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RADIO INTERFEROMETRIC ARRAY - SCMA

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Abstract. This paper gives the analysis and the solution to a radio interferometric array - SCMA (Serbian Centimeter/Millimeter Array). SCMA will be made of three experimental dish antennas, with an aim to study the high energy processes mainly in the Solar atmosphere, by application of detection and localization in the microwave frequency range (1-50 GHz). System design is modeled as e-VLBI configuration, providing the maximum angular resolution of $\approx 0\overset{''}{.}004$. We simulate interferometric observations using models of idealized radio sources. The corresponding data were reconstructed using the CLEAN algorithm. The research shows that SCMA observing efficiency of non-thermal, thermal and line emissions of Galactic and cosmic scale sources is theoretically affirmed.

1. INTRODUCTION

The radio interferometers are commonly classified into two types depending on baseline dimension (see e.g. Rohlfs et al., 2009). Very Long Baseline Interferometers (VLBI or e-VLBI, for example VLBA and EVN) are characterized with very high angular resolution (long baselines ~1000 km), which results in longer integration times due to lower sensitivity. VLBI technique is intended for observations of very bright extragalactic and large cosmic scale objects. Second stream comprises rather compact antenna arrays (such as VLA or now EVLA, ALMA, ATCA, etc.) with short baselines ~$(0.01 - 100)$ km. Despite their lower (relative to VLBI) angular resolution, higher sensitivity enables them to resolve galactic and surrounding sources morphology in a matter of minutes, and sometimes seconds, reffered to as snapshots.

Creation of the hybrid type experimental solution could provide equilibrium state between observing requests, satisfying both Galactic and deep field radio imaging. Even at low frequencies (e.g., 1.42 GHz), array with baselines of multiplets order of 100 km provides imaging at resolution comparable to that of a compact array observing at high frequencies. On the other hand, the higher frequency band combined with the mid-range baselines, gives a good trade-off between sensitivity and resolution, improving the first within acceptable frame of the second. Presented solution to radio interferometric array is therefore complementary to both types of array.

We now consider some theoretical background, involving elementary radio antenna and interferometry equations, and then apply it in the case of three components array (SCMA).

2. THEORY FUNDAMENTALS AND ARRAY MODELING

For single dish (parabolic antenna), resolution and sensitivity are the main effective characteristics, that directly define behaviour of an array as coherent system and constrain its performance. Considering field of view of an array or single dish resolution (main beam), one can make use of the FNBW (First Null Beam Width):

$$\theta_{\mathrm{FNBW}} = 1.22 \cdot \frac{\lambda}{D}, \tag{1}$$

where D represents the antenna diameter and λ the wavelength of observed emission, meaning that signals from within a circle of angular radius that equals FNBW (with respect to pointing direction), will mostly contribute to overall power detected.

Sensitivity of antenna is determined relative to size of effective aperture which defines amount of flux collected by antenna dish, and Johnson-Nyquist noise temperature of receiver itself. Sky and ambient temperatures are outer and technology independant factors. The source signal power is included in overall sky emission. The system temperature then represents contribution from all components: receiver, sky and ambient (Rohlfs et al., 2009).

Relative to single antenna sensitivity, array equation is additionaly modified by the number of possible antenna pairs (baselines) and correlator efficiency (Yatagai et al., 2011):

$$\sigma_T = \frac{\rho \cdot T_{\mathrm{sys}}}{\eta_{\mathrm{c}} \sqrt{N(N-1) n_{\mathrm{p}} \tau_{\mathrm{int}} \Delta\nu}}, \quad T_{\mathrm{sys}} = T_{\mathrm{rx}} + T_{\mathrm{sky}} + T_{\mathrm{amb}}. \tag{2}$$

