

DYNAMICS AND KINEMATICS OF CELESTIAL BODIES AND SYSTEMS

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Abstract. The most important results achieved and published by the participants in the project "Dynamics and Kinematics of Celestial Bodies and Systems" from 2011 to 2017 are presented. The results are numerous and significant, they cover the following four fields: dynamics of Solar-System minor bodies, observations of double and multiple stars and analysis of orbital motion, kinematical properties of stars from the solar neighborhood and photometric observations of WEBT objects and morphology of quasars.

1. INTRODUCTION

Project No. 176011 "Dynamics and Kinematics of Celestial Bodies and Systems" is financed by the Ministry of Education, Science and Technological Development of the Republic of Serbia. In its realisation the following researchers have taken part: Dr. Rade Pavlović, Dr. Zorica Cvetković, Dr. Goran Damljanović, M. D. Jovanović (joined the project in April 2015), Dr. Zoran Knežević (first leader of this project, retired in August 2016), Ivana Milić Žitnik, Dr. Slobodan Ninković (retired in August 2017), Dr. Milan Stojanović and Dr. Nataša Todorović from the Astronomical Observatory in Belgrade, Prof. Dr. Mike Kuzmanoski (retired in October 2011), Dr. Bojan Novaković and Dr. Dušan Marčeta (joined the project in January 2017) from the Faculty of Mathematics of the Belgrade University. During the project realisation three PhD theses were defended: B. Novaković, N. Todorović and M. Stojanović, whereas one thesis is very near its completion (I. Milić Žitnik). Between 2011 and 2016 the project participants published 47 papers in prestigious international journals. The project research comprises the following topics: dynamics of small solar system bodies, observations of double and multiple stars and analysis of orbital motion, statistical investigation of local kinematics of our Galaxy in solar neighbourhood and astrometry and photometry of quasar and Whole Earth Blazar Telescope (WEBT) objects.

2. DYNAMICS OF SMALL SOLAR SYSTEM BODIES

Most of our research of this topic in the period of the project realisation has been devoted to the various problems of asteroid dynamics and to the identification, interpretation, age determination and study of dynamical and physical properties of individual asteroid families.

The discovery of new small Solar System bodies in the images taken by modern wide-field surveys has been made more efficient by introducing an algorithm to first identify observations belonging to known Solar System objects and remove them for the observation files (Knežević and Milani 2012, Milani et al. 2012). This is an attribution problem that occurs when a well constrained least squares orbit already exists for the object and the new data are sparse. The new algorithm introduces quality metrics to control biases in the astrometric residuals which arise from the stellar catalogs used in the reduction of asteroid observations. It has been shown that a simple debiasing with regional catalog biases removed significantly improves the results. The attribution algorithm was tested using data from the PanSTARRS-1 survey that relied on the 2MASS star catalog for the astrometric reduction. Small but statistically significant biases of up to 0.1 arcsec have been found in the data. The false attribution rate was $< 1/1000$, while the attribution efficiency is rated as consistent with 100 %.

To cope with the problem of rapidly increasing number of asteroids with accurate orbits in asteroid family classification, a new approach combining the Hierarchical Clustering Method (HCM) with a method to add new members to existing families has been proposed (Milani et al. 2014). By first segmenting the problem and selecting from the catalog a smaller number of large asteroids, a number of core families is identified; to these the next layer of smaller objects is attributed. Next, all the already identified family members are removed from the catalog, and the HCM is applied to the rest. This provides satellite families of the previously found cores, as well as the new independent families consisting mainly of small asteroids. These two cases are discriminated by another step of attribution of new members and by merging intersecting families. The resulting initial classification contained 128 families and 87,095 members. The membership is then increased automatically with each update of the proper elements catalog; changes in the list of families occur more seldom, while only once in a while it is necessary to repeat the whole classification procedure from the scratch.

The results from the classification are then analyzed, using information on asteroid physical properties, in particular albedos and color indexes. This allowed us to solve some difficult cases of families overlapping in the proper elements space, and to obtain information on the geometrical properties of the family forming impact events. The families formed by one or more cratering events turned out to be more numerous than previously believed, and some examples of cratering families (Mas-salia, Vesta, Eunomia) were analyzed, which show internal structures interpreted as multiple collisions.

