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TESTING EXTENDED THEORY OF GRAVITY BY Sgr A*

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Abstract. Here we analyze an Extended Gravity Theory model, in which there is non-minimal coupling between the geometry and a scalar field. We derived a particular theory among the class of scalar-tensor theories of gravity, and then tested it by studying dynamics of S2 star around supermassive black hole at the Galactic Center. We also discuss the Newtonian limit of this class of scalar-tensor theories of gravity, as well as its parameters. To constrain these parameters, we compare the observed orbit of S2 star with our simulated orbit which we obtained theoretically with the derived potential.

1. INTRODUCTION

In this study, we consider possible signatures for a Scalar Tensor (ST) theory within the Galactic Central Parsec, not tested at these scales yet. This theory of gravity contains two arbitrary functions of the scalar field: $F(\phi)$, which underlines a non-minimal coupling between the scalar field and the geometry, and $V(\phi)$, which implies a self-interaction of the field. The specific form of these functions is determined by the parameters (m, n, ξ, λ). In order to constrain these parameters observationally, we derived the modified gravitational potential of the central object in the weak field limit to simulate orbits of S2 star, and then we compared the results with the set of S2 star orbit observations obtained by the Very Large Telescope (VLT). This is a continuation of our previous studies where we considered different extended gravities models: $f(R)$ gravity (Borka et al. 2012, Zakharov et al. 2014, Zakharov et al. 2016), $f(R, \phi)$ gravity (Borka et al. 2013, Capozziello et al. 2014, Borka et al. 2016).

Sagittarius A (or Sgr A) is a complex radio source that consists of three components, which overlap: (1) Sgr A East (the supernova remnant), (2) Sgr A West (the spiral structure), (3) Sgr A* (a very bright compact radio source at the center of the spiral) (Ghez et al. 2000, Ghez et al. 2008, Genzel et al. 2010). Sgr A* is very compact and motionless source, and its location coincides with the dynamical center of the Galaxy. The massive black hole Sgr A* at the Galactic center (GC) is surrounded by a cluster of stars orbiting around it: S-star cluster. Light from these stars is bent by the gravitational field of the black hole. S-stars are orbiting with large velocities ($v > 1000$ km/s), and have very eccentric orbits around central supermassive black hole (SMBH) at GC. S2 star is one of the brightest members of the S-star cluster. It has about 15 Solar masses and seven times its diameter, with orbital period of approximately 15.8 yr (Genzel et al. 2010, Gillessen et al. 2012). There are a few indications that S2 star orbit really deviates from the Newtonian case. Some recent studies (Gillessen et al. 2009a, Gillessen et al. 2009b, Meyer et al. 2012, Gillessen et al. 2017, Boehle et al. 2016) provide evidence that the orbit of S2 star is not closing.

2. THEORY

Extended Theories of Gravity (ETGs) are physical theories that attempt to describe the phenomena of gravitation in competition to Einstein's theory of general relativity, by preserving the undoubtedly positive results of Einstein's theory. Instead of introducing Dark Matter (DM), a hypothetical type of matter, some theories, which modify the laws of gravity, could explain in a very natural fashion several astrophysical and cosmological observations: for short distances, Solar system, spiral galaxies, galaxy clusters and cosmology. See reviews in: Capozziello & Faraoni 2010, Capozziello & De Laurentis 2011, Nojiri & Odintsov 2011, Capozziello & De Laurentis 2012, Clifton et al. 2012.

The ST theory of gravity contains the metric tensor $g_{\mu\nu}$ and a fundamental scalar field ϕ (Capozziello et al. 1996). The coupling $F(\phi)$ and the potential $V(\phi)$ are generic functions of the scalar field ϕ. In this study we take the action of the form:

$$S = S_M + \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[F(\phi)R + \frac{3}{2\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)]. \tag{1}$$

We choose a specific form for $F(\phi) = \xi\phi^m$, $V(\phi) = \lambda\phi^n$, where ξ is a coupling constant, λ gives the self-interaction potential strength, m and n are arbitrary parameters.

We obtained solutions for few different cases $h_{00} = 0.5\Phi$, where Φ is Newtonian potential.

In case of $n \neq 0$ and $n \neq 2m$, we obtain:

$$h_{00} \simeq \frac{\kappa^2}{4\pi\xi\varphi_0^m}\frac{M}{r} - \frac{\lambda}{2\xi}\varphi_0^{n-m}r^2 - \frac{\kappa^2 m^2 M}{3(1 - m^2\varphi_0^{m-1}\xi)}\frac{e^{-pr}}{4\pi r}, \tag{2}$$

where κ is a coupling constant and M is central mass.