The minimum stochastic source signal power (σ_T) detectable by array depends on the system temperature T_{sys}, the used frequency bandwidth $\Delta\nu$, the number of polarizations n_{p} and the integration time τ_{int}; the constant ρ equals $\frac{2k}{A_{\mathrm{eff}}}$, where A_{eff} is the effective aperture of single antenna and k is the Boltzmann constant, N is the number of array components and η_{c} is the correlator efficiency. The switching technique used to eliminate receiver noise contribution, requires additional factor of $\sqrt{2}$ to be included. Interferometric array sensitivity is indeed $\sqrt{(N-1)/2}$ times higher than sensitivity of all antennas working together in a single dish mode. Making the use of equation (2), and considering receiver technology constraints (Brown et al., 2000, Vila Vilaro, 2011, McKinnon et al., 2006), SCMA sensitivity is calculated and presented in table 1.

Table 1: SCMA continuum and line sensitivity of array at different band frequencies, assuming total observing time of $\tau_{\mathrm{obs}} = 12^{\mathrm{h}}$ and on-source integration time of $\tau_{\mathrm{int}} = 6^{\mathrm{h}}$

ν (GHz)	T_{sys} (K)	$\sigma_{\mathrm{continuum}}$ (mJy)	$\Delta\nu$ (GHz)	σ_{line} (mJy)	Δv (km/s)
43	152	0.17	4.0	28.3	1
23.7	96	0.11	4.0	24.6	1
10	71	0.11	2.0	19.8	2
5	60	0.09	2.0	15.0	5
1.72	45	0.10	1.0	19.2	5

The atmospheric emission contribution to T_{sky} is simulated by *am* code (Paine, 2011). Ambient temperature is assumed to be 290 K and antenna diameter 10 m. The aperture efficiency η_{a} is calculated using Ruze formula and is fixed at 0.70.

Completely different approach is used to describe interferometric array resolution and formation of 2D images. It is the general case of complex (continual and discrete) process called aperture synthesis. Antennas are described as points, but in reality they are spread by wavefront diffraction, that consequently results in convolution of the single dish power distribution A_ν and function of antenna positions, refered to as the synthetised beam B_ν,

$$B_\nu^{\mathrm{c}} = A_\nu * B_\nu. \tag{3}$$

It is obvious that array corresponds to unfilled aperture, and discreteness is what makes imaging possible. Cross-correlation coefficients of antenna pair signals are then observable, and their values are used in image reconstruction by application of DSP techniques (Smith, 1997), especially Fast Fourier Transform (FFT). In theory, two domains are defined; they correspond to derivation that the source brightness distribution function is sampled in a Fourier domain via all array baselines as sampling points, which in reverse corresponds to convolution of given functions in spatial domain, according to convolution theorem (see, e.g., Rohlfs, 2009, Urošević & Milogradov-Turin, 2007):

$$I_\nu^{\mathrm{D}}(l,m) = I_\nu(l,m) * B_\nu^{\mathrm{c}}(l,m) = \iint V_\nu(u,v) S_\nu^{\mathrm{c}}(u,v)\ e^{2\pi i(ul+vm)}\ dldm. \tag{4}$$

Column integrated spectral source brightness distribution $I_\nu(l,m)$ can be considered as coming from the plane defined by (l,m) coordinates, refering to as an image plane. Array baselines are distributed and oriented over Earth's surface, that is the Fourier plane domain, also known as $u-v$ plane. When considering long baselines (VLBI), the third coordinate w must be accounted for in addition to (u,v) domain. Function $B_\nu^{\mathrm{c}}(l,m)$ represents the dirty or synthetised beam, that is the response of interferometric array to the point source in a far field approximation (Fraunhofer diffraction), refered to as Point Spread Function (PSF). Functions $V_\nu(u,v)$ and $S_\nu^{\mathrm{c}}(u,v)$ are visibility, and $u-v$ coverage, both refering to as Fourier transform of $I_\nu(l,m)$ and $B_\nu^{\mathrm{c}}(l,m)$ respectively. Observing process starts with antenna signal detection, continuing with correlation of antenna pair signals and appropriate mapping in time, in relation to the baseline orientation. The acquired data consist of samples in the $u-v$ plane. Greater integration time results in a better $u-v$ coverage and higher sensitivity (2). Earth's rotation technique is commonly used in boosting the quality of imaging. Samples are discrete in time (and amplitude), and Fourier transform backs it in the image plane providing the dirty image $I_\nu^{\mathrm{D}}(l,m)$ of observed source brightness distribution (4).

