Publ. Astron. Obs. Belgrade No. 96 (2017), 51 - 60 *Invited Review*

GEOPHYSICAL FLUIDS, GEOMAGNETIC JERKS, AND THEIR IMPACT ON EARTH ORIENTATION

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Abstract. Geophysical fluids (atmosphere, oceans, and to some extent also continental water) have significant impact on Earth orientation parameters. Dominant is the excitation of polar motion and speed of rotation, much smaller but now measurable influence can be found also in precession/nutation. Recently Malkin (2013) found a correlation between the observed changes of Free Core Nutation parameters (phase, amplitude) and geomagnetic jerks (rapid changes of the secular variations of geomagnetic field). In our recent study (Vondrák & Ron 2014) we tested this hypothesis and found that if the numerical integration of Brzeziński broad-band Liouville equations (Brzeziński 1994) of atmospheric/oceanic excitations is re-initialized at the epochs close to geomagnetic jerks, the agreement between the integrated and observed celestial pole offsets is improved significantly. This approach tacitly assumes that the influence of geomagnetic jerks has a stepwise character, which is physically not acceptable. Therefore we introduce a simple continuous excitation function (having a "double ramp", or triangular, shape), centered on the epochs of geomagnetic jerks, and estimate its amplitude to fit best the integrated pole positions to its observed positions. The combined results of numerical integration of atmospheric/oceanic excitations plus this newly introduced excitation are then compared with the observed celestial pole offsets. The comparison shows that this approach improves the agreement between the two time series significantly.

1. INTRODUCTION

Earth rotation, in a wider sense, means the total orientation of the body in space (precession-nutation, polar motion, proper rotation), affected by

- external torques by the Moon, Sun, and to a lesser extent also by planets;
- geophysical influences (internal composition of the Earth, transfer of mass at core-mantle boundary, oceans, hydrosphere, atmosphere, magnetic coupling. . .).

Earth rotation has a fundamental importance in astronomy, especially for transformation between rotating terrestrial and non-rotating celestial reference systems, but also in many other applications, as, e.g., space navigation, geodesy, geophysics etc. . .

Precession was known already to Hipparchus (second century B.C.), polar motion was theoretically predicted by Euler (1765), observationally first detected by Kűstner

(1884/5), and its two main components of about 12 and 14 months determined by Chandler (1891). Since 1899 International Latitude Service (ILS) was set up to monitor polar motion, later replaced by International Polar Motion Service (IPMS) and finally by International Earth Rotation and Reference Systems Service (IERS). Nutation was observed by Bradley and theoretically explained by Euler, in the middle of the 18th century; since then systematic improvement of the model of nutation took place. Secular deceleration of Moon's motion, observed already by Halley (1695) and later studied by Laplace (18th century), was implied to be linked with the decelerating speed of rotation of the Earth by G. Darwin (end of 19th century). Only in the first half of the 20th century decadal and seasonal variations of Earth's speed of rotation were observed.

In the following, we shall first give a short description of the theory of Earth rotation to show how much the geophysical excitations can influence different Earth orientation parameters, and then we shall concentrate on nutation and its excitation by geophysical effects.

2. CONCISE THEORY OF EARTH ROTATION

Earth rotation can be simply described as a time-dependent relation between two reference systems (see Fig. 1):

1. xyz - rotating system, connected with the Earth,

2. XYZ - non-rotating system, linked to extragalactic objects.

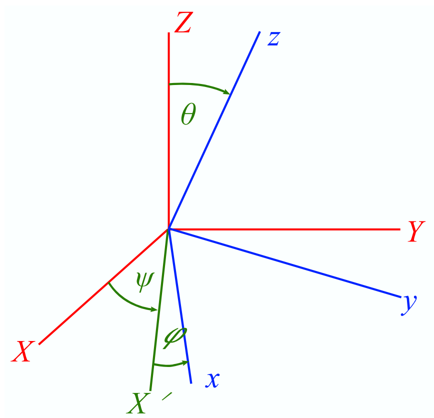

Figure 1: Transformation between rotating and non-rotating reference systems.

