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CALCULATION OF MEAN DENSITY OF SOLAR PLANETS BY MODIFIED SAVICH-KASHANIN METHOD

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Abstract. Savich and Kashanin in 1960-ties developed a mathematical model to calculate the mean density of solar planets. The model used van der Waals (vdW) equation of state for real gases, from which it follows that the volume of a substance at absolute zero temperature (V_0) is one third of volume at critical point (V_c), i.e. V_0=Vc/3. However, empirical data published more recently confirmed that V_0=Vc/4. Using this empirical fact we modified mathematical model of Savich and Kashanin thus enabling the more correct calculation of planet's mean densities.

1. MATERIAL DENSITY CHANGES ACCORDING TO SAVICH-KASHANIN THEORY

Savich and Kashanin believe that by the compressing of matter, its density alternates between intervals of gradual and abrupt changes (Figure 1) (Savić 1961, 1978, Savić and Kašanin 1962). The density is gradually changed from d_1^0 to d_1* in the pressure range from p_0* to p_1*. At the pressure p_1*, there is a jump of density from d_1* to d_2^0. Again, up to the pressure p_2* there is the interval of gradual change of density, and again there is a jump of density, etc... Substances can only have those values of density that correspond to intervals of 1, 2, 3, 4... Each interval corresponds to one phase state of matter. The density is gradually changing within a definite phase state. The transition from one phase to another is like a jump in terms of changes in density.

The causes of these alternating stepwise and gradual changes in the density of matter, Savich and Kashanin looked for in the combination of quantum-mechanical phenomena, which describes the structure and properties of atoms. When atoms approach each other, there arises the moment when the atoms are close enough to each other that their outer electron orbits "touch". Further compression is possible only if electrons leave their former paths and rebound from the atoms seeking a new space for their movement. Atoms, stripped due to these runaway electrons, can further approach each other until again "touch" the remaining outer electrons.

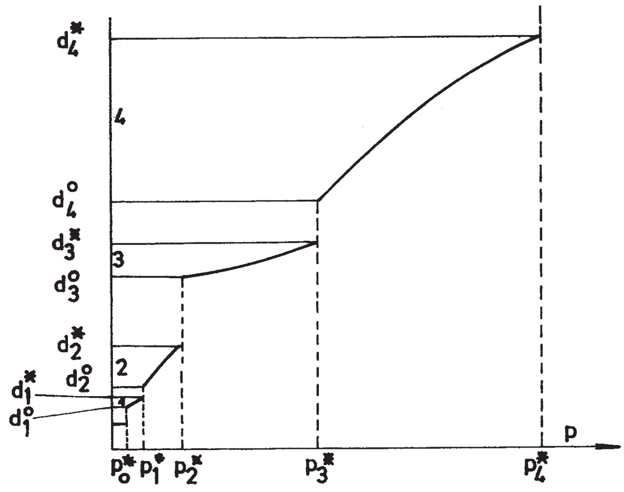

Figure 1: Changes of density of matter (d) with the change of pressure (p) according to Savich-Kashanin theory.

"Excitation and ejection of electrons under the influence of pressure leads to a number of new phenomena in macro-systems. We see that large and ultra-high pressure disrupts the inner micro-structure of the electron shells of chemical elements by pushing and ejecting electrons from them. Since the electrons are deployed by discrete, spaced levels, which are sharply separated from each other..., their ejection under this pressure will be in jumps. Accordingly, **material densities under pressure must be changed in jumps or sharp transitions from one value to another**. Due to the layered structure of the electron shells, by the displacement and ejection of electrons by the pressure, densities of the materials, as well as the properties of the macro-systems of particles, must exhibit abrupt changes" (Savić, 1978, p. 70).

Savich and Kashanin show the function in the form of a staircase diagram, which describes the change in density of matter at the beginning and at the end of certain phases (Figure 2).

The law of Savich and Kashanin for the stepwise change of density was not directly derived from the quantum-mechanical model of the atom. They empirically come to the law, however, that the density of matter at the end of subsequent phases changes abruptly, according to Eq. (1a). Density at the beginning of some phase d_i^0 is calculated by multiplying the density at the end of a that phase d_i*with parameter α, where α = 3/5 and α = 5/6 for the even and the odd phases, respectively, Eqs. (1b and 1c).