System solution to SCMA is presented in Figure 1, refering to as e-VLBI type array with non-real time streaming of digitized baseband signals via RF link toward the correlator, where data processing and storage take place. Array will consist of the three 10 m Cassegrain antennas, with Alt-Azimuth mounting, distributed along 35.21° inclined north-south direction. Three possible baseline lengths of an array are equal to 100, 200 and 300 km, which makes the discrete Fourier transform more efficient. Single baseline resolution is calculated similar to (1):

$$\theta_{\mathrm{sb}} \approx \frac{\lambda}{b}, \tag{5}$$

where b now stands for the baseline length, and θ_{sb} refers to FNBW angle of the synthetized beam. Its inverse value represents spatial frequency in the $u - v$ plane, acting as selective set of very narrow bandpass Finit Impulse Response (FIR) filters. This means that only spatial structures within narrow bands of angular sizes relative to (5) will be shown on the dirty image.

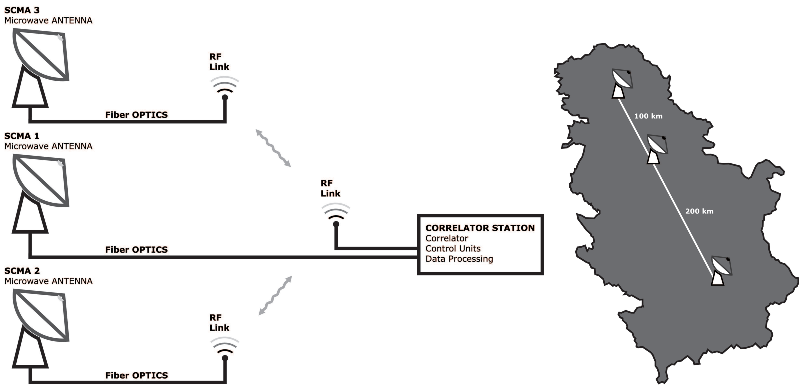

Figure 1: System diagram and distribution of SCMA.

3. SIMULATIONS

Concerning theoretical background derived in a previous section, simulations are done by numericaly solving the equation (4) using FFT and image processing algorithms (Smith, 1997, Press et al., 2007). The $u - v$ plane coverage is formed by making use of equation (5) in relation to hour angle range of observed object. Declination is kept fixed at 90° for all simulations. Models used in simulations are idealized and noise contribution is not considered. Figure 2 shows FFT spectrum of simulated disc source, and projected baselines (filtered components) or $u - v$ plane coverage, for observing period t = (-4,+4) h, at 1.42 GHz frequency.

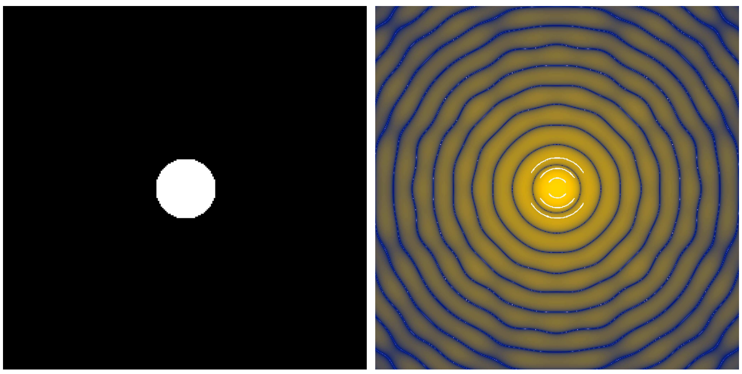

Figure 2: Simulated disc source (left) and its FFT spectra with $u - v$ plane coverage (right). Source image size is $0\overset{\prime}{.}1 \times 0\overset{\prime}{.}1$ and observing frequency is 1.42 GHz.

