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CONDENSED MATTER PHYSICS AND IMPACT CRATER FORMATION

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Abstract. Various kinds of craters exist on solid bodies in the planetary system and some of them are due to impacts into the surfaces of the objects concerned. Impact craters are "by default" analyzed within the "scaling theory", based on dimensional analysis. The same problem can be analyzed by using standard laws of condensed matter physics. In this contribution the two approaches will be compared, and possibilities for future work discussed to some extent.

1. INTRODUCTION

Surfaces of solid bodies in the solar system are filled with craters of various sizes. Some of them are of volcanic origin, while others (which are the subject of the present paper) are the results of impacts of small bodies into the target surfaces. The existence of impact craters is an expectable consequence of the fact that a multitude of small solid bodies, remaining from the epoch of formation o the planetary system, is orbiting the Sun. Their study is attractive for two important reasons: on the fundamental level, analyzing these craters gives the opportunity of inferring conclusions about the impactors which made them. The "applied" interest in impacts and impact craters is much more important: the impact of a sufficiently large object into a sufficiently densely populated region on the Earth would provoke a catastrophe. Accordingly, developing the possibilities of predicting the place of an impact, the size of the possible crater, heating or melting of the target material or the height of a possible tsunami, is extremely interesting and important for humanity.

This paper is devoted to a comparison of two approaches to the problem of impacts and the resulting craters: the scaling theory (Holsapple 1993) and the approach based on standard laws of condensed matter physics (Celebonovic 2013). Each of the following two sections is devoted to brief outlines of each of these approaches, the section after to a comparison of their possibilities, and finally the conclusions.

2. THE SCALING THEORY

The crucial term in this approach is the notion of scaling. Scaling is defined as the application of some relation (called the scaling law) to predict the outcome of one event from the results of another. Parameters which are different between the two events are called *scaled* variables. It can also mean predicting the dependence of the outcome of a problem on its parameters (Holsapple 1993).

The form of scaling laws can be determined in three ways: by impact experiments, analytical calculations and approximate theoretical solutions.

The basic principle of impact experiments is very simple: projectiles of varied composition and mass are fired with different speeds into targets of differing chemical composition, and data are measured on the resulting craters. Such experiments are being performed for decades (some examples are Oberbeck 1971; Fujiwara et al., 1977; Baldwin et al., 2007; Suzuki et al., 2012) and they have given various interesting results. However, a common problem with all these experiments is that the projectiles are launched in them with velocities below those of interest for studies of creation of large craters. In a similar kind of experiments, solid targets are shock compressed by the impact of short-lived laser beams. For a recent report on a newly developed experimental platform for such experiments see, for example, (Gauthier et al., 2014).

Laws of physics needed for theoretical studies of impacts and the formation of craters are well known; these are the basic laws of classical physics,conservation of mass,momentum and energy, supplemented by knowledge on the equation of state (EOS) of the materials of the target and the impactor. However, this kind of work encounters a problem: lack of detailed knowledge on phase transitions in materials of ill defined chemical composition. The point here is that if the impactor is sufficiently massive and the speed of impact sufficiently high, at the moment of impact a transition solid $\rightarrow$ plasma occurs; the plasma cools rapidly, and the process ends-up in the domain of condensed matter physics.

Approximate theoretical solutions are based on a simple idea: the initial phase of the problem is approximated as a "point source" of shock waves propagating throughout the target after the impact. This approach was developed for studies of the effects of nuclear explosions. For details see (Holsapple 1993; Nellis, 2000) and references given there.

A good example of a scaling law is the problem of formation of a crater of volume V, resulting from the impact of an impactor of radius r, speed v and mass density ρ_1 into a target (planet) having surface gravity g, material strength X, and mass density ρ. Material strength is loosely defined as the ability of a material to whitstand load without failure. All material properties can be expressed as combinations of the dimensions of stress and mass density. This implies (Holsapple,1993) that the volume of an impact crater can be expressed as

$$V = f[\{r, v, \rho_1\}, \{\rho, X\}, g] \tag{1}$$

where the first three variables describe the impactor, the following two the material making up the planet, and the surface gravity of the planet. This expression is completely general, and scaling models are derived from it by dimensional analysis.

It follows from equation (1) that

$$\frac{\rho V}{m} = f_1[\frac{gr}{v^2}, \frac{X}{\rho v^2}, \frac{\rho}{\rho_1}] \tag{2}$$

where $m = \frac{4\pi}{3}\rho_1 r^3$ is the mass of the impactor. The quantity on the left-hand side is the ratio of the mass of the material within the crater to the mass of the impactor.It is usually called cratering efficiency and denoted by π_V. The first term in the function is the ratio of the lithostatic pressure ρgr to the initial dynamic pressure ρv^2 generated by the impactor. The lithostatic pressure at a certain depth is defined as the pressure exerted by the material above it. This ratio is denoted by π_2; the second term is the ratio of the material strength to the dynamical pressure, denoted by π_3. The final term is the ratio of the mass densities.