In case of $n = 2m$, we obtain:

$$h_{00} \simeq \frac{\kappa^2 M}{4\pi r}[\frac{3 - 3m^2\varphi_0^{m-1}\xi - m^2\xi\varphi_0^m}{3\xi\varphi_0^m(1 - m^2\varphi_0^{m-1}\xi)}] - \frac{\lambda\varphi_0^m}{2\xi}r^2. \tag{3}$$

In case of $n = 1$, we obtain:

$$h_{00} \simeq \frac{\kappa^2 M}{4\pi r}\left[\frac{3 - 3m^2\varphi_0^{m-1}\xi - m^2\xi\varphi_0^m}{3\xi\varphi_0^m(1 - m^2\varphi_0^{m-1}\xi)}\right] - \frac{\lambda\varphi_0^{1-m}}{2\xi}r^2. \tag{4}$$

3. RESULTS AND DISCUSSION

We derived the modified gravitational potential of the central object in the weak field limit to simulate orbits of S2 star, and then we compared the results with the set of S2 star observations obtained by VLT. The ST gravitation potential in the weak field limit can be written in the form:

$$U_{ST} = \frac{\widetilde{G}}{\xi\varphi_0^m}\frac{M}{r} - \frac{\lambda}{4\xi}\varphi_0^{n-m}r^2 - \frac{\widetilde{G}m^2 M}{3(1 - m^2\varphi_0^{m-1}\xi)}\frac{e^{-pr}}{r}, \tag{5}$$

where p is function of the ST gravity parameters ξ, λ, m and n:

$$p = \sqrt{\frac{\lambda n\varphi_0^{n-1}(2m - \lambda n)}{3(m^2\xi\varphi_0^{m-1} - 1)}}, \tag{6}$$

and $\widetilde{G}$ is related with a gravitation constant G_N through relation:

$$\tilde{G} = -\left[\frac{3(1 - m^2\varphi_0^{m-1}\xi)\xi\varphi_0^m}{3 - \xi(3m^2\varphi_0^{m-1} + m^2\varphi_0^m)}\right]G_N. \tag{7}$$

In order to constrain parameters λ, ξ, m and n observationally, we performed two-body simulations of S2 star orbit in ST gravity potential by numerical integration of the following two differential equations of motion:

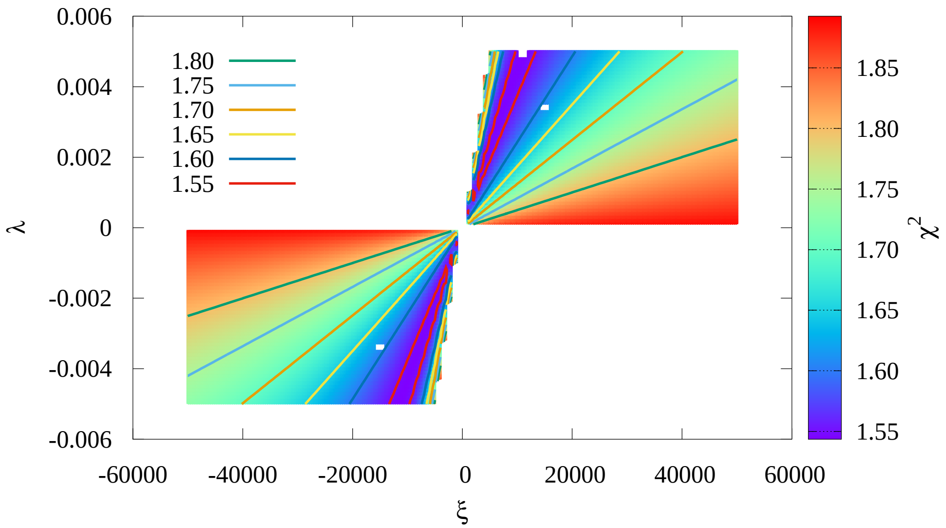
$$\dot{\overrightarrow{r}} = \overrightarrow{v}, \qquad \mu\ddot{\overrightarrow{r}} = -\overrightarrow{\bigtriangledown}U_{ST}(\overrightarrow{r}), \tag{8}$$

where $\mu = M_{BH} \cdot m_S/(M_{BH} + m_S)$ is the reduced mass in the two-body problem (m_{BH} being the mass of the central black hole and m_S the mass of the S2 star).

The positions of the S2 star along its true orbit are calculated at the observed epochs using two-body simulations in the ST gravity potential, assuming that distance to the S2 star is $d = 8.3$ kpc and mass of central black hole $M_{BH} = 4.3\ \times 10^6 M_S$ (Gillessen et al. 2009a).

All the above two-body simulations in ST gravity potential resulted with the true orbits of S2-like stars, i.e. the simulated positions of S2-like stars. In order to compare them with observed positions, the first step is to project them to the observer's sky plane, i.e. to calculate the corresponding apparent orbits (x, y).

