analyzed how the time spent inside the resonance depends on orbital eccentricity and found a precise functional relation that describes dependence of the average time on the eccentricity, on the strength of the resonance and on the semi-major axis drift speed.

1. INTRODUCTION

The dynamics of asteroids is ruled by interaction between gravitational and non-gravitational forces. It is well known that the most important gravitational mechanism is orbital resonance, especially mean motion resonance (MMR) and, on the other side, the most important non-gravitational effect is Yarkovsky. Very important consequence of MMR is slow evolution (Nesvorný & Morbidelli 1998) in some orbital elements of asteroids. During the last two decades, the Yarkovsky effect has been used to study and explain many unsolved problems in dynamics of asteroids (Vokrouhlický et al. 2015). The Yarkovsky thermal effect is a non-gravitational force due to the anisotropic emission of thermal energy by a rotating body around source of heat (Farinella & Vokrouhlický 1999). For a detailed understanding of the Yarkovsky effect role in the evolution of asteroids, an analysis of the interaction between the Yarkovsky-drifting orbits and MMRs would be very useful (Vokrouhlický et al. 2001). This interaction happens when an asteroid due to the modification of its orbital semi-major axis (caused by the Yarkovsky effect) reach the resonance. The resonance induces a periodic oscillations in the asteroid's semi-major axis around its center. The Yarkovsky effect exactly causes the permanent (secular) evolution of the semi-major axis. As a result of their interaction the mean semi-major axis drift speed is modified with respect to the one caused solely by Yarkovsky. This motivated us to study the effect of different MMRs on an asteroid's semi-major axis changing due to the Yarkovsky effect. The most important results of our research were presented in Milić Žitnik & Novaković (2016) when we had derived functional relation among the time that asteroid spent inside MMR, the strength of the resonance and the semi-major axis drift speed. Soon after, we made an extended analysis of interaction between these two effects that improved previous functional relation including orbital eccentricity in calculating time that asteroid spent in MMR (Milić Žitnik 2016).

2. METHODS

Our methods were explained in Milić Žitnik & Novaković (2015, 2016) in details, so we refer the reader to these papers for additional explanations. Here, we will describe our methodology very shortly. In order to study the aforementioned interaction, the orbital motion of test particles across the resonances is numerically simulated. We performed a set of numerical integrations of 72 000 test asteroids in order to examine the semi-major axis drift delay inside the MMR in a public domain integrator *ORBIT9* (Milani & Nobili 1988). The orbital motion of test objects was simulated between 40 and 120 Myr, depending on the resonance's strength and on the Yarkovsky drift speed. All our analysis are obtained using mean proper orbital elements, that are mostly free from the short-periodic perturbations. The mean proper elements are obtained directly from the *ORBIT9* which has an option to perform online digital filtering in order to remove short-periodic oscillations. We decided to test a range of believable values of the Yarkovsky effect for kilometer-sized Main belt asteroids (Vokrouhlický et al. 2015). The orbit of every test asteroid was propagated using ten equidistant values of da/dt from -4×10^{-5} to -2.0×10^{-3} AU/Myr. In order to compare results for different resonances, we need to chose MMRs whose mutual comparison is direct, so we used two-body resonances always with the Jupiter. We analyzed 12 isolated MMRs with Jupiter, because that is the most massive planet in our Solar system (Figure 1).

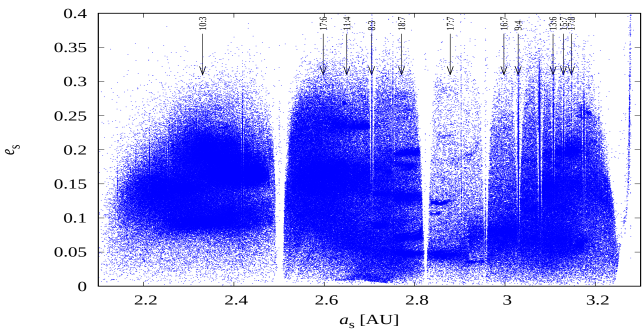

Figure 1: Locations of 11 MMRs with Jupiter shown in the proper semi-major axis vs. the proper eccentricity plane (except 7:3).