In the subsequent papers (Knežević et al. 2014, Milani et al. 2016), the operation of the automated attribution of newly numbered asteroids has been demonstrated, and the inclusion of the multiopposition asteroids into the classification procedure described. Enough evidence was found to perform 9 mergers of the previously independent families. Also, using by an improved method of estimation of the expected

family growth in the less populous regions (e.g. at high inclination) reliable decision was possible on rejection of one tiny group as a probable statistical fluke. Thus the current list is reduced to 115 families.

Asteroid family ages (Spoto et al. 2015) have been determined by a new rigorous method, consisting of a least squares fit of the two sides of a V-shape, formed by a size dependent non gravitational Yarkovsky drift, in the proper semimajor axis, inverse diameter plane. An advanced error model for the uncertainties of asteroid diameters, an iterative outlier rejection scheme and quality control were also applied, and the best available Yarkovsky measurement was used to estimate a calibration of the Yarkovsky effect for each family. The results are presented separately for the families originated in fragmentation or cratering events, for the young, compact families and for the truncated, one-sided families. For all the computed ages the corresponding uncertainties are provided. The ages of several families have been estimated for the first time, in other cases the accuracy of the estimate has been improved.

The results were as follows: ages for old families were successfully determined, some useful results were obtained for young and ancient families, however, a little evidence was found for primordial ones. In two cases two separate dynamical families were found to form a single V-shape with similar slopes, thus indicating a single collisional event, while in three cases dynamical families were shown to be formed in multiple collisional events: for these different slopes for the two sides of the V-shape resulted in distinct ages. Two families exhibit a conspicuous subfamily, such that it was possible to determine the slope of a distinct V-shape, thus the age of the secondary collision. The family ages were derived with a uniform methodology, thus they could be compared among different families, providing a first example of collisional chronology of the asteroid main belt.

Additional 10 collisional ages were estimated for 9 families for which for different reasons the previous attempts failed (Milani et al. 2017). In general, these are difficult cases that required dedicated effort, such as a new family classifications for asteroids in mean motion resonances, in particular the 1/1 and 2/1 with Jupiter, as well as a revision of the classification inside the 3/2 resonance. Asteroid families affected by secular resonances and those infested by interlopers were also considered. Overall, 53 ages for a total of 49 families were computed.

Two papers (Paolicchi & Knežević 2016, Paolicchi et al. 2017) were dedicated to revealing the footprints of the YORP non gravitational effect in asteroid families, which manifest itself as a depletion of objects ("YORP¹ eye") in the central part of the family, visible in the absolute magnitude vs. semimajor axis V-plot. Not all the V-plots exhibit the expected depletion, thus the concept of the YORP eye and a general method of analysis have been introduced to tackle the problem. It is shown that the effect may sometimes be located in the low H tail, and thus difficult to detect. Moreover, it may be hindered by several anomalous physical properties of the family (asymmetry, cratering origin, multiple collisions history and so on). With a new method of analysis, the footprints of the effect are clearly identified for most of the analyzed families, obtaining also an independent estimate of their ages. A very good agreement was obtained between these ages and those estimated on the basis of the Yarkovsky slopes of the V-plot – a result which supports both methods and the underlying physics.

¹YarkovskyO'KeefeRadzievskiiPaddack effect

For the strongly asymmetric families, for which it is difficult to assess the existence and location of a YORP eye, a special "mirroring tool" technique has been developed and applied in combination with the age determination method. The results show that the mirroring tool can, in some cases, be satisfactorily effective. A better calibration against the results obtained by means of other techniques proved to be necessary, including the age determination techniques based on the analysis of the size dependent Yarkovsky-driven family spreading in semi-major axis.