If all the parameters of eq.(2) were known, or could be measured or calculated, it would not be a particular problem to determine the volume of an impact crater. As this is far from being the case, solutions of this equation are usually studied in two limiting cases: the "strength" regime and the "gravity" regime. The "strength" regime is the situation in which the strength of the surface material is larger than the lithostatic pressure. Practically speaking, this implies impators with diameters of approximately one meter.This means that

$$\frac{\rho V}{m} = f_1[\frac{X}{\rho v^2}] \tag{3}$$

where it was assumed that the ratio of the densities is approximately one. In this regime, the volume of the impact crater increases linearly with the volume of the impactor, its mass and its energy. Any dimension of the crater increases with the radius of the impactor. In the opposite case, when the diameter of the impactor is of the order of a kilometer or more, the lithostatic pressure is bigger than the material strength,meaning that

$$\frac{\rho V}{m} = f_1[\frac{gr}{v^2}] \tag{4}$$

This is the definition of the "gravity" regime. Various experiments (discussed in Holsapple,1993) have been performed on the dependence of π_V on π_2, the result being an exact power law. This can be explained, as discussed in (Holsapple,1993) by the assumption that whenever there is a dependence on the impactor size and speed, it is actually the dependence on its kinetic energy. This idea was used in the early sixties, in scaling from a nuclear event called "Teapot ESS" to the creation of the Meteor Crater in Arizona.

The idea that the consequences of an impact depend on the kinetic energy of the impactor is equivalent to the "point source" approximation. The kinetic energy is given by $\frac{1}{2}mv^2$. Taking the cube root, introducing the mass density, and dropping the numerical factor, one gets the function

$$C = r\rho^{1/3}v^{2/3} \tag{5}$$

which can be generalized to

$$C = r\rho^{\mu}v^{\nu} \tag{6}$$

Using this, equation (1) becomes

$$V = f[r\rho^{\mu} v^{\nu}, \rho_1, X, g] \tag{7}$$

It can be shown by dimensional analysis (Holsapple, 1993) that in the strength regime the volume of a crater is given by

$$V \propto \frac{m}{\rho_1} \times (\frac{\rho_1 v^2}{X})^{3\mu/2} \times (\frac{\rho}{\rho_1})^{1-3\nu} \tag{8}$$

and a similar expression can be derived for the gravity regime. Values of scaling exponents can be determined in impact cratering experiments (such as Suzuki, 2012).Once they are known for a given material (or materials) calculations referring to the formation of the impact craters become possible.

3. CONDENSED MATTER PHYSICS

Surfaces of objects in the solar system on which impact craters exist are solid. As the impactors are solid objects, the question is what (if anything) can be concluded about the impacts by using laws of condensed matter physics. The aim of this section is to outline these possibilities, using recent results of the present author.

The first step in analyzing impact craters by the use of solid state physics, is to determine the minimal velocity which a projectile must have in order to form a crater. This was studied in (Celebonovic and Sochay, 2010), where the condition for the formation of a crater was defined as the equality of the kinetic energy of a unit volume of the material of the impactor with the internal energy of the unit volume of the material of the target. It was shown that this speed is given by

$$v^2 = \frac{\pi^2}{5\rho_1} \frac{(k_B T)^4}{\hbar^3} (\frac{\partial P}{\partial \rho})^{-3/2} \tag{9}$$

where ρ_1 is the mass density of the impactor, T the temperature of the target, and P, ρ are the pressure and mass density of the material of the surface of the target. The dimensions of the impactor and of the resulting crater were not taken into account. As a test,this expression was applied to the case of an impactor made up entirely of olivine $(Mg, Fe)_2SiO_4$. It was shown that the minimal impact speed of such an object should be 16.3 km/s. For comparison, note that the impact velocity of a real object, asteroid 99942 Apophis, is estimated to be between 13 and 20 km/s, which means that two completely different methods: celestial mechanics and condensed matter physics give very similar results. 99942 Apophis is an interesting object for such a comparison, because celestial mechanics indicates that there exists a small but non-zero probability that it collides with the Earth on April 13, 2036 (Giorgini et.al., 2008). Similar results have been reached for the asteroid 1950DA, for which a probability of impact exists for March, 2880 (Farnocchia and Chesley, 2014).