In Figure 3, the dirty image is shown and power dissipation is evident. Deconvolving of the dirty image through ~400 loops of CLEAN algorithm (Högbom, 1974), provides image that highly matches to the simulated source.

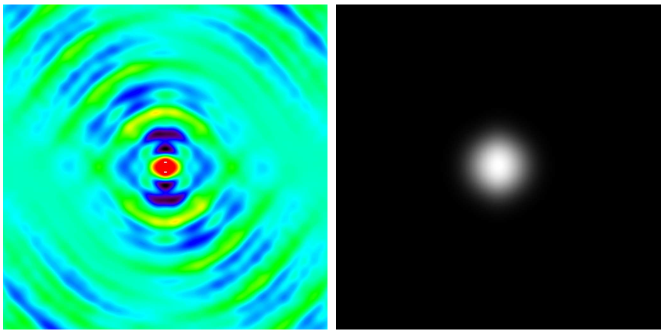

Figure 3: The dirty image (left) and deconvolved clean image (right). Image size is $0'.1 \times 0'.1$ and observing frequency 1.42 GHz.

We now consider some realistic source by making the use of Cassiopeia A model, but keeping its size in consistence with SNRs in M82, of about $0''.25$ (≈ 4 pc) in diameter (Figure 4).

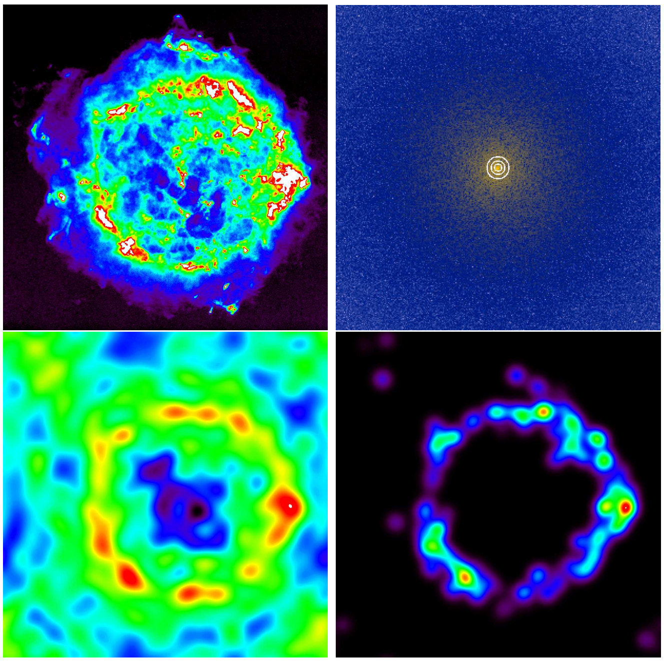

Figure 4: Cass A source model (top-left), $u - v$ coverage (top-right), dirty (bottom-left) and clean (bottom-right) images respectively. Image size is $0''.25 \times 0''.25$, hour angle $t = (-6, +6)$h and frequency 10 GHz.