In Novaković *et al.* (2015) it was reported on the significant role of a so far overlooked dynamical aspect, namely, a secular resonance between the dwarf planet Ceres and other asteroids (Figure 1). They demonstrate that this type of secular resonance can be the dominant dynamical factor in certain regions of the main asteroid belt. Specifically, a dynamical analysis of the asteroids belonging to the (1726) Hoffmeister family was performed. To identify what kind of dynamical mechanisms are actually at work in this part of the main asteroid belt, i.e. to isolate the main perturber(s), they study the evolution of this family in time. The study is accomplished using numerical integrations of test particles performed within different dynamical models. The obtained results reveal that the post-impact evolution of the Hoffmeister asteroid family is a direct consequence of the nodal secular resonance with Ceres. This leads us to the conclusion that similar effects must exist in other parts of the asteroid belt. In this respect, the obtained results shed light on an important and entirely new aspect of the long-term dynamics of small bodies. Ceres' fingerprint in asteroid dynamics, expressed through the discovered secular resonance effect, completely changes our understanding of the way in which perturbations by Ceres-like objects affect the orbits of nearby bodies.

More detailed consideration of the role of the dwarf planet Ceres on the secular dynamics of the asteroid main belt could be found in Novaković *et al.* (2016). Specifically, they examine the post impact evolution of asteroid families due to the interaction of their members with the linear nodal secular resonance with Ceres. First, they find the location of this resonance and identify which asteroid

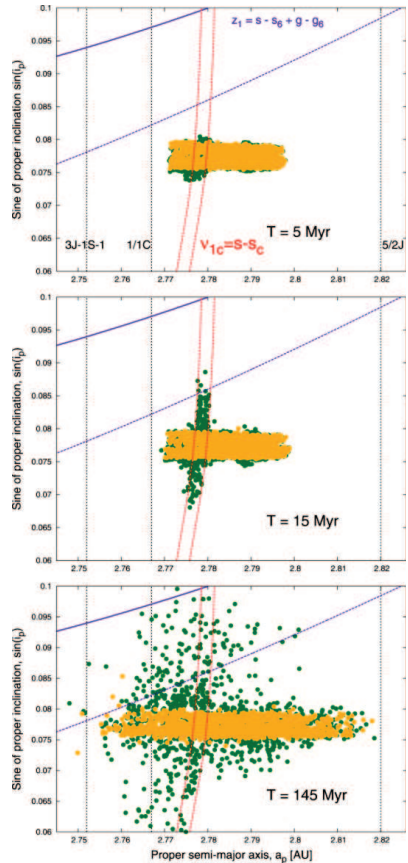


Figure 1: Evolution of the Hoffmeister family in the space of proper orbital elements. The three panels show the distribution of the test particles after 5, 15, and 145 Myr of the evolution, from top to bottom, respectively. The orange dots represent the evolution of the particles within the dynamical model that includes the four giant planets, from Jupiter to Neptune, and accounts for the Yarkovsky effect. The dark green dots show the evolution when Ceres is added to the previous model.

families are crossed by its path. Next, they summarize their results for three asteroid families, namely (1726) Hoffmeister, (1128) Astrid and (1521) Seinajoki which have irregular distributions of their members in the space of the proper elements, indicative of the resonance effect. They confirm this by performing a set of numerical simulations, showcasing that the perturbing action of Ceres through its linear nodal secular resonance is essential to reproduce the actual shape of the families.

In Milić Žitnik and Novaković (2015), two specific characteristics of the Phocaea region were studied. The first was the presence of the secular resonances involving the inner planets. The results show that some of these are present, and relevant for the dynamics of asteroids in this region. The most important seems to be the $s-s_4+g_3-g_7$ resonance. The second one was the role of the $7/2$ mean motion resonance with Jupiter as a border of the region in terms of semimajor axis. In particular, they check whether or not this resonance could be crossed under the combined influence of gravitational and non-gravitational forces. The obtained results show that a significant fraction of test particles successfully transit across the resonance, without being removed from the region. Moreover, they found that most of the asteroids below a few hundred meters in diameter should be able to cross the $7/2$ resonance. This means, despite being relatively effective in pumping up asteroid eccentricities in this region, that this resonance is not an absolute dynamical boundary. More details can be found in Milić Žitnik (2018).

Using sophisticated numerical methods, very precise stability maps for some parts of the Solar System could be produced. In this methodology, using the so-called Fast Lyapunov Indicator (FLI), a quick and efficient chaos detection tool was introduced in the late 90s. The first application FLI to the Solar System using this method was performed in Todorović and Novaković (2015) where they mapped the region of the asteroid family, Pallas. They identified not only the most important resonances in the domain, but also many weak resonances such as the $1-1$ mean motion resonance with the dwarf planet Ceres or the secular resonance nu_6 whose presence in the Pallas region was not detected in the earlier studies. Also, they have tested how the choice of initial orbital angles affects the computed maps.