The final result of any impact is a crater. If the impact is strong enough (if the kinetic energy of the impactor is high enough), and if the target has a suitable value of the heat capacity, a consequence of the impact will be heating of the target. Depending on the kinetic energy of the impactor, the target may heat enough so as to melt, and possibly even evaporate at the point of impact. In this regime condensed matter physics obviously cannot be applied. Regardless of the amount of heating in the impact, the outcome is always the same: a certain quantity of material of the target gets "pushed aside" at the point of impact, thus creating a crater of given dimensions. The aim of the calculations outlined here is to draw conclusions about the impactor using measurable dimensions of the crater and various parameters of the target. Such an approach corresponds to what has earlier been named "the inverse" problem (Holsapple, 2003), where the aim was to deduce the impactor size and speed by analyzing the volume of impact melt.

Formation of impact craters was recently discussed as the following analogous problem in condensed matter physics: how big must be the kinetic energy of the impactor in order to produce a hole of given dimensions in a target material with known parameters (Celebonovic, 2013)? It was assumed that the material of the target is a crystal, that one of the usual types of bonding exists in it, and that as a consequence of the impact the target does not melt, so that condensed matter physics can be applied. The problem of heating in impacts has recently been discussed in (Celebonovic, 2012).

This calculation is based on a simple physical idea: the kinetic energy of the impactor must be greater than or equal to the internal energy of some volume, denoted by V_2, of the material of the target. The kinetic energy of the impactor of mass m_1 and speed v_1 is obviously

$$E_k = \frac{1}{2} m_1 v_1^2 \tag{10}$$

and the internal energy E_I consists of three components: the cohesion energy E_C, the thermal energy E_T and $E_H(T)$ - the energy required for heating the material at the point of impact by an amount ΔT. Therefore,

$$E_I = E_C + E_T + E_H(T) \tag{11}$$

and the condition for the formation of an impact crater as a consequence of an impact is

$$E_I = E_k \tag{12}$$

The details of the calculation are avaliable in (Celebonovic, 2013) and the final result for the energy condition which must be satisfied to enable the formation of an impact crater is given by

$$\begin{aligned} &3k_B T_1 N\nu[1 - \frac{3}{8}\frac{T_D}{T_1} - \frac{1}{20}(\frac{T_D}{T})^2 + \frac{1}{10}(\frac{T_D^2}{TT_1}) \\ &+(\frac{1}{560})(\frac{T_D}{T})^4 - \frac{1}{420}\frac{T^4}{T^3 T_1} - \frac{3\bar{u}^2\rho\Omega_m}{np\nu k_B T_1}] = \frac{2\pi\rho_1}{3} r_1^3 v_1^2 \end{aligned} \tag{13}$$

The number N is equal to the ratio of the volume of the crater, and the volume of the elementary crystal cell, v_e: $N = V/v_e$. The meaning of other symbols is: k_B Boltzmann's constant, T the initial temperature of the target, T_1 the temperature to which the target heats, T_D the Debye temperature of the target, ρ_1, r_1 v_1 - mass density, radius and impact velocity of the projectile, p, n - parameters of the interatomic interaction potential in the material of the target, ν the number of particles in the elementary crystal cell, $\bar{u}$ the speed of sound in the material of the target and Ω_m is the volume per particle pair.

Equation (13) may at first sight look very complicated. In fact, it is simply an expression of the energy conservation law. Its main result is that it links parameters of the impactor, with those of the material of the target, which was the aim of the calculation.

This expression was applied to a well known case - the Barringer crater in Arizona, for which most of the experimental parameters are known. Assuming that the material of the crater is pure Forsterite (Mg_2SiO_4), and making plausible assumptions about other parameters of Eq. (13), it was obtained that $v_1 \cong 41\,\mathrm{km/s}$, which is far larger than existing estimates. Assuming that only 10 percent of the material is Forsterite, and keeping all the other parameters constant, gave the value of $v_1 \cong 15\,\mathrm{km/s}$, for the impact speed, which is much closer to the results of celestial mechanics. Details of this calculation are avaliable in (Celebonovic, 2013).

The calculation outlined above was performed using the notion of cohesive energy of solids. The problem is that the cohesive energy is a very "impractcal"quantity: it is defined as the energy needed to transform a sample of a solid into a gas of widely separated atoms (Marder, 2010). As a consequence of this definition, it is difficult to measure experimentally and it is not related to the strength of solids measurable in experiments.

A much more "practical" notion is the stress. It is defined as the ratio of the force applied to a body to the cross section of the surface of a body normal to the direction of the force. After an impact,a crater will form if stress in the material becomes sufficiently high for the formation of a fracture.