Mutual orientation of both systems is defined by three Euler angles (ψ - precession angle, θ - nutation angle, and φ - angle of proper rotation). Three consecutive rotations are necessary to go from XYZ to xyz: around Z-axis by ψ, then around new X'-axis by $-\theta$, and finally around z-axis by φ. According to laws of mechanics, time derivative of angular momentum of the Earth $\mathbf{H}$ must be equal to external torque $\mathbf{L}$, exerted by external forces (by the Moon, Sun, and planets). Expressed in a rotating system, the corresponding equation reads

$$\frac{\mathrm{d}\mathbf{H}}{\mathrm{d}t} + \omega \times \mathbf{H} = \mathbf{L}, \tag{1}$$

where $\omega = (\omega_1, \omega_2, \omega_3)^T$ stands for the vector of rotation. For a non-rigid body it holds $\mathbf{H} = \mathbf{C}\omega + \mathbf{h}$, in which $\mathbf{C}$ is the tensor of inertia and $\mathbf{h}$ is the relative angular momentum. Hence Liouville equations follow

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{C}\omega + \mathbf{h}) + \omega \times (\mathbf{C}\omega + \mathbf{h}) = \mathbf{L}. \tag{2}$$

Taking into account that

$$\mathbf{C} = \begin{pmatrix} A + c_{11} & c_{12} & c_{13} \\ c_{12} & A + c_{22} & c_{23} \\ c_{13} & c_{23} & C + c_{33} \end{pmatrix}, \quad \mathbf{h} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \tag{3}$$

(A and C being mean values of principal moments of inertia) and denoting $h = h_1 + \mathrm{i}h_2$, $c = c_{13} + \mathrm{i}c_{23}$, $L = L_1 + \mathrm{i}L_2$, $m = (\omega_1 + \mathrm{i}\omega_2)/\Omega$, $m_3 = \omega_3/\Omega$, in which Ω is the mean speed of Earth's rotation, we obtain linearized Liouville equations, the first one being given in complex form:

$$\begin{aligned} m + \mathrm{i}\dot{m}/\sigma_E &= \Psi \\ \dot{m}_3 &= \dot{\Psi}_3 \end{aligned} \tag{4}$$

with $\Psi = [\Omega^2 c + \Omega h - \mathrm{i}(\Omega\dot{c} + \dot{h} - L)]/\Omega^2(C - A)$, $\Psi_3 = -(\Omega\dot{c}_3 + \dot{h}_3 - L_3)/C\Omega$, called excitation functions, $\sigma_E = \Omega(C - A)/A$ is the Euler frequency. If we put $c_{ij} = h_i = 0$, the Eqs (4) become Euler equations, valid for rigid Earth. By solving Liouville equations, we obtain the position of the vector of immediate rotation m, i.e., polar motion components $x = m_1$, $y = -m_2$, and relative change of speed of rotation m_3.

The solution for *polar motion* has a free component, which has a period of 305 days for rigid Earth (Euler period), but its observed value for real Earth is 435 days (Chandler period). Forced components are mostly seasonal; geophysical influence, that is responsible for this motion, becomes dominant, because the changes of tensor of inertia $\mathbf{C}$ and relative angular momentum $\mathbf{h}$ are long-periodic in terrestrial system. External forces $\mathbf{L}$ have minimal effect since they are short-periodic and therefore strongly suppressed during integration.

Speed of rotation is constant for rigid Earth. For non-rigid Earth, the external torques (through zonal tidal deformation causing long-periodic changes of $\mathbf{C}$) and geophysical excitations, that are also long-periodic, are almost equal.

Position of axis z in non-rotating celestial system (angles ψ, θ) and angle of proper rotation φ are then given by integrating Euler kinematic equations

$$\begin{aligned} \dot{\psi}\sin\theta &= -\omega_1 \sin\varphi - \omega_2 \cos\varphi \\ \dot{\theta} &= -\omega_1 \cos\varphi + \omega_2 \sin\varphi \\ \dot{\varphi} &= \omega_3 - \dot{\psi}\cos\theta. \end{aligned} \tag{5}$$

During the integration, geophysical effects in precession-nutation are suppressed (they are short-periodic in celestial system) and external torques become dominant (being long-periodic).

