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ORBIT DETERMINATION AND PARAMETER ESTIMATION: EXTENDED KALMAN FILTER (EKF) VERSUS LEAST SQUARES ORBIT DETERMINATION (LSQOD)

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Abstract. In the age of intensive exploring of the solar system, many professionals and non-professionals become interested in calculating the basic data regarding the solar system planets. We have considered some concepts of the physical ephemeris calculation for the natural and artificial solar system bodies. As an effective result, during a conference session, we presented an oral explanation of an interactive program for practical calculation of the physical ephemeris of the planets as a problem of general interest. As a specific example, in this article the readers can find the theoretical and practical elements and procedure explanation for two useful methods of the satellite orbit determination: LSQOD and EKF.

1. INTRODUCTION

The precise determination of a spacecraft orbit is a necessary condition for carrying out any spaceflight mission, such as, performing orbital transfer, orbital rendez-vous, gravitational fly-by, atmospheric reentry, etc.

There are a lot of preconditions that should be satisfied, before any orbit calculation can be done. The most important are: choosing an appropriate coordinate system and applying an adequate theoretical and practical model.

2. CHOICE OF THE COORDINATE SYSTEM

It is important to chose an appropriate coordinate system because an optimal choice allows easier determination of the most important parameters and makes the calculation process more easy. For example, tracking of a geostationary satellite in the Earth-rotating equatorial coordinate system enables treating of any changes in the coordinates as orbital disturbances that should be corrected by some on-board action. On the other hand, tracking of an interplanetary probe in that coordinate system does not have much sense. Therefore, the choice of the coordinate system depends mostly on the orbit type and the type of results that are required. Generally, coordinate systems can be classified according to several principles. The first one is based on the position of the coordinate system center. In the case of Earth-orbiting spacecrafts, they can be divided into two groups: geocentric and spacecraft centered. Furthermore, geocentric coordinate systems can be inertial and non-inertial often connected with the Earth rotation. The inertial coordinate systems are suitable for calculation of an orbital maneuver while non-inertial coordinate systems are used for determination of the spacecraft position with respect to a ground site. Most commonly used inertial geocentric coordinate systems are equatorial ones.

For the purpose of determining the satellite orbit the horizontal coordinate system is most commonly used because any ground measurement of the position of a satellite usually has azimuth, elevation and range as the output.

3. CHOICE OF THE MATHEMATICAL MODEL

The choice of an appropriate mathematical model depends on two things:

- accuracy level that is required,
- resource and time consuming limitations.

Depending on the accuracy level that is required, a number of important parameters can be included in the model: atmospheric drag, solar pressure, Earth thermal radiation pressure, gravitational anomalies, third body perturbations etc. On the other hand, the choice of the mathematical model, is also influenced by the resource and time consuming limitations. Nowadays, there is a lot of space missions that include spacecrafts of very small and limited masses (femto satellites have only up to 100 grams) and every gram is very important and expensive. Time consuming problems appear because there are many situations when it is necessary to make decisions very fast and there is no much time for a lot of calculation. Therefore, in a particular case, it is necessary to find an optimal relation between these two opposed conditions.

4. METHODS FOR ORBIT DETERMINATION

Two methods for orbit determination are presented here: least square orbit determination (LSQOD) and extended Kalman filter (EKF) (Vallado 2007). The main difference between these two methods is the fact that LSQOD gives the orbit prediction after evaluation of an entire group of measurements while the extended Kalman filter gives the improved prediction after evaluation of every new measurement.

4. 1. LEAST SQUARES ORBIT DETERMINATION (LSQOD)

In the least squares method there are three quantities of interest:

- True value
- Measured value
- Estimated value

In mathematical notation

$$\begin{array}{ccll} \tilde{y} & = [\tilde{y_1}...\tilde{y_n}]^T & - \text{measured } y \text{ values} \\ f(x) & = [f_1...f_n]^T & - \text{independent functions} \\ x & = [x_1...x_n]^T & - \text{true } x \text{ values} \\ \nu & = [\nu_1...\nu_n]^T & - \text{measurement errors} \\ \hat{y} & = [\hat{y_1}...\hat{y_n}]^T & - \text{estimated } y \text{ values} \\ e & = [e_1...e_n]^T & - \text{residual errors} \\ \hat{x} & = [\hat{x_1}...\hat{x_n}]^T & - \text{estimated } y \text{ values} \end{array} \qquad (1)$$

The goal of the least squares method is to find the minimum of the loss function

$$J = e^T e = [\tilde{y} - f(\hat{x})]^T W [\tilde{y} - f(\hat{x})] \tag{2}$$

where W is a weight matrix. The algorithm for orbit determination using least squares method is shown in the figure bellow.