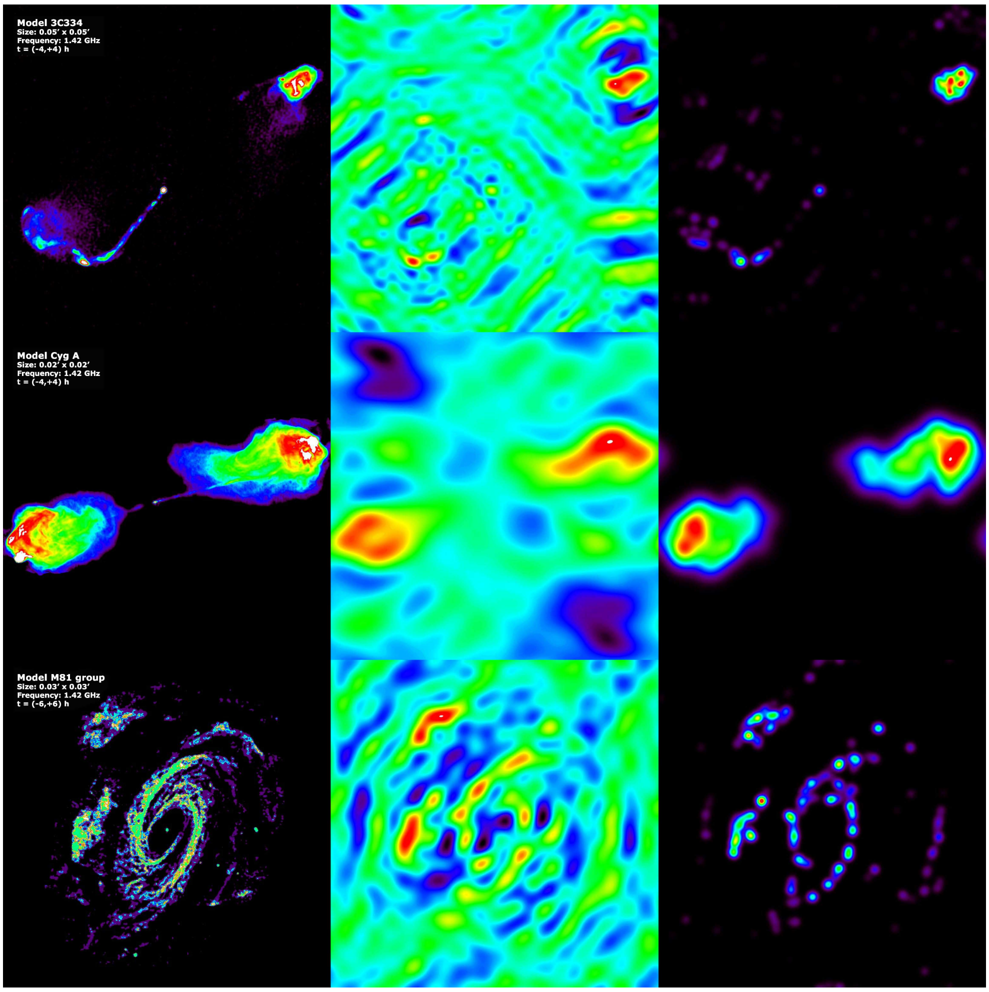

Figure 5: Simulation results of cosmic scale morphology imaging, at 1.42 GHz and less then half a day observing time. Left column contains source models, center and right columns represent dirty and clean images respectively.

Simulation results of some extragalactic source models with different angular size at 1.42 GHz frequency, are presented in Figure 5.

4. DISCUSSION AND CONCLUSIONS

The solution presented here for the potential applicability of SCMA is used for further development of experimental base related to the prospective European Union (IPA) funded project, which carrier is University of Belgrade. Referring to system performance of the array and simulations done, it can be concluded that this experimental, hybrid type solution to a radio interferometric array, should provide balance between observing requests. Instead of cosmology scale radio imaging in great details at centimeter band, this solution could satisfy similar quality of imaging as a trade-off between sensitivity and resolution. Combination of mosaic imaging technique and receiver multichannel parallel signal acquisition and correlation ability, should make this simple array far more efficient in resolving the small scale structure of Galactic and Solar system objects, at small costs of increase of integration time. Once again, we would like to emphasize that our analysis is related to the idealized simulations, with neglecting of noise influence in the system. Due to this, our results are at the significantly higher level (in a sense of image quality) than they would be appeared in practice. By the other hand, taking into account that the projected baseline on the sky changes with time, which means that each baseline describes an elliptic curve in the $u-v$ plane and therefore provides a better $u-v$ coverage, the results are expected to be improved.

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