2. 1. NUTATION MODELS

Usually, the nutation model is derived in two steps:

Step 1. Rotation of rigid Earth (Euler equations) is solved, under the influence of external torques (Moon, Sun, planets).

Step 2. Reaction of non-rigid parts of the Earth (visco-elastic mantle, fluid outer core, rigid inner core ...) on the external forces is calculated via frequency-dependent transfer function, which is the ratio between the amplitude of nutation and its value for rigid Earth.

Another option is the direct solution of Liouville equations with appropriate Earth model, but this approach (though theoretically more correct) has not led to satisfactory results so far.

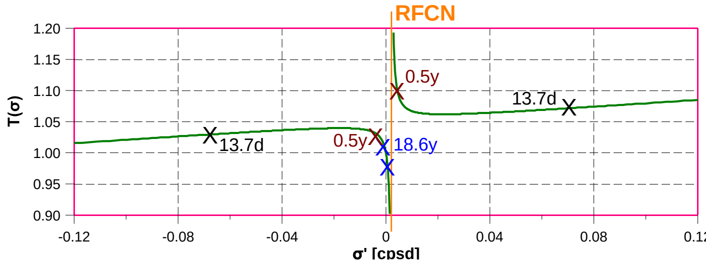

Figure 2: MHB transfer function - real part.

The most recent IAU2000 model of nutation is valid since 2003. It is based on solutions by Souchay et al. (1999) for the rigid Earth and Mathews et al. (2002) for transfer function. Rather complicated Earth model is used; visco-elastic mantle, outer fluid and inner rigid cores, atmosphere, oceans, electromagnetic coupling between outer core and mantle and inner and outer core are considered. The model contains 1360 periodic terms. Corresponding Mathews-Herring-Buffet (MHB) transfer function in complex form, whose numerical parameters were fixed to fit VLBI observations of celestial pole offsets, is

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_\circ \left[1 + (1+\sigma) \left(Q_\circ + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right], \tag{6}$$

where σ is the frequency of nutation (in terrestrial frame), e_R is the dynamical ellipticity of the rigid Earth, N, Q are complex constants, and s_j are complex resonance frequencies, corresponding to: 1. Chandler wobble – CW ($P_{ter.} \doteq 435$d); 2. Retrograde Free Core Nutation – RCFN ($P_{cel.} \doteq 430$d); 3. Prograde Free Core Nutation – PFCN ($P_{cel.} \doteq 1020$d); 4. Inner Core Wobble – ICW ($P_{ter.} \doteq 2400$d).

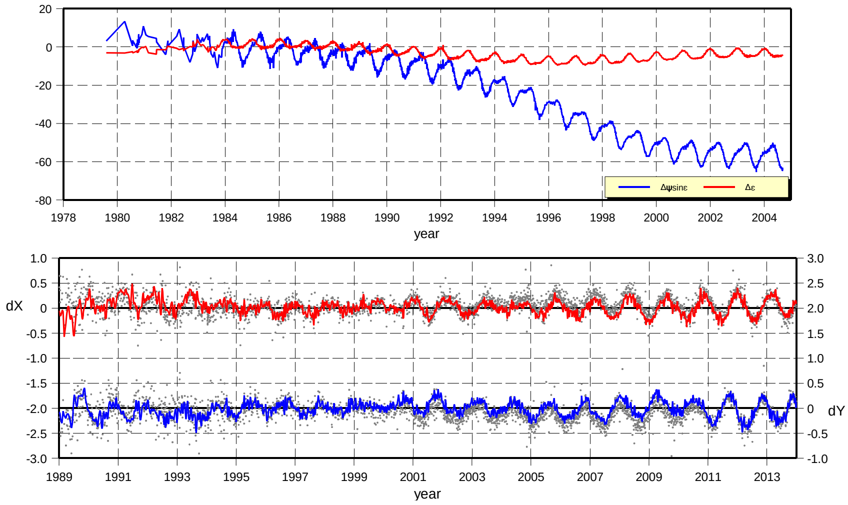

Figure 3: Celestial pole offsets from IAU1980 (top) and IAU2000 (bottom) models.

