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EFFECTS OF RADIATION ON CONVECTIVE INSTABILITY IN STRATIFIED STELLAR PLASMAS

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Abstract. The effect of black body radiation on convective stability in stellar plasmas is twofold. It enters analytical expressions for adiabatic processes and for plasma density distribution in hydrostatic equilibrium. We derive and analyze the condition for onset of convective instability assuming small adiabatic displacements of fluid particles in stratified plasma with prescribed model temperature profiles.

1. INTRODUCTION

Stellar plasma is subject to gravitational stratification as well as to radiation forces arising from the radiative energy flux emanating from the region where the essential nuclear reactions are taking place. The resulting temperature profile in a plasma medium follows from processes included and described by the radiative transfer equations which are not the aim of this presentation. In what follows, we assume a given temperature model profile which is taken locally linear and with temperature decreasing with the distance from the zone of nuclear energy generation.

The mere existence of a temperature gradient in gravitationally stratified plasma, initially in static equilibrium, yields a possibility for convective instability to set in which initiates macroscopic turbulent motions that play important role in stellar structure. In particular, convective motions are the essence of dynamo mechanisms that generate magnetic fields with a variety of consequences they have in stellar dynamics, these motions also change the type and intensity of thermal energy transport toward the surface of a star and have influence the differential rotation of a star. It is therefore important to investigate the conditions for instability to appear taking into account contributions of radiative fluxes in gas dynamics. In this sense, we derive the instability criterion and discuss its consequences in some typical stellar plasma environments.

2. EQUILIBRIUM STATE

The considered plasma is assumed a perfect gas of uniform composition in external gravitational field meaning that self-gravitation effects are not included in our treatment. In other words, the gravitational acceleration is either locally constant or it has the inverse-distance-squared dependence as will be assumed in this contribution. For the sake of simplicity, we ignore macroscopic global rotations of the plasma system that can then be taken spherically symmetric and initially in hydrostatic equilibrium. The only forces acting upon plasma are thus the gravitational force and forces due to radiation taken as a black body radiation in an optically thick medium.

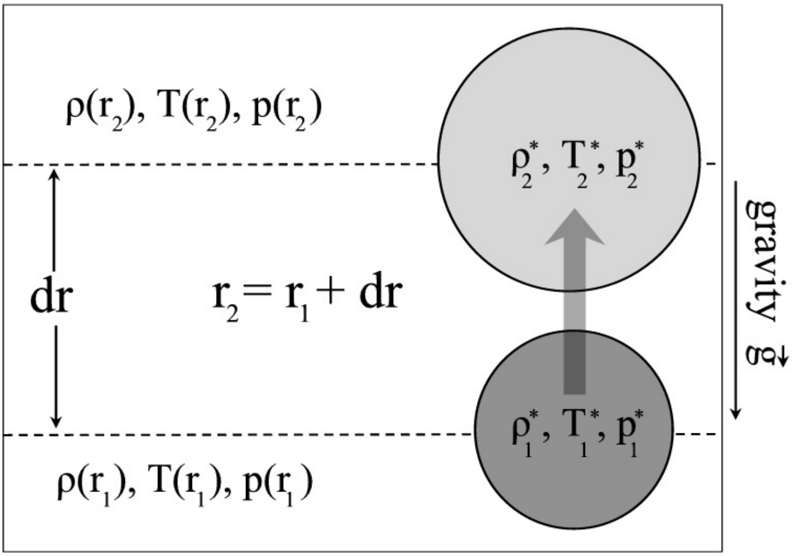

Figure 1: Schematic view of vertical displacement of a distinct fluid parcel causing changes of its thermodynamic properties.

The initial hydrostatic equilibrium is therefore given by:

$$\frac{dp(r)}{dr} + \rho(r)g(r) = 0 \quad \text{where:} \quad p(r) = \rho R_M T(r) + \frac{a_R T^4(r)}{3}$$

$$\frac{\rho(r)}{dr} + \left[\frac{g(r)}{R_M T(r)} + \frac{1}{T(r)}\frac{dT(r)}{dr}\right]\rho(r) + \frac{4a_R}{3}\frac{T^2(r)}{R_M}\frac{dT(r)}{dr} = 0. \qquad (1)$$

Here, the physical constants have their standard meanings and the expression for gravitational acceleration g(r) relative to some reference level r_0 is taken as given bellow:

$$g(r) = \frac{g(r_0) r_0^2}{r^2}, \quad R_M = \frac{R}{A}, \quad M = A\,\mathrm{m}_1,$$

$$R = 8.31\ \times \mathrm{JK}^{-1}\mathrm{mol}^{-1}, \quad A = 6.022\ \times 10^{23}\mathrm{mol}^{-1}$$

$$\sigma = 5.67\ \times 10^{-8}\mathrm{Wm}^{-2}\mathrm{K}^{-4}, \quad a_R = \frac{4\sigma}{c} = 7.56\ \times 10^{-16}\,\mathrm{Jm}^{-3}\mathrm{K}^{-4}.$$