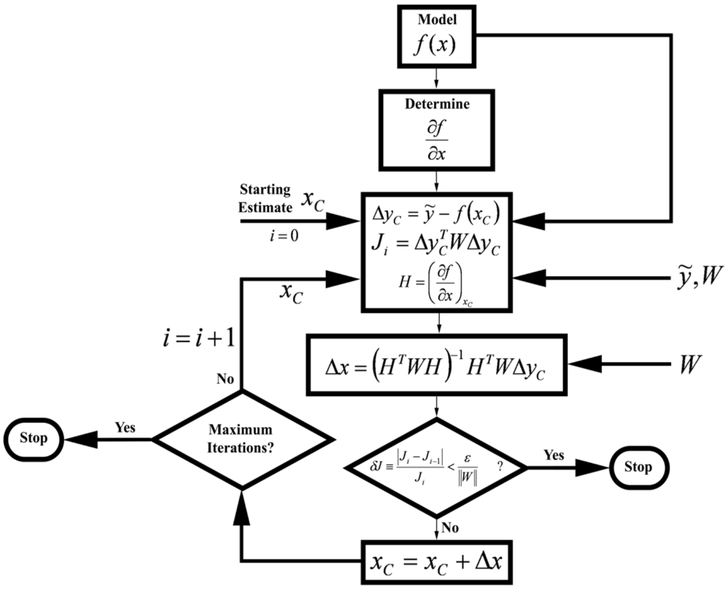

Figure 1: LSQOD algorithm.

As you can see, there are two possible criteria for termination of the calculation. The first one is when the convergence criterion is reached, so that Δx is small enough to satisfy our starting condition, and the second one is when the previously specified maximum number of iterations is reached.

4. 2. EXTENDED KALMAN FILTER (EKF)

The Kalman filter was introduced in 1960 by Rudolf E. Kalman. It is a set of mathematical equations that provide an efficient computational (recursive) means to estimate the state of a process, from a series of noisy measurements, in a way that minimizes the mean squared error. The Kalman filter model assumes that the true state at a time k is evolved from the state at $(k-1)$. This means that only the estimated state from the previous time step and the current measurement are needed to compute an estimate for the current state. In contrast to batch estimation techniques, no history of observations and/or estimates are required.

The basic Kalman filter is limited to the assumption of linearity. However, most non-trivial systems are non-linear. The non-linearity can be associated either with the process model or with the observation model or with both. In the extended Kalman filter, the state transition and observation models need not to be linear functions of the state, so the EKF is a non-linear version of the Kalman filter. In mathematical notation

$$\begin{array}{lrcl}
\text{Model} & \dot{x}(t) & = & f(x(t),u(t),t)+G(t)w(t), w(t)\sim N(0,Q(t)) \\
 & \tilde{y}_k & = & h(x_k)+\nu_k, \nu_k \sim N(0,R_k) \\
\text{Initialize} & \hat{x}(t_o) & = & \hat{x}_o \\
 & P_o & = & E\{\tilde{x}(t_o)\tilde{x}^T(t_o)\} \\
\text{Gain} & K_k & = & P_k^- H_k^T(x_k^-)[H_k(\hat{x}_k^-)P_k^- H_k^T(\hat{x}_k^-)+R_k]^{-1} \\
 & H_k(x_k^-) & \equiv & \left(\frac{\partial h}{\partial x}\right)_{\hat{x}(t)} \\
\text{Update} & \hat{x}_k^+ & = & \hat{x}_k^- + K_k[\tilde{y}_k - h(\hat{x}_k^-)] \\
 & P_k^+ & = & [I-K_k H_k(\hat{x}_k^-)]P_k^- \\
\text{Propagation} & \dot{\hat{x}}(t) & = & f(\hat{x}(t),u(t),t) \\
 & \dot{P}(t) & = & F(\hat{x}(t),t)P(t)+P(t)F^T(\hat{x}(t),t)+G(t)Q(t)G^T \\
 & F(\hat{x}(t),t) & \equiv & \left(\frac{\partial f}{\partial x}\right)_{\hat{x}(t)}
\end{array} \tag{3}$$

The algorithm for orbit determination using the extended Kalman filter is shown in Fig. 2.