$d_{i-1}* = 2\ d_i*$ (1a)
$d_i^0 = \alpha \cdot d_i*$ (1b)
α = 3/5 and α = 5/6 for the even and the odd phases, respectively (1c)

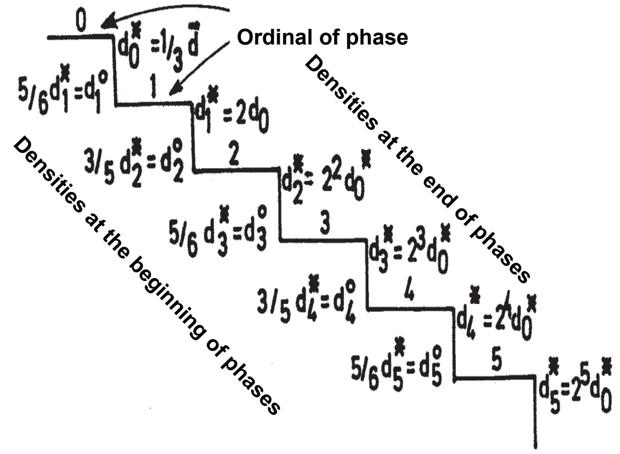

Figure 2: The density at the beginning of d_i^0 at the end d_i* of individual phases (i = 1, 2, 3...) according to the Savich-Kashanin theory.

Savich and Kashanin calculated these values of parameter α by taking into account the van der Waals (vdW) equation of state for real gases (2).

$$(P + a/V^2)\ (V - b) = RT \qquad (2)$$

P, V and T are pressure, specific volume and absolute temperature, respectively, a and b are the so-called van der Waals constants, R is the universal gas constant.

It follows from (2) that the constant b, so-called "covolume", is the specific volume V_0, which a gas would have at absolute zero (T = 0 K). (3). It is also known that it follows from (2) that covolume b is equal to one third of critical volume V_c, for given material at critical point. Respecting equation (2), Savich and Kashanin took into account relation (3) and calculated the values of parameter α as in Eq. (1c). It should be noted that relation (3) is one of the important assumptions built into obtaining the mathematical model of Savich and Kashanin (Figure 2).

$$b = V_0 = V_c/3 \qquad (3)$$

It should be borne in mind that the specific volume of matter (V) is equal to the reciprocal of the density (d), Eq. (4), so it is easy to calculate the value of one of them if the other is known.

$$V = 1/d \qquad (4)$$

Another important assumption of Savich and Kashanin used to derive their mathematical model is that matter at the end of the zero-phase has a volume, or density, which corresponds to the critical point (5).

$$d_0^* = d_c = 1/V_c \qquad (5)$$

Savich and Kashanin applied the above mathematical model (Figure 2) to calculate the mean density of planets in the solar system and the results of their calculations they compared with astronomical data available by that time. According to their calculations some planets should have a density of 0.67 g/cm^3, which approximately corresponds to the density of Saturn (0.65 g/cm^3). For one group of planets the calculated density was 1.33 g/cm^3; which corresponds to Jupiter (1.34 g/cm^3), Uranus (1.36 g/cm^3) and Neptune (1.32 g/cm^3). For the second group of planets the calculated density was 5.33 g/cm^3; which corresponds to Earth (5.52 g/cm^3), Venus (5.21 g/cm^3) and Mercury (5.6 g/cm^3). The agreement of calculated and measured values is very good. A large discrepancy is only in the case of Mars: the calculated value is 5.33 g/cm^3, while the empirically estimated value is 3.94 g/cm^3. Savich and Kashanin believed that their calculation was correct, and the discrepancy with the observed value of the density indicated a possible error of astronomical data for the radius of Mars.

2. ADAPTATION OF STEPWISE MATHEMATICAL MODEL BY ACTUAL EMPIRICAL DATA

Analyzing the mathematical model of Savich and Kashanin we have noticed that some of their assumptions are not consistent with the recent empirical data.

Empirical data (Filippov 1978) show that the relations (3) are not correct, but the volume of material at critical point (V_c) is twice the value of covolume (b) (Dean 1979) and four-fold higher value than the volume of matter at absolute zero temperature (V_0), Eq. (6).

$$V_c = 2\ b = 4\ V_0 \qquad (6)$$

Analysis of the compressed gaseous ethylene showed that the different phases are indeed formed (Stoiljković 1981). However, the density of ethylene at the critical point corresponds to the end of the first phase ($d_1^* = d_c = 1/V_c$), but not to the end of the zero phase, as it is proposed by Savich and Kashanin in relation (5).