Graphical representation of MHB transfer function (its real part, corresponding to amplitudes) is displayed in Fig. 2, in which the argument is the frequency σ' in celestial frame (cycles per sidereal day). Important nutation terms are marked with crosses, and the dominant RFCN resonance is shown as a vertical line.

Figure 3 demonstrates how much the recent model of nutation IAU2000 improved the agreement with the observations, when compared with the previous one, IAU1980 (Wahr, 1981). The figure depicts the celestial pole offsets (i.e., the differences of the observations from the adopted model) in milliarcseconds (mas) – notice the difference of vertical scale of both plots. Bottom plot shows both the individual observed values (dots) and the filtered data used in our subsequent calculations (full line) – see Section 4 below. The newest model IAU2000 agrees with observations on the level of only $\pm$0.2mas, the dominant term has a period of about 435-460d, and a variable amplitude of about 0.1mas, corresponding to RFCN which is not included in the nutation model IAU2000. Important is also quasi-seasonal term with a similar amplitude to RFCN, excited by geophysical processes (atmosphere, oceans...), as we shall demonstrate below.

3. GEOPHYSICAL EXCITATIONS OF NUTATION

In order to compute the effect of geophysical excitation, we use numerical integration of Brzeziński's broad-band Liouville equations (Brzeziński 1994) in celestial frame, based on an Earth model that is simpler than the one used by Mathews et al. (2002) – it accounts for only visco-elastic mantle and fluid outer core, and consequently has only two dominant resonances (Chandler and RFCN). It reads, in complex form

$$\begin{aligned}\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f P = &- \sigma_C \left\{\sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w)\right.\\ &+ \left. \mathrm{i}\left[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w\right]\right\}, \qquad (7)\end{aligned}$$

where $P = \mathrm{d}X + \mathrm{i}\mathrm{d}Y$ is the motion of Earth's spin axis in celestial system due to excitation. σ'_C, σ'_f are Chandler and RFCN frequency in celestial frame, σ_C is Chandler frequency in terrestrial frame, χ'_p, χ'_w are the effective angular momentum functions (pressure and wind terms, respectively) in celestial frame, and $a_p = 9.509 \times 10^{-2}$, $a_w = 5.489 \times 10^{-4}$ are numerical constants, expressing different reaction on pressure/wind terms.

The effective angular momentum functions χ are dimensionless quantities expressing the excitations by the atmosphere (oceans), defined by Barnes et al. (1983). χ_p are calculated from air pressure changes measured at Earth's surface, χ_w from the velocity of the wind measured at different altitudes. Here we use only their two equatorial components, expressed as complex quantity $\chi = \chi_1 + \mathrm{i}\chi_2$. Because they are available from meteorological centra in terrestrial frame, they must be transformed into celestial frame, using a simple formula $\chi' = -\chi \mathrm{e}^{\mathrm{i}\phi}$, where ϕ is the Greenwich sidereal time.

Corresponding transfer function (in frequency domain) between excitation and nutation is

$$T_{p,w}(\sigma) = \sigma_C \left(\frac{1}{\sigma'_C - \sigma} + \frac{a_{p,w}}{\sigma'_f - \sigma} \right), \quad P_{p,w}(\sigma) = T_{p,w}(\sigma)\chi'_{p,w}(\sigma). \tag{8}$$

The two resonant frequencies mentioned above, rapid Chandler σ'_C and slow RFCN σ'_f with different response for pressure and wind terms, are evident. Transfer function is a practical tool for comparing the spectrum of geophysical excitations $\chi'(\sigma)$ with the spectrum of celestial pole offsets $P(\sigma)$.