Powerful MMRs with Jupiter (2:1, 3:1 or 5:2) are not appropriate for our methods. These resonances quickly throw out asteroids locked inside them (Vokrouhlický et al. 2001) due to close approach with planets. Also, due to their large width they overlap weaker nearby resonances, thus do not satisfy the condition to be isolated (Figure 2). Also, in resonances 3:1, 5:2 and 7:3 exist overlapping of secular resonances that causes increasing in eccentricity to 1 (Moons & Morbidelli 1995).

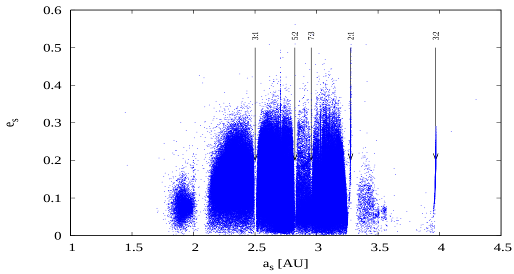

Figure 2: Some of the strongest two-body resonances with Jupiter shown in the proper semi- major axis vs. the proper eccentricity plane, that we excluded from our results.

To estimate strength of resonances, we applied a numerical method proposed by Gallardo (2006). Strength of our MMRs spreads over a range of even seven orders of magnitudes. We used our numerical method to estimate resonance width because of nature of our work, in order to measure time spent inside a resonance. The initial positions of our test asteroids resembled a shape of a MMR in the mean semi-major axis vs. the mean eccentricity plane. In Figure 3 are presented borders of resonance 9:4 as an example of our determination of borders.

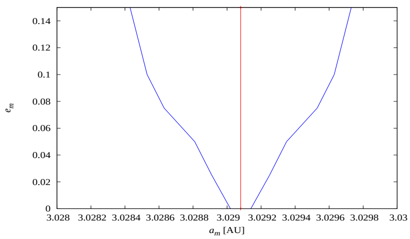

Figure 3: Inner and outer borders of resonance 9:4 in the mean semi-major axis vs. the mean eccentricity plane (broken lines). The center of this resonance is located 3.0291 AU from the Sun (vertical line).

In order to measure the time spent inside a resonance it was necessary to determine the instants of entering, t_1, and exiting, t_2, from the resonance (Figure 4). Further, if Δt and $\Delta \mathrm{a}$ are defined as $\Delta t = t_2 - t_1$ and $\Delta \mathrm{a} = \mathrm{a}_2 - \mathrm{a}_1$, where a_1 and a_2 are semi-major axes at moments t_1 and t_2 respectively, then the time interval dtr used in our analysis is defined with (Milić Žitnik & Novaković 2016):

$$\mathrm{d}tr = \Delta t - \frac{\Delta \mathrm{a}}{\mathrm{da}/\mathrm{d}t}. \tag{1}$$

We used dtr instead of Δt because dtr is not sensitive to the criteria for resonance entering and exiting. It follows from Equation (1): when Δt increases than Δa is increasing, so dtr measures time for which asteroid crossed strictly one whole resonance and also measures speed up or slow down of that asteroid. It is very important to say that in this way, we bypassed problem with determination instant t_2, that exists only in some cases, and as a consequence is not precisely enough determination of time interval Δt.

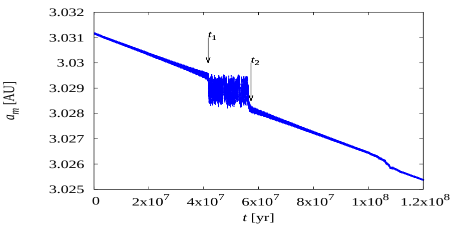

Figure 4: An example of behaviour of test asteroid with the slowest Yarkovsky drift speed da/dt=-4×10^{-5} AU/Myr, that entering in resonance 9:4 at the instant t_1=41 602 600 years and exiting at the instant t_2=57 253 000 years.

3. RESULTS

Now, we present the results of our numerical investigation about estimation effect of the resonances on the semi-major axis drift. We were considered only asteroids that crossed MMRs. Our results had discovered that exists function between the average time $\langle \mathrm{d}tr \rangle$, the strength of the resonance SR and the semi-major axis drift speed da/dt. For 9 (out of 10) values of da/dt analyzed, we found that $\langle \mathrm{d}tr \rangle$ increases when SR is increasing. For the slowest drift speed an opposite trend exists and all values $\langle \mathrm{d}tr \rangle$ are negative (Figure 5). This result might indicate that below some limiting value of da/dt objects typically rapidly jump across the resonance, so we excluded this value from all further analysis presented here. However, behaviour of asteroids with small Yarkovsky drift speeds will be theme of our future work. The same trend exists for the strongest resonance 7:3 (Figure 5), that we excluded also from the results presented here. All asteroids that crossed 7:3 have negative values of the average time $\langle \mathrm{d}tr \rangle$. This is the very interesting result that should be further investigated in the future.