In the theory of Savich and Kashanin the initial state of matter is the rarefied gas that is condensed into forming Sun and planets. This means that the beginning of the zero phase should have a density, which is close to zero. However, in their staircase model (Figure 2) the density at the beginning at the zero phase has some definite value higher than zero, i.e. $d_0^0=(3/5)\ d_0^* >> 0$.

Due to these empirical facts, it was necessary to adapt the mathematical model so that it will be consistent with these empirical facts. This adaptation and theoretical derivation of our model is presented by Stoiljković and Jovanović (1983) and Stoiljković et al. (1995) in which the ratio of the characteristic volumes of matter to the critical volume is shown. The same model can be represented by the density of the matter, which is the reciprocal of the volume, Eq. (4). Hence, the adapted theoretical mathematical model was obtained that shows the relationship of mean density of the planet to mean density of the Sun (Figure 3).

According to our staircase model, the condensation starts with the gaseous matter where the density is close to zero and then the density increases in the phase transition from zero to the first phase, then to the second, then to the third phase and so on. The coefficients α that describe the ratio of density at the beginning to at the end of some phases are not 3/5 or 5/6, as in Savich-Kashanin theory (1c), but have the values according to Eq. (7).

$$\alpha\,(i) = d_i^0/d_i^* \quad = 2^{-1/i} \tag{7}$$

Where the number of the phase is i = −1, 0, 1, 2, 3...

Bearing in mind that the mean density of the Sun is equal to 1.41 g/cm^3, the values of mean density of the planets can be calculated by our model (Figure 3). Agreement with empirical data (Syunyaev 1986) was very good (Table 1). In addition, unlike the staircase model of Savich and Kashanin (Figure 2), our model shows that mean density of Mars should be about 4 g/cm^3, which is close to the empirical data.

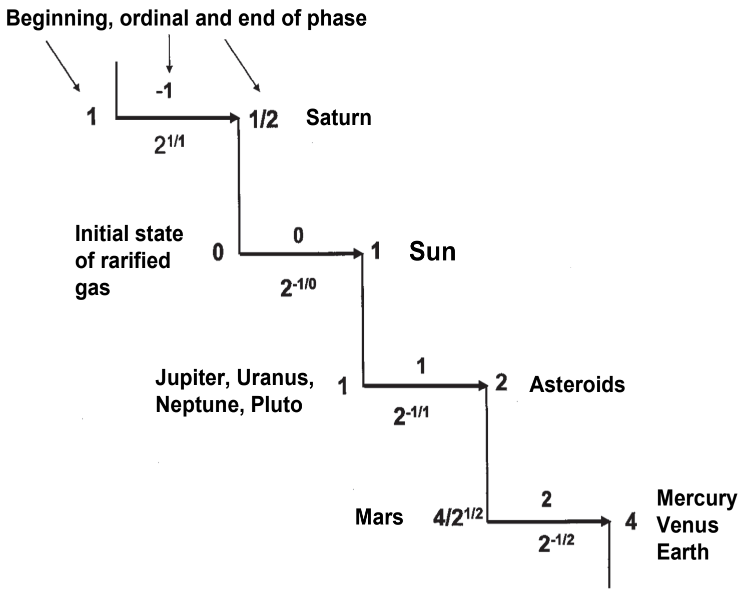

Figure 3: Our theoretical staircase model – the ratio of mean density of the planets with that of the Sun (d_s = 1.41 g/cm^3) (Stoiljković 1986).

Based on our staircase model, a celestial body (or bodies) with mean density 2.8 g/cm^3 could exist in the Solar system. Indeed, it corresponds to the asteroids whose density is in the range of 2.0 to 3.5 g/cm^3.

Although Pluto was recently removed from the list of planets, we also calculated by our model that its mean density would be 1.41 g/cm^3 which is in accordance with empirical data, i.e. 1.75 g/cm^3 (though in the literature one can find other, very different, values).