3. 1. NUMERICAL INTEGRATION OF BROAD-BAND LIOUVILLE EQUATIONS

Equation (7) is a second-order differential equation in complex form. In order to facilitate its numerical integration, it is split into two first-order equations, by using a simple substitution $y_1 = P,\ y_2 = \dot{P} - \mathrm{i}\sigma'_C P$. Thus we have a system of two complex differential equations

$$\begin{aligned} \dot{y}_1 &= \mathrm{i}\sigma'_C y_1 + y_2 \\ \dot{y}_2 &= \mathrm{i}\sigma'_f y_2 - \sigma_C \left\{ \sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) \right. \\ &+ \left. \mathrm{i}\left[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w\right] \right\}, \end{aligned} \tag{9}$$

which we numerically integrate by using fourth-order Runge-Kutta method with 6-hour steps. We use the Fortran subroutine `rk4` (Press et al. 1992) that we re-wrote into complex form. Initial values $y_1(0) = P(0)$ and $y_2(0) = \mathrm{i}(\sigma'_f - \sigma'_C)P(0)$ are chosen so that the quasi-diurnal free motion disappears, and the best rms fit to observations is obtained. It is necessary to say that the choice of initial pole position influences only the amplitude and phase of RFCN; the forced motion remains intact by the choice.

4. DATA USED AND RESULTS

4. 1. DATA USED

In our recent study (Vondrák and Ron 2014) we compared different sources of geophysical excitations (European ECMWF for the atmosphere and OMCT for the oceans,

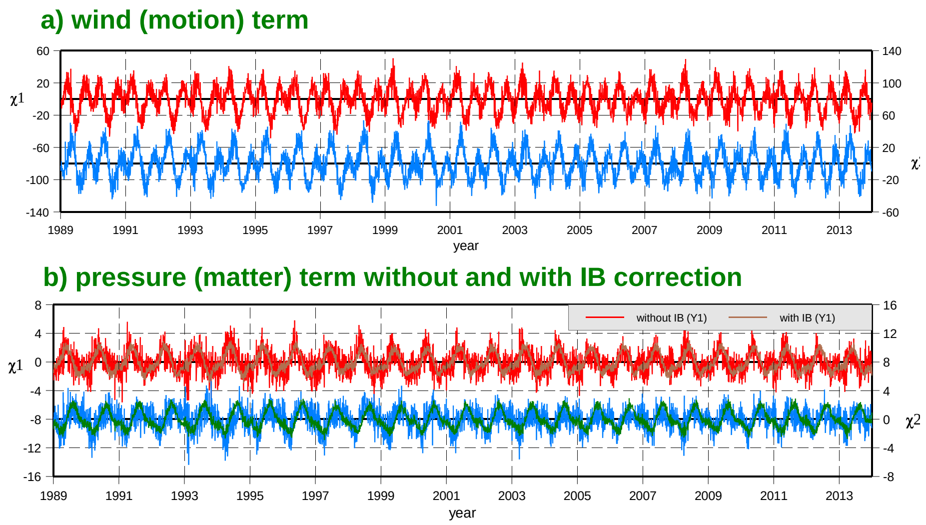

Figure 4: NCEP effective angular momentum functions.

American NCEP/NCAR for the atmosphere with and without Inverted Barometer – IB correction) and found that the best fit with VLBI-based observations of celestial pole offsets is obtained for NCEP/NCAR excitations with IB correction. European models yield systematically larger amplitudes, if compared with the observations. Consequently, we show in this paper only the results based on NCEP/NCAR atmospheric excitations in 1989.0–2014.0, given in terrestrial frame in 6-hour intervals. Prior to their use, they were re-calculated into celestial reference system, and smoothed (Vondrák 1977) to contain only periods longer than 10 days. The input data are shown in Fig. 4.

The integrated values are then compared with the observed values of celestial pole offsets, provided by International VLBI Service for Geodesy and Astrometry (IVS) as a combination from all their analysis centers, solution `ivs13q4X`, covering the same time interval, i.e. 1989.0-2014.0. These data are given in unequal intervals (from 1 to 7 days), so they were first filtered to contain only periods between 60 and 6000 days, and then interpolated to ten-day equidistant epochs. IAU2000 nutation model contains an empirical term with annual period (Sun-synchronous correction) that is supposed to account for the effects of geophysical fluids. In order to be directly comparable with integrated geophysical excitations, this term was removed from the celestial pole offsets. The input data are displayed in bottom plot of Fig. 3.