In Figure 6 we used a logarithmic scale to show the correlation between $\langle \mathrm{d}tr \rangle$ and SR (left panel) and between $\langle \mathrm{d}tr \rangle$ and da/dt (right panel). It is obvious that $\langle \mathrm{d}tr \rangle$ time increases while resonance strength SR is increasing. In the log-log plane, this dependence is almost linear, displaying an exponential relation between $\langle \mathrm{d}tr \rangle$ and SR. Some deviation from this trend might exists for weaker resonances (left panel in Figure 6), because of poor signal-to-noise ratio in calculation of $\langle \mathrm{d}tr \rangle$. Similar linear dependence $\langle \mathrm{d}tr \rangle$ shows with changes in da/dt, but in this case with an opposite trend (right panel in Figure 6) and an exponential relation between $\langle \mathrm{d}tr \rangle$ and da/dt was again suggested.

According to Milić Žitnik & Novaković (2016), there is an unique functional relation between $\langle \mathrm{d}tr \rangle$, SR and da/dt, that follows from the above described results:

$$\langle \mathrm{d}tr \rangle = c_1\ (SR)^{\beta}\ (\frac{\mathrm{da}}{\mathrm{d}t})^{\gamma}. \tag{2}$$

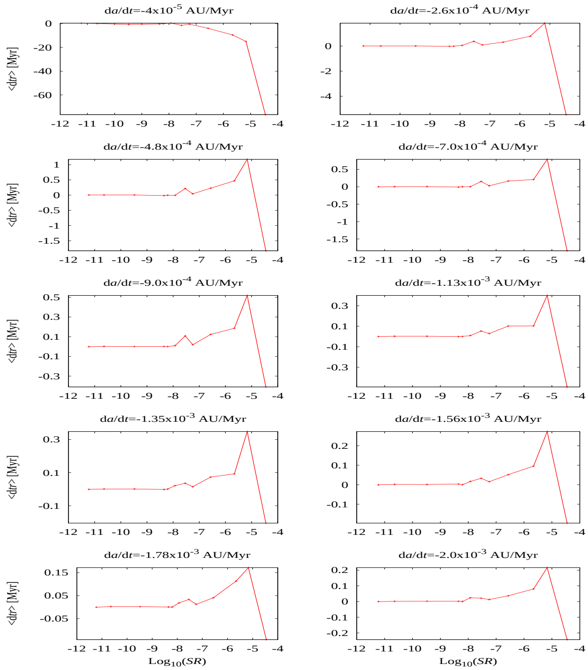

Figure 5: Changes of average time $\langle \mathrm{d}tr \rangle$ in MMRs as a function of $\mathrm{Log}_{10}SR$. Here are shown asteroids that crossed 12 MMRs for every of 10 different values of Yarkovsky drift speed.

These unknown parameters (c_1, β, γ) could be found by numerically fitting data. We found that it is the most convenient to apply the method of least squares fitting using Equation (3) to the data shown in Figure 6:

$$\log_{10}(\langle \mathrm{d}tr \rangle) = \beta \log_{10}(SR) + \gamma \log_{10}(\frac{\mathrm{da}}{\mathrm{d}t}) + c_2. \tag{3}$$

We found fitting parameters that describe the best relation between $\langle \mathrm{d}tr \rangle$, SR and $\mathrm{da}/\mathrm{d}t$ are: $\beta = 0.44 \pm 0.03$, $\gamma = -1.09 \pm 0.20$ and $c_2 = 4.35 \pm 0.66$. Data presented in Figure 6 indicate that the trend of $\langle \mathrm{d}tr \rangle$ might change for smaller values of SR. In order to check it, we repeated the same fitting procedure to the data that exclude the five weakest resonances and the values of the parameters are obtained: $\beta = 0.47 \pm 0.04$, $\gamma = -0.97 \pm 0.15$, $c_2 = 5.11 \pm 0.54$. Conclusion is that the two sets of parameters are statistically the same and we decided to use fitting parameters for 11 resonances in Equation (3). Equation (3) is valid only for eccentricities around 0.1 (approximately for $0.08 \leq e \leq 0.12$), for which SR was estimated. It is well known that SR depends on eccentricity (Gallardo 2006, Lykawka & Mukai 2007). This problem we have bypassed by involving one more parameter in Equation (3) that depends on eccentricity. After that, we calculated SR for equidistant values of eccentricity $0.025 \leq e \leq 0.4$ with step of 0.025. We took these boundaries for