Table 1. Mean density of planets in the solar system

Planet	Mean density (g/cm^3)	
	Empirical data (Syunyaev 1986)	Calculated using our staircase model (Figure 3) (Stoiljković 1986)
Mercury	5.4	5.64
Venus	5.2	5.64
Earth	5.5	5.64
Mars	3.9	4.00
Jupiter	1.3	1.41
Saturn	0.7	0.71
Uranus	1.6	1.41
Neptune	1.7	1.41

The trend of densification is indicated by an arrow in Figure 3. If Saturn followed the same trend, then there would be no condensation, but a state of spreading rarefied gas. In other words, according to our model, planet Saturn could not have been condensed.

There is a hypothesis that the Sun has a twin star, which is called Nemesis. Mean density of Nemesis would be 79.21 g/cm^3 (Galaksija 1988). It can be calculated by extrapolation of our staircase model that there could be a body with density of 80.63 g/cm^3, which is in good agreement with the above data for Nemesis.

CONCLUDING REMARKS

Our modification of mathematical model given by Savich and Kashanin enables the more exact calculation of planet's mean densities. Furthermore, the same modified model enables calculation of densities of matter in its characteristics states (i.e. critical point, triple point, absolute zero temperature...) (Stoiljković et al 1995) as well as to predict the supra-molecular structure of fluids (real gases and liquids) (Stoiljković et al 1981, 1988; Radičević et al 1995), which has a great importance in physics and chemistry, too. The more detailed scientific and philosophical insight of Savich-Kahanin theory and its connection with Roger Boscovich's theory of natural philosophy has been published, too (Stoiljković 1979, 2005, 2010, 2014).

References

Dean, J. A.: 1979, "Lange's Handbook of Chemistry", 12th edition, McGraw-Hill, New York.

Filippov, L. P.: 1978, "Podobye svoistv veshchestv", Izdael'stvo Moskovskogo universiteta, Moskva.

Galaksija: 1988, No. 4, 33.

Radičević, R., Korugić, Lj., Stoiljković, D., Jovanović, S.: 1995, "Supermolecular organization and characteristic moments of the polymerization of methyl methacrylate", *J. Serb. Chem. Soc.*, **60**, 347-363.

Savić, P.: 1961, "O nastanku rotacije sistema i pojedinih nebeskih tela", *Glas SANU*, CCXLV, **21**, 37-43.

Savić, P., Kašanin, R.: 1962, "The behaviour of the materials under high pressures", Serbian academy of sciences and arts, Monographs, Vol. **351**, Section for Natural Sciences and Mathematics, No. 29, Beograd.

Savić, P.: 1978, "Od atoma do nebeskih tela - poreklo rotacije nebeskih tela", second edition, "Radivoj Ćirpanov", Novi Sad.

Stoiljković, D.: 1979, "Dijalektičko-materijalistička osnova teorije Savić-Kašanin o ponašanju materije pri visokim pritiscima i o nastanku rotacije nebeskih tela", *Dijalektika*, **14**, 137

Stoiljković, D.: 1981, "Mehanizam i kinetika polimerizacije etilena pri visokom pritisku", doktorska disertacija, Tehnološko-metalurški fakultet, Beograd.

Stoiljković, D., Jovanović, S.: 1981, "The mechanism of the high pressure free-radical polymerization of ethylene", *J. Polym. Sci, Polymer Chem. Ed.*, **19,** 741-747.

Stoiljković, D., Jovanović, S.: 1983, "Relations between characteristic volumes of matter", *Bull. Soc. Chim., Beograd*, **48**, 49-54.

Stoiljković, D.: 1986, "Characteristic volumers of matter – Estimation, meaning and significance ", plenary lecture, V Meeting of chemists of Vojvodina, Kikinda, P-3.

Stoiljković, D., Jovanović, S.: 1988, "Supermolecular organization and polymerization of compressed ethylene", *Acta Polymerica*, **39**, 670-676.

Stoiljković, D., Macanović, R., Pošarac, D.: 1995, "The correlation between characteristic volumes of matter - a mathematical model and its physical meaning", *J. Serb. Chem. Soc.*, **60**, 15.

Stoiljković, D.: 2005, "Sažimanje materije - Odjeci Boškovićevih shvatanja u teoriji Savić-Kašanin", *Vasiona*, **53** (4) 178-184.

Stoiljković, D.: 2010, 2014, "Roger Boscovich – The founder of modern science", (serbian edition: Istraživačka stanica Petnica 2010; english edition: Lulu publisher 2014)

Syunyaev, R. A.: 1986, "Fizika kosmosa", Sovetskaya entsiklopediya, Moskva.