The same methodology was applied to the $5:2$ mean motion resonance with Jupiter in Todorović (2017). The computation time of the maps was significantly shortened, which enabled the detection of chaotic structures inside the resonance. We have observed the so-called normally hyperbolic invariant manifolds, often identified as natural transportation routes in the Solar System. In order to verify this hypothesis, some test particles along the structures were chosen and their orbital evolution was tracked over 5 million years. As many as 99.5 % of the particles interacted with the resonance and migrated close to the Earth. Let us mention that in the earlier studies the rate was below 10 percent. A large fraction of particles was removed from the Solar System by an unknown direction defined by a perihelion distance line close to $q \sim 0.26$ AU whose origin is still unexplained. And finally, we have counted the test bodies reaching the orbit of the asteroid Phaethon, the parent body of Geminides meteor showers and observed that $5:2$ mean motion resonance (MMR) – Phaethon dynamical link is several times stronger than the previous results which was confirmed through the high transportation efficiency of the observed structures.

3. OBSERVATIONS OF DOUBLE AND MULTIPLE STARS AND ANALYSIS OF ORBITAL MOTION

Binary stars have been studied for decades for the purpose of accurate determination of stellar masses, verification of the evolutionary models and star formation theories. Washington Double Star Catalog (WDS)² contains data for more than 120,000 star pairs, but only for a small fraction of them (about 2%) there are orbital solutions. Because of this, as many as possible new observations of double stars should be performed.

With CCD observations it is not possible to resolve systems with small separation between the components. Closer pairs are monitored using high angular resolution techniques such as speckle interferometry, adaptive optics, etc. Wide stellar systems have large orbital periods in general. For many of them there are few observations, the observations over a short orbital arc or they have a low accuracy. Our programme is to observe such pairs. In the framework of the collaboration between the Bulgarian Academy of Sciences and Serbian Academy of Sciences and Arts our joint research project "Investigation of visual double and multiple stars" was started in 2012. This collaboration has enabled us to utilise in addition to our telescopes at Vidojevica, also the 2 m telescope at NAO Rozhen. In this period more than 25,000 CCD frames were obtained. This resulted in the publishing of five measuring series of double and multiple stars Cvetković *et al.* (2011, 2015, 2016 i 2017) and Pavlović *et al.* (2013). All our results (measurements) have been included in the WDS Catalog.

Newly purchased very quick EMCCD Andor iXon 897 camera (Pavlović *et al.* (2018), in this publication) will be use as a part of the equipment for the speckle-interferometric technique of observing double stars with separations less than 1 arcsec. But, for the beginning it will be utilized for obtaining frames by applying lucky imaging.

We examine the dependence of correlation coefficients of orbital elements on the length of the orbital arc covered by measurements, on measurements of different accuracies, and on the number of measurements. The obtained correlation coefficients for the orbital elements are found to decrease with the orbital arc length covered by measurements, they are independent of the measurement precision, and they do not depend on the number of measurements for long arcs and they decrease with the

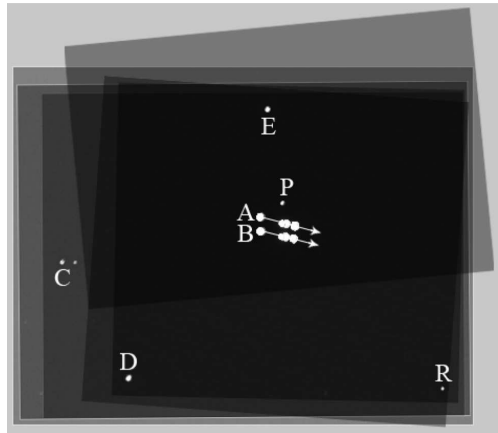


Figure 2: CCD frames of the multiple system ADS 48 are overlapped in order to have at the same position images of the components C, D, E, P, and R for which the configuration is invariable. The motion of pair AB in the view field is clearly seen; the direction and sense are indicated by the arrow.