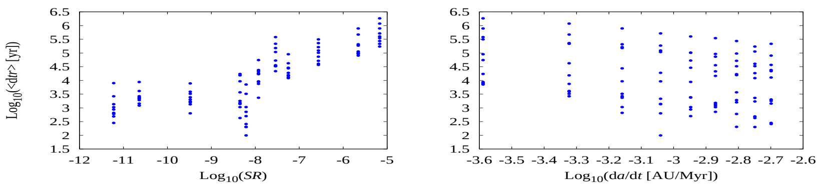

Figure 6: Dependence of average time delay caused by the resonance $\langle \mathrm{d}tr \rangle$ on the 11 resonance strength SR (left panel) and on the 9 semi-major axis drift speed $\mathrm{da}/\mathrm{d}t$ (right panel).

eccentricity, because most of the asteroids have values of eccentricity in this range. Than, we calculated unknown fitting parameters for these new values of e and SR. Unknown coefficient β defines the relation between e and SR. We got that β depends on eccentricity linearly, $\beta = ae + b$. The parameters a and b could be found by the least-squares method of fitting the obtained data as shown in left panel in Figure 7. We found that their values are $a = 2.06 \pm 0.02$ and $b = 0.24 \pm 0.01$. The parameter γ has the same value for all eccentricity ≈ 1.09 (see Table 3 in Milić Žitnik 2016) because it depends only on the Yarkovsky drift speed. Values of c_2 increases with increasing eccentricity except for $e = 0.025$ (right panel in Figure 7). This function has some oscillations around linear trend. So, we did not look for precise functional relation between e and c_2 and decided to use values of c_2 for appropriate interval of e from Table 3 (see Milić Žitnik 2016). At the end, we got general equation that includes asteroid's eccentricity:

$$\log_{10}(\langle \mathrm{d}tr \rangle) = (2.06e + 0.24)\log_{10}(SR) - 1.09\log_{10}(\frac{\mathrm{da}}{\mathrm{d}t}) + c_2. \tag{4}$$

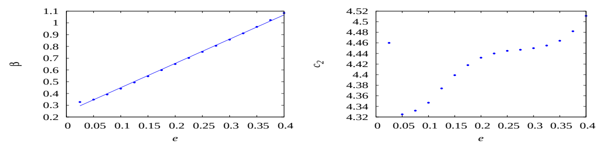

Figure 7: Dependence between e and β (left panel) and between e and c_2 (right panel) for resonance's strength calculated for $0.025 \leq e \leq 0.4$.

In order to understand accuracy of Equation (4), also in order to define its limitations, we calculated standard errors of $\langle \mathrm{d}tr \rangle$ from this equation. We got standard error $\sigma(\langle \mathrm{d}tr \rangle)$ from the total differential of the first order of Equation (4). In this way, we had:

$$\sigma(\langle \mathrm{d}tr \rangle) = \langle \mathrm{d}tr \rangle \times \ln(10) \times [\mathrm{d}\beta \log_{10}(SR) + \mathrm{d}\gamma \log_{10}(\mathrm{da}/\mathrm{d}t) + \mathrm{d}c_2]. \tag{5}$$

With Equation (5) we calculated 3σ standard errors for $\langle \mathrm{d}tr \rangle$ (Figure 8). We concluded that acceptable disagreement between the results obtained by the Equation