Recently Malkin (2013) showed that the changes of amplitude and phase of RFCN occur near the epochs of geomagnetic jerks (GMJ). GMJ are rapid changes of the second time derivative of intensity of the Earth's magnetic field, typically lasting from several months to a year (Mandea et al. 2010). We tested this in our recent study (Vondrák & Ron 2014) by re-initializing the numerical integration at the epochs of GMJ and found that the agreement with observations improved significantly. Here we use slightly different approach, since sudden stepwise changes of pole position are physically not acceptable. Instead, we use a continuous additional excitation of

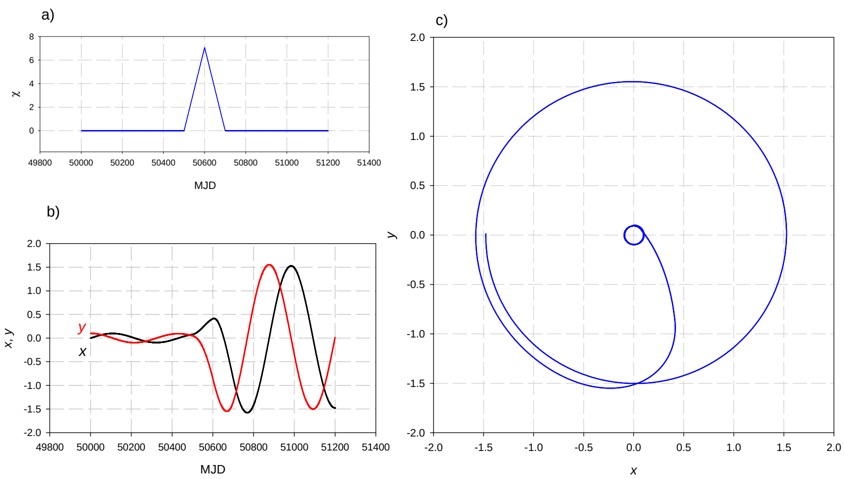

Figure 5: Schematic excitation and its effect in integrated pole position.

'double ramp' (or triangle) shape, centered at GMJ epochs and lasting 200 days. This simulated schematic excitation and its calculated effect on celestial pole position is depicted in Fig. 5; excitation a) causes continuous growth of the amplitude and change of phase during the 200 days covering the excitatio n, as shown in plots b) and c). Here we use the fixed GMJ epochs 1991.0, 1994.0, 1999.0, 2003.5, 2004.7 and 2007.5, taken over by Malkin (2013). Only their complex amplitudes are estimated to fit best to the observed celestial pole offsets.

4. 2. RESULTS

The results of numerical integrations and their fit to observed values are shown in Figs 6 and 7; GMJ epochs in Fig. 7 are marked by arrows. Fig. 6 displays significant differences, both in phase and amplitude, large values of rms fit and low correlations reflect this fact. The improvement of the fit when GMJ excitations are added, both in rms and correlation, between the two series is evident in Fig. 7. Also the solution with IB correction yields better agreement, in lower parts of both figures.

If we make a least-square fit to derive the complex amplitudes of annual and semi-annual terms from integrated values of Fig. 7, we get the results summarized in Tab. 1. The arguments of both terms are identical with those of nutation terms (l' for annual, $2F - 2D + 2\Omega$ for semi-annual), corresponding periods are 365.26 and 182.62 days, respectively. For comparison, the same terms obtained from the fit to IVS celestial pole offsets and the term that is the part of IAU2000 nutation model are shown in the lower part of the table. All these values mutually agree on the level of several tens of microarcseconds.