²<http://www.astro.gsu.edu/wds/>

number of measurements for short arcs. In addition, we plan to use this for developing a new method to compute the orbit grades.

We discuss the software developed for the purpose of determining the relative coordinates (position angle and separation) for visual double or multiple stars. It is based on application of Fourier transforms in treating CCD frames of these systems. The objective was to determine the relative coordinates automatically to an extent as large as possible. In this way the time needed for the reduction of many CCD frames becomes shorter. The capabilities and limitations of the software are examined. Besides, the possibility of improving is also considered. The software has been tested and checked on a sample consisting of CCD frames of 165 double or multiple stars obtained with the 2 m telescope at NAO Rozhen in Bulgaria. The results have been compared with the corresponding results obtained by applying different software and the agreement is found to be very good.

Using the measurements obtained by us from the CCD observations at NAO Rozhen (NAOR) and Astronomical Station Vidojevica (ASV) we have calculated the first orbits and masses for about 20 binaries and more than 30 linear solutions were presented for the first time. These orbits have been included in the Sixth Catalog of Orbits of Visual Binary Stars³ and the linear solutions have been included in the Catalog of Rectilinear Elements⁴. Two linear elements, X_A and Y_A , are used to calculate the velocity V of relative motion of the secondary with respect to the primary. We can calculate the relative proper motion, μ_{rel} , for the same pairs by using the proper motion in right ascension and the proper motion in declination. Then, we can compare the proper motion μ_{rel} with the values of the relative velocity, V , for all components in linear solutions. An agreement between them is an argument in favour of pairs not being gravitationally bound, i. e. they are optical pairs.

Also, we studied an interesting multiple system, ADS 48, using the data obtained at the Astronomical Observatory of Belgrade between 1994 and 1996, and at Rozhen and at the Vidojevica Station during the last eight years. Our aim was to establish which of the seven components are gravitationally bound, i.e. have an orbital motion around the mass center, and which of them are mutually very distant in space so that only their projections are close in the field of view. The detailed analysis of the system ADS 48 is given in the paper Cvetković et al. (2012a). The conclusions combined with the criteria based on celestial mechanics lead us to the following: i) within the system ADS 48 only stars A and B are gravitationally bound (Figure 2); ii) component F has common proper motion with A and B, but is not bound to them; iii) all the other components considered here form optical pairs with AB.

During the autumn of 2011, we observed the same objects at both NAOR and ASV. We noticed that the measured separations (ρ_{NAOR} , ρ_{ASV}) differ for the same pairs of stars and the differences increase with increasing angular separation. Therefore, we measured the angular separations between the images of stars visible in our CCD frames. The separation depends on the angle corresponding to one pixel, i.e. the focal length of the telescope. We determined the focal length of the 60 cm telescope at the ASV and for the 2 m NAOR telescope more precisely. The differences are relatively small: of the order of 1.4%. For pairs of stars with angular separations smaller than 10 arcseconds, the differences are approximately equal to measurement

³<http://www.astro.gsu.edu/wds/orb6.html>

⁴<http://www.astro.gsu.edu/wds/lin1.html>

errors. Our observational programme of double and multiple stars contains mainly pairs with angular separations less than 10 arcseconds and therefore small deviations in separations resulting from inaccurate telescope focal length could not be previously noted. Much more about determining the focal length of the 60 cm telescope at the ASV more is given in the paper Cvetković *et al.* (2012b), and for the 2 m NAOR telescope in the paper Cvetković *et al.* (2013).

4. KINEMATICAL PROPERTIES OF STARS FROM THE SOLAR NEIGHBORHOOD

The Galactocentric orbit of a Milky Way star is obtained by applying the Lagrange equations. If, as usually, for the Milky Way potential the axial symmetry is assumed, the motion of a star can be described by solving the following two differential equations

$$\begin{aligned}\ddot{R} - J_Z^2/R^3 &= \frac{\partial \Pi}{\partial R} \\ \ddot{Z} &= \frac{\partial \Pi}{\partial Z}.\end{aligned}\tag{1}$$

The designations are: Π - potential, R - distance to the symmetry axis, $|Z|$ - distance to the symmetry plane, J_Z - component of specific angular momentum ($J_Z = \text{const}$).