We chose some values for ϕ_0, m and n, with the following conditions: ϕ_0 is positive real number close to 1, m and n are integer numbers, for which: $n \neq 2m$ and $n \neq 0$. The initial values for true position (x_0, y_0) and orbital velocity $(\dot{x}_0, \dot{y}_0)$ of S2 star at the epoch of the first observation are specified and the positions (x_i, y_i) and velocities $(\dot{x}_i, \dot{y}_i)$ of S2 star along its true orbit are calculated for all observed epochs by numerical integration of equations of motion in the ST gravity potential. The observed



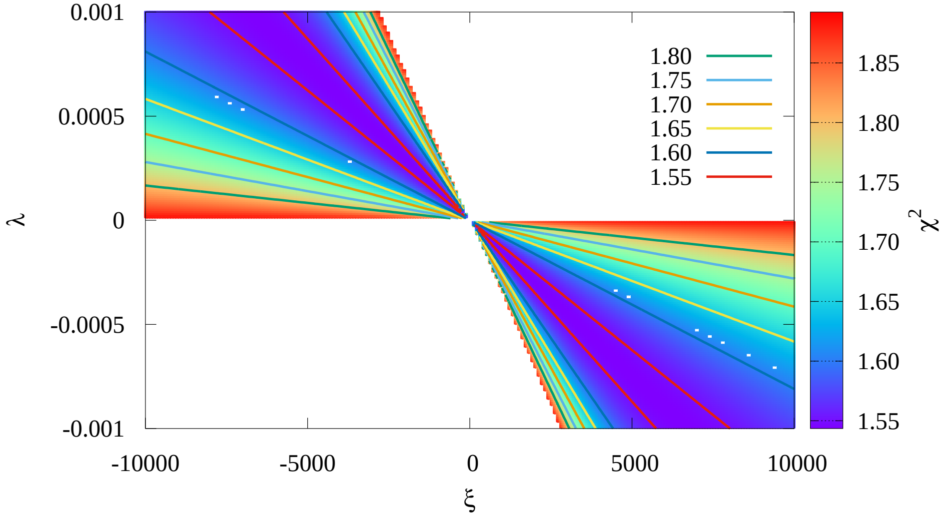


Figure 1: *Top*: The map of the reduced χ^2 over the parameter space $\{\xi, \lambda\}$ of ST gravity in case of NTT/VLT observations of S2 star which give at least the same (χ^2 =1.89) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. Figure represents case for $(m, n) = (2, 2)$. *Bottom*: The case for $(m, n) = (1, 3)$. A few contours are presented for specific values of reduced χ^2 given in the upper left part of the top figure, and in the upper right part of the bottom figure.

values we denote with (x_i^o, y_i^o), the calculated with (x_i^c, y_i^c), and the variances are σ_{xi}, σ_{yi}. The reduced χ^2 of the fit is estimated according to the following expression:

$$\chi^2 = \frac{1}{2N - \nu} \sum_{i=1}^{N} \left[\left(\frac{x_i^o - x_i^c}{\sigma_{xi}} \right)^2 + \left(\frac{y_i^o - y_i^c}{\sigma_{yi}} \right)^2 \right]. \qquad (9)$$

Table 1: χ^2 for different values of m, n, ξ and λ gravity parameters (for all these calculations we used $\phi_0 = 1$).

χ^2	m	n	$\{\xi, \lambda\}$
1.5434359	1	1	-11000, -0.0049
1.5434454	1	3	33000, -0.0049
1.5434454	2	1	-9000, -0.0050
1.5434363	2	2	11000, 0.0049
1.5434474	2	3	-1000, -0.0006
1.5434336	3	1	1000, 0.0008
1.5434383	3	2	1000, 0.0005
1.5434454	3	3	33000, -0.0049
1.5434317	4	1	4000, 0.0041
1.5434478	4	2	-1000, -0.0006
1.5434454	4	3	33000, -0.0049

We vary the parameters ξ and λ over some intervals, and search for those solutions which for the simulated orbits in ST gravity give at least the same ($\chi^2 = 1.89$) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. We repeat the procedure for different combinations of parameters m and n (see some examples in Table 1). For more detailed description about fitting procedure and numerics see in papers (Moré et al. 1980, Borka et al. 2013).

The map of the reduced χ^2 over the parameter space $\{\xi, \lambda\}$ for $(m, n) = (2, 2)$ is given in Fig. 1 (top). This map shows an area of all the parameter values $\{\xi, \lambda\}$ for which the simulated orbits of S2 star give at least the same or better fits than the Keplerian orbits. The map of the reduced χ^2 over the parameter space $\{\xi, \lambda\}$, but for $(m, n) = (1, 3)$, is given in Fig. 1 (bottom). According to Fig. 1 we can notice that different choice of parameters m and n position of the region of allowed values of the parameters $\{\xi, \lambda\}$.

4. CONCLUSIONS

In this paper, orbit of S2 star has been investigated in the framework of the ST gravity potentials. Using the observed positions of S2 star around the GC we constrained the parameters of these gravity potentials.

We derived a particular theory among the class of ST theories of gravity, and then tested it by studying dynamics of S2 star around SMBH at GC.

We also discuss the Newtonian limit of this class of scalar-tensor theories of gravity, as well as its parameters.

We constrained the parameters of ST gravitational potential.

To constrain these parameters, we compare the observed orbit of S2 star with our simulated orbit which we obtained theoretically with the derived potential.

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