5. CONCLUSIONS

We demonstrate that the geophysical effects in nutation are significant and now measurable. Best agreement of integrated excitations with observed celestial pole offsets is obtained if NCEP/NCAR atmospheric angular momentum functions with Inverted

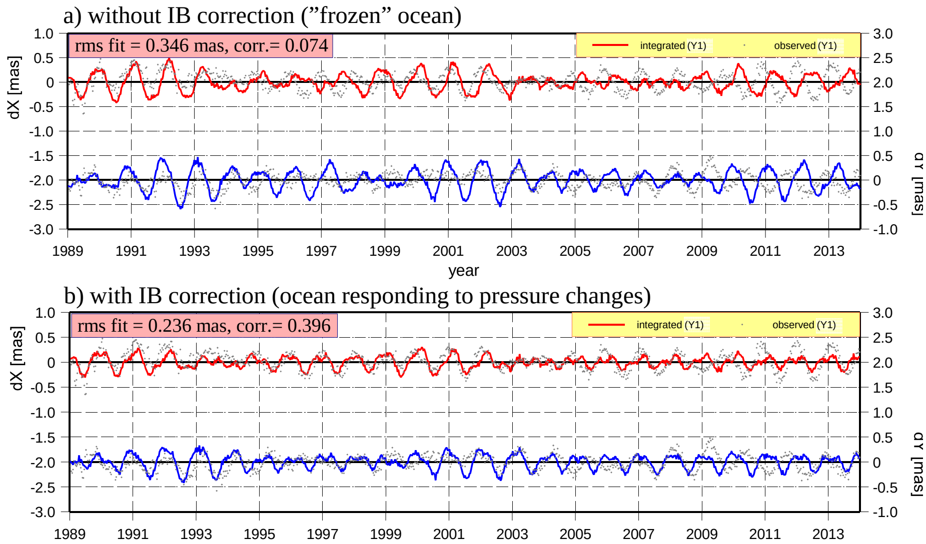

Figure 6: Integrated nutation with NCEP excitation.

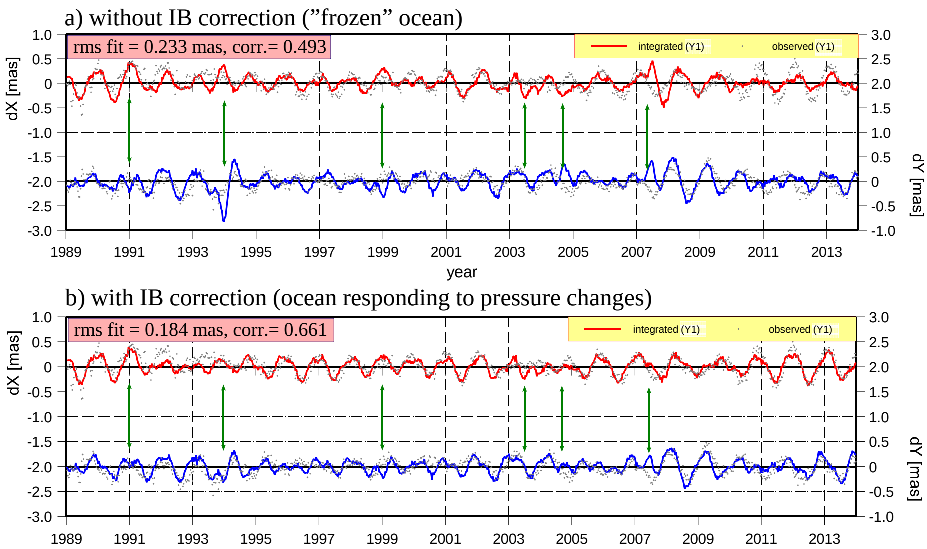

Figure 7: Integrated nutation with NCEP + GMJ excitation.

Table 1: Seasonal geophysical effects in nutation [μas]

solution	annual				semi-annual			
	prograde		retrograde		prograde		retrograde	
	Re	Im	Re	Im	Re	Im	Re	Im
NCEP	−16	+81	−52	−8	−24	+50	0	−10
NCEP IB	−47	+90	−43	+37	−8	+69	0	−6
IVS	−17	+96	−11	+50	+3	+31	−16	−24
Sun-Synchr.	−10	+108	0	0	0	0	0	0

Barometer correction are used, and the fit is further improved if additional excitations at the epochs of geomagnetic jerks are added. However, we do not offer physical explanation of the mechanism how GMJ could lead to the changes in nutation, we only demonstrate here that there is a remarkable coincidence between the two phenomena.

Acknowledgment

This study was made thanks to the grant No. 13-15943S, awarded by the Grant Agency of the Czech Republic.

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