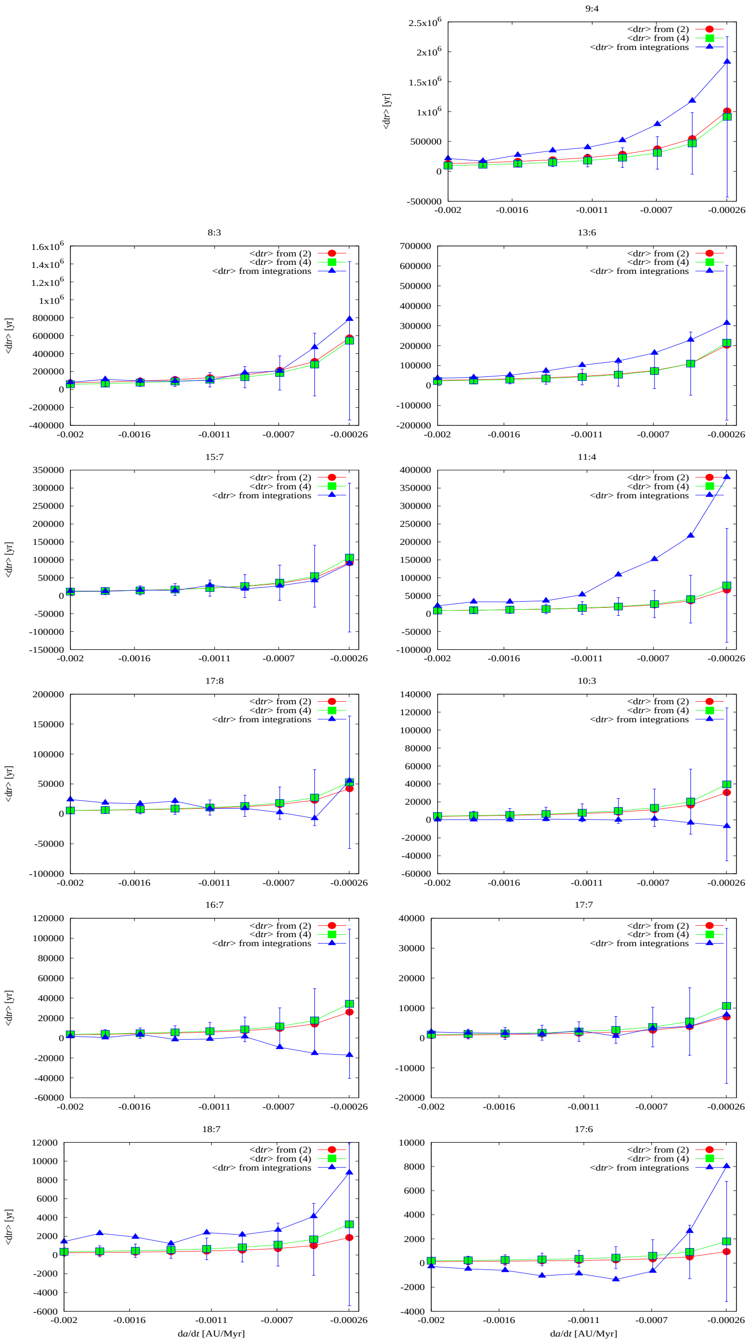

Figure 8: Values $\langle \mathrm{d}tr \rangle$ for 9 the largest Yarkovsky drift speeds calculated with Equation (2) and Equation (4), also from numerical integrations. Results from Equation (4) are presented with 3σ interval of standard error calculated using Equation (5).

(4) (with $e = 0.1$), Equation (2) (with average rounded values: $\beta = 0.5$, $\gamma = -1$ and $c_1 = 10^{-1}$) and numerical integrations, can be explained with poor signal-to-noise ratio in determination of single values dtr in weak MMRs (see Figure 6) and because these equations represent approximation of $\langle \mathrm{d}tr \rangle$.

Based on the previous analysis of results (Figure 8), we came to the conclusion that Equation (4) is possible to use for asteroids in MMR with strength $[6 \times 10^{-12}, 6.7 \times 10^{-6}]$ and with Yarkovsky drift speed $[2.6 \times 10^{-4}, 2 \times 10^{-3}]$ AU/Myr.

4. CONCLUSIONS AND FUTURE WORK

This paper briefly review results presented in Milić Žitnik & Novaković (2016) and Milić Žitnik (2016) about the average time spent inside a resonance, the strength of the resonance, eccentricity, the semi-major axis drift speed. Now, it would be easy to calculate the average time that an object spent inside a MMR with given the resonance's strength, the Yarkovsky drift speed and an asteroid's eccentricity. These equations can be applied only to asteroids that entered and exited from MMRs. Work on the remaining issues continues. For instance, we plan to examine the possibility of finding a functional relation between dtr and da/dt, SR, e, i.

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