The density distribution for a prescribed temperature profile T(r) now follows from Eq. (1) whose analytical solution has the following form:

$$\rho(r) = \mathrm{e}^{-\int_{r_0}^{r}\left[\frac{dT(\xi)}{d\xi} + \frac{g(\xi)}{R_M T(\xi)}\right]d\xi} \times \left\{\rho(r_0) - \int_{r_0}^{r} \frac{4a_R T^2(\xi)}{R_M}\frac{dT(\xi)}{d\xi}\mathrm{e}^{\int_{r_0}^{\xi}\left[\frac{dT(\xi')}{d\xi'} + \frac{g(\xi')}{R_M T(\xi')}\right]d\xi'}\right\} \tag{2}$$

The above expression clearly indicates a less steep density decrease with *r* due to the black-body radiation force in a plasma medium with a typical negative temperature gradient. In terms of potential energy, this means that such a radiation tends to increases the potential energy of the system which makes it less stable.

3. CONVECTIVE INSTABILITY CRITERION

Now, we briefly derive a criterion for convective instability for stratified plasma in a black-body radiation field. To do this, let us consider a blob of perfect gas displaced radially by *dr* in the outward direction with respect of the central region of a star where the major fraction of its mass is located (domain $r<r_0$). During the displacement, the parcel's volume is taken to expand adiabatically with its pressure $p^*(r)$ balanced by the pressure $p(r)$ of the ambient gas as the boundary condition:

$$p(r) = p^\star(r) \quad \rightarrow \quad \frac{dp^\star(r)}{dr} = \frac{dp(r)}{dr}. \tag{3}$$

At the same time, the parcel's density $\rho^*(r)$ differs from $\rho(r)$ of the surroundings while the parcel is being displaced (Fig. 1). This difference is given by:

$$\Delta\rho \equiv \rho^\star(r_2) - \rho(r_2) = \left[\frac{d\rho^\star(r)}{dr} - \frac{d\rho(r)}{dr}\right] dr \tag{4}$$

as $\rho^*(r_1) = \rho(r_1)$ at the starting position. The same argument holds also for temperature changes. The sign of $\Delta\rho$ now determines the local stability of the medium so that the inequality $\Delta\rho < 0$ represents *the local convective instability criterion*.

Finally, to obtain the instability criterion Eq. (4) explicitly, it is necessary to derive expressions for adiabatic changes of thermodynamic quantities, i.e. for the derivative $d\rho^*/dr$ in Eq. (4). Starting from the definition of an adiabatic process in general (Shih-I Pai 1966):

$$\delta Q \equiv \left(\frac{\partial E}{\partial T}\right) dT + \left(\frac{\partial E}{\partial \rho}\right) d\rho + pd\left[\frac{1}{\rho(r)}\right] = 0$$

and taking into account the presence of the radiation filed meaning that the internal energy E and the total pressure (gas and radiation) p are given by:

$$E = c_V T(r) + \frac{a_R}{\rho(r)} T^4(r); \quad p(r) = \rho(r) R_M T(r) + \frac{a_R}{3} T^4(r)$$

we obtain for the total pressure and density variation within the fluid parcel:

$$\frac{dp^\star}{dr} - \Gamma(r)\frac{p^\star(r)}{\rho^\star(r)}\frac{d\rho^\star}{dr} = 0 \tag{5}$$

which reduces the instability criterion Eq. (4) to the following form:

$$\frac{dT(r)}{dr} \quad < \quad -\frac{g(r)}{R_M}\frac{b(r)[\Gamma(r) - b(r)]}{[4 - 3b(r)]\Gamma(r)}$$

$$\Gamma(r) = b(r) + \frac{(\gamma - 1)[4 - 3b(r)]^2}{b(r) + 12(\gamma - 1)[1 - b(r)]}, \quad b(r) = \frac{3\rho(r)R_M}{a_R T^3(r) + 3\rho(r)R_M}, \quad \gamma = \frac{c_p}{c_v}.$$

We see that this criterion becomes the known Schwarzchild condition if $b=1$, i.e. when radiation effects are negligible. In the opposite case when radiation forces dominate, i.e. for $b=0$, all negative temperature gradients yield convective instability meaning that a strong radiation field destabilizes the considered system. More discussion on the instability criterion for intermediate conditions $0<b<1$ will be presented at the conference.

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References

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