The critical value of the stress needed for the occurence of a fracture in a material is given by (Tiley, 2004)

$$\sigma_C = \frac{1}{2}\left(\frac{E\chi\tau}{r_0 w}\right)^{1/2} \tag{14}$$

where E is Young's modulus of the material, χ is the surface energy, τ is the radius of curvature of the crack, r_0 the interatomic distance at which the stress becomes zero and w is the length of a crack which preexists in the material. Defined in this way, σ_C has the dimensions of pressure.

At the moment of impact, the kinetic energy of the impactor is used for fracturing and heating the material of the target. Therefore:

$$E_k = \sigma_C V + C_V V(T_1 - T_0) \tag{15}$$

where V is the volume of the crater formed as a result of the impact, C_V is the heat capacity of the target material and T_0 the initial temperature of the target. In accordance with recent experiments (Suzuki et al.,2012) the volume of the crater is approximated by

$$V = \frac{1}{3}\pi b^2 c \tag{16}$$

where b is the radius of the "opening" of the crater and c denotes its depth.

It will be assumed that the impactor is a sphere of radius r_1, made up of a material of density ρ_1 having impact velocity v_1. Its kinetic energy is given by $E_k = \frac{2\pi}{3}\rho_1 r_1^3 v_1^2$. It follows from Eq. (15) that

$$T_1 = T_0 + \frac{1}{C_V}\left(\frac{E_k}{V} - \sigma_C\right) \tag{17}$$

and after some algebra (Celebonovic, 2014)

$$V = \frac{2\pi}{3} \frac{\rho_1 r_1^3 v_1^2}{\alpha C_V T_0 + \sigma_C} \tag{18}$$

where $T_1 - T_0 = \alpha T_0$. Equation (18) can be expressed as

$$V = \frac{E_k}{\alpha C_V T_0 + \sigma_C} \tag{19}$$

implying that the crater volume is a linear function of the kinetic energy of the impactor. On the other hand, raw experimental data on crater volumes and the impactor energies in (Suzuki et al., 2012), can be fitted by an equation of the form $V[m^3] = V_0 \times \mathrm{Exp}[E[J]/c]$, with $V_0 = (4 \pm 2) \times 10^{-7} m^3$ and $c = (583 \pm 56) J$. For sufficiently low energies E, this exponential expression reduces to the form $V - V_0 \cong (V_0/c)E$. Combining with results of the calculations reported here, it follows that $V_0/c = 1/(\alpha C_V T_0 + \sigma_C)$ The implication is that the results of the calculations reported here are relevant to low kinetic energies of the impactors.

Experiments such as (Grady and Lipkin, 1980) have shown that measured data for various materials can be fitted by an expression of the form $\sigma_C = a\dot{\epsilon}^n$ where $\dot{\epsilon}$ is the strain rate, and a, n are material dependent constants. This implies that the volume of an impact crater also depends on the rate of strain to which the target material is exposed at the point of impact.

Calculations outlined here open up the possibility of making various estimates of physical quantities occuring in the equations. Using known experimental data, and taking that the most abundant mineral at the site of the Barringer crater is SiO_2 it was shown in (Celebonovic, 2014) that at the moment of impact the site heated up to $T_1 \approx 1300K$. For another terrestrial entity, the Kamil crater on the border of Egypt and Sudan, it was shown that $\sigma_C \cong 1.56 \times 10^8$ J/m^3.

4. COMPARING THE TWO APPROACHES

In this contribution we have outlined to some extent two approaches to the problem of the impact craters on the surfaces of solid bodies in the planetary system. One is the so called scaling theory and the other is standard condensed matter physics. Both approaches have a similar goal: using avaliable experimental data, and relevant laws of physics, draw as much conclusions as possible on the impacts and the impactors.

Scaling theory aims at linking the craters of "celestial" origin with those resulting from man made,classical or nuclear explosions. Scaling in such a way gives encouraging results. This approach is very general,which is excellent, but there exists the problem of treatment of phase transitions. However,the main method of work with scaling theories is dimensional analysis. One of the results of the scaling approach is that the volume of a crater formed after an impact depends also on the mass density of the target. The same conclusion can be reached within the condensed matter physics approach (Celebonovic, 2014).

The approach based on condensed matter physics is rigorously based on well known physical laws. However, by its very nature, this approach has an inherent limitation: it can treat either slow impacts of "not very massive" projectiles, or the final phase (in which heating effects have cooled down). Future work in this approach could go along two lines: including in more details the effects of heating, and thus enabling the study of the "hot phase" of the formation of a crater, and exploring the upper mass limits of this approach and introducing (if it turns out to be necessary) some possible new factors which influence the final outcome.

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