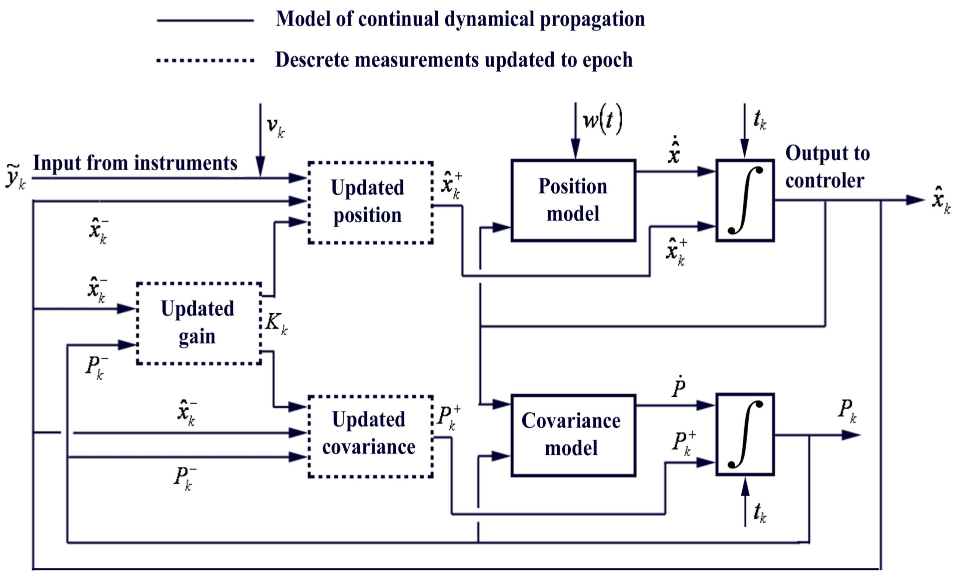

Figure 2: EKF algorithm.

There are two parts of this algorithm. The first part represents discrete measurements updated to the epoch and the second one represents the model of continual dynamical propagation. After every new measurement, the new Kalman gain, updated covariance and updated position are calculated.

Table 1: Coordinates of the HST

UTC	$A[^o]$	$E[^o]$	$\rho[km]$
10:42:00	140.50	-37.07	8577.517
10:44:00	135.49	-39.84	9016.998
10:46:00	130.53	-42.58	9439.921
10:48:00	125.52	-45.28	9840.469
10:50:00	120.39	-47.91	10210.323
10:52:00	115.07	-50.47	10563.606
10:54:00	109.46	-52.93	10881.29
10:56:00	103.50	-55.25	11167.061
10:58:00	97.11	-57.41	11418.971
11:00:00	90.20	59.38	11636.214

Table 2: Convergence history for the LSQOD

	Initial guess	1. iteration	2. iteration	3. iteration
x_o	400	396.141	396.150	396.150
y_o	-6400	-6409.396	-6409.368	-6409.368
z_o	-2670	-2665.134	-2665.139	-2665.139
$\dot{x_o}$	7.1	7.153	7.153	7.153
$\dot{y_o}$	1.3	1.341	1.341	1.341
$\dot{z_o}$	-2.1	-2.128	-2.128	-2.128

5. COMPARISON OF THE METHODS

We analyzed 10 sets of HST (Hubble Space Telescope) coordinates in order to compare the initial conditions estimated by LSQOD and EKF. Since the main goal is to compare these two methods, we have used the simplest mathematical model which includes only the inverse square law in the equation of motion of the satellite.

The coordinates of HST are taken from www.n2yo.com and shown in Table 1.

In Table 2, a dramatic convergence of the LSQOD can be seen after just two iterations.

Table 3 shows the most descriptive characteristics of the EKF where the estimation

Table 3: Convergence history for the EKF

	Initial guess	1	2	9	10
x_o	400	400	393.256	395.691	396.150
y_o	-6400	-6400	-6408.462	-6409.284	-6409.368
z_o	-2670	-2670	-2662.006	-2664.563	-2665.139
$\dot{x_o}$	7.1	7.108	7.164	7.154	7.153
$\dot{y_o}$	1.3	1.267	1.338	1.340	1.341
$\dot{z_o}$	-2.1	-2.071	-2.137	-2.129	-2.128

of the initial conditions is improved after every new measurement, and after the 10th measurement it reaches the same accuracy as LSQOD in 2 iterations.

The most important advantage of EKF against LSQOD is that EKF can use a previous estimation to evaluate a new one for an extended number of observations, while in LSQOD we must find the estimation again from the entire set of measurements. In the case of on-board calculations on strongly weight limited satellites this is of paramount importance.

6. CONCLUSIONS

As shown in the example above, in the case when it is necessary to have an improved estimation after every new measurement, EKF is more suitable because it includes significantly less calculations than LSQOD and thus, it is much faster and has lower resource requirements.

It is important to say that both methods could have problems with numerical stability. A lot of algorithms which include different types of matrix factorization are developed for overcoming this problem. This can make the calculation process more time consuming and thus, increase resource requirements. EKF is more sensitive to this problem because its main advantage over LSQOD is its possibility to work fast and with lower resource load.

Regarding all mentioned above, LSQOD is used for the determination of the passive satellite orbits and for the estimation of the geodetic parameters from satellite orbits. On the other hand, EKF is more suitable for real-time applications because of its possibility of improving the estimation after every new measurement, in a very short time and without need of evaluating all previous measurements. This fact makes it a very useful mathematical tool for on-board calculation on strongly weight limited satellites.

References

Lang, K. R.: 1991, Astrophysical Data: Planets and Stars, ed., Springer, New York.

Seidelmann, P. K.: 1992, Explanatory supplement to the Astronomical Almanac, ed., U.S. Naval Observatory, Washington, D.C.

Šegan, S.: 2006, Račun efemerida, priv. commun.

Vallado, D. A.: 2007, Fundamentals of Astrodynamics and Applications, ed., Springer, New York.