In order to solve system of equations (1) one needs the potential. The most favourable situation is if the potential is given analytically. Then the system is solved numerically (by applying a well known procedure). Since the steady state is also assumed, the energy conservation can serve for the purpose of algorithm control.

Since the Milky Way has a composite structure, its potential should be given as a sum of the contributing potentials. Each contributing potential is due to a subsystem of the Milky Way. Very often as relevant subsystems the bulge, the disc and the dark halo are assumed.

In the case of the bulge the flattening may be neglected, i. e. the spherical symmetry is sufficient. There exists the problem of a cusp within the core. A general mass distribution involving different possibilities for the cusp has been proposed (Ninković 2014).

In the case of the disc the Miyamoto-Nagai formula has been usually used. However, as incompatible with the exponential model (almost generally assumed for the disc), this formula requires to be modified. In the framework of our project two modifications were proposed (Ninković 2015, 2017a).

In the case of the subsystem formed by the dark matter the spherical symmetry has been generally adopted. There exists the problem of cuspy core versus almost constant density one. A discussion comprising this topic wherein a specific mass distribution with a core of almost constant density is proposed can be found in Ninković (2017b).

Each contributing potential contains some constants referred to as parameters. The consequence is that the total number of parameters for a potential consisting of three components is not small. The values adopted for the parameters are related to the values of the quantities known as Milky Way constants (Galactocentric distance of the Sun, Oort constants, etc.). The values of these constants are known within some limits. Therefore, their varying can be of interest. However, in the case of a mass distribution model which may contain almost 10 parameters, the varying procedure is not simple. In addition, the numerical procedure used in the orbit determination

also requires time. Fortunately, if a star of the thin disc is considered, then the Galactocentric orbit is simply described (e. g. Stojanović 2015). In other words, the orbital eccentricity becomes the principal orbit parameter. It can be simply related to the assumed quantities where the $A/|B|$ ratio (A and B are the Oort constants) is the most influential parameter (Ninković 2011). In this way it becomes possible to determine the orbital eccentricity (also R_{\odot}/R_m , R_m is arithmetic mean between extremal distances) for many stars of the thin disc within a rather short time in the conditions of varying $A/|B|$ (Stojanović 2015).

With the simple formulae derived by Ninković a new formula which explains the ratio of the mean squares of the random velocity for thin disc stars has been obtained. It contains the old formula (usually referred to as epicyclic one) as its special case. A brief description of the new formula is given in Ninković (2018).

5. PHOTOMETRIC OBSERVATIONS OF WEBT OBJECTS AND MORPHOLOGY OF QUASARS

From the beginning of the project, CCD observations of quasi stellar objects (QSOs) visible in the optical domain were started in accordance with the tasks of the astrometric mission Gaia. QSOs, as one type of objects with active galactic nuclei (AGN), having a compact radio and optical core, without complex structures and stable flux, are of interest to Gaia. This is the reason for monitoring such QSOs in the optical domain, and to follow the changes of their morphology and photometry over time. The position stability of QSOs depends on the structures and photometry of QSOs and it makes the morphology and photometry investigations of these objects very important for astrometry and astrophysics. These observations are used for construction of the relation between radio and optical reference systems. The observational results concerning AGN objects can be used to study their physical features. Our observations are part of a more general project of astrophotometric and astrophysical studies of extragalactic radio sources for the purpose of obtaining more reliable reference systems.

In the framework of the PhD thesis by M. Jovanović the observations on three telescopes were initiated: ASV 1.4m and 60cm and 2m RCC Rozhen NAO BAS are used. The very first results can be found in Jovanović et al. (2018).

6. CONCLUSIONS

Here a brief summary of the results achieved by the participants of project No. 176011 in the period 2011–2017 is presented. We shall continue the research in all the four fields mentioned above, and it is planned to include new participants in the project.

For the future we plan to procure an additional CCD detector and new optical equipment which, together with the existing Andor iXon ultra 897 CCD camera, would be attached to the 1.4 m telescope "Milanković" and enable us to begin speckle interferometric measurements at the Astronomical Station Vidojevica.

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