Publ. Astron. Obs. Belgrade No. 86 (2009), 173 - 176 *Contributed paper*

SOME REMARKS CONCERNING THE ACCELERATED ORBITAL MOTION OF CELESTIAL BODIES

I. LUKAČEVIĆ
Department of Astronomy, Faculty of Mathematics, University of Belgrade, Studentski trg 16, 11000 Belgrade, Serbia

Abstract. A general explanation of the accelerated orbital motion of celestial bodies is proposed. That explanation is based on nonstatic solutions, conformally equivalent to basic static ones, within the framework of Rosen's bimetric gravitation theory (a criticized theory). As the Birkhoff's theorem for spherically symmetric gravitational fields, is not valid in that theory, there is a possibility to explain different phenomena of the orbital motion related to close binaries, but also to the secularly accelerated planets of the Solar system.

1. INTRODUCTION

The purpose of this communication is to complete and to extend the results of a paper published many years ago (Lukačević and Ćatović 1992), related to nonstatic gravitational fields in Rosen's bimetric gravitation theory (Lukačević 1986). In particular, an unnecessary assumption will be dismissed.

Rosen's bimetric gravitational theory has been the object of strong criticism, as the consequences of the existence of emission weak gravitational waves were incompatible with it. Nevertheless, there is an alternative possibility offered by the theory, which could, if further developed, be of some use for the explanation of the phenomenon of the accelerated motion of close binaries. To that there are two reasons, first, after many years gravitational waves have not been identified, and second, the nonstatic solutions, conformal to the basic static solution in free space, are connected with the wave equation, i.e. the conformal factor is the exponential function of a solution of the wave equation in the Minkowskian metric.

The possibilities offered by the existence of aforementioned fields, conformal to basic ones, impossible in classical relativity, can also be considered in connection with the secular acceleration of planets in the Solar system. The different explanations of that phenomenon are briefly exposed and commented in a paper of Schroeder and Treder (2008).

2. THE SPHERICALLY SYMMETRIC GRAVITATIONAL FIELD IN ROSEN'S BIMETRIC GRAVITATION THEORY

We shall begin with the Schwarzschild solution of classical relativity

$$\varepsilon ds^2 = \frac{d\rho^2}{1-\frac{2m}{\rho}} + \rho^2(d\vartheta^2 + \sin^2\vartheta d\lambda^2) - (1-\frac{2m}{\rho})c^2dt^2 \tag{1a}$$

$$\varepsilon = \pm 1 \ .$$

Transformed to isotropic coordinates

$$\rho = r(1+\frac{m}{2r})^2$$

that solution reads

$$\varepsilon ds^2 = (1+\frac{m}{2r})^4[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\lambda^2)] - (\frac{2r-m}{2r+m})^2c^2dt^2 \ . \tag{1b}$$

In bimetric gravitation theory the spherically symmetric gravitational field in isotropic coordinates reads

$$\varepsilon ds^2 = e^{\frac{2M}{r}}[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\lambda^2)] - e^{-\frac{2M}{r}}c^2dt^2 \ . \tag{2}$$

It is interesting to note that the metric (2) agrees with the Schwarzschild solution (1b) to the second order terms for the space-like part and to the third order terms for the time-like part of the metric, developed in potential series (Rosen 1979).

The conformally equivalent metric of (2) has the form

$$\varepsilon d\tilde{s}^2 = e^{2(\Phi+Mr^{-1})}[dr^2 + r^2(d\vartheta^2 + sin^2\vartheta d\lambda^2)] - e^{2(\Phi-Mr^{-1})}c^2dt^2 \ . \tag{3}$$

Φ being the solution of the wave equation (Lukačević 1986) in the Minkowskian background metric (the metric of the completely energy-free space-time)

$$\gamma^{\alpha\beta}\Phi_{|\alpha\beta} = 0 \ . \tag{4}$$

If we assume that Φ depends on r and t only as was made in Lukačević and Ćatović (1992) the above equation will take the comparatively simple and well-known form

$$\frac{\partial^2\Phi}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} = 0 \ . \tag{5}$$

Let us assume first the more general case of (4) with $\Phi = \Phi(r,\vartheta,t)$, that is, an axially symmetric field conformally equivalent to the basic spherically symmetric one. If we formulate the differential equations of time-like geodesics (the orbit of celestial bodies in the metric (3))

$$\frac{d}{d\tilde{s}^2}\frac{\partial L}{\partial\dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0 \qquad (\alpha = 1,2,3,4) \tag{6}$$

with the help of the Lagrangian L, defined by (3), we shall at once obtain the "Keplerian" integral

$$e^{2(\Phi+Mr^{-1})}r^2\sin^2\vartheta\frac{d\lambda}{d\tilde{s}}=l=const\ . \tag{7}$$

Using this relation we shall substitute the angular variable λ to the proper time $\tilde{s}$. That substitution is made in order to have, instead of $\tilde{s}$, a geometrically objective quantity related to the central body-gravitational source. The remaining three equations of the system (6) then take the form

$$(f'\equiv\frac{df}{d\lambda})$$

$$r''-2r^{-1}r'^2-2\text{ctg}\vartheta\ r'\ \vartheta'-r\vartheta'^2-4Mr^{-2}e^{-4Mr^{-2}}t'^2-$$

$$-r\sin^2\vartheta-l^{-2}e^{2(\Phi+Mr^{-1})}r^4\sin^4\vartheta(\frac{\partial\Phi}{\partial r}-Mr^{-2})=0 \tag{8}$$

$$\vartheta''-2\text{ctg}\vartheta\ \vartheta'^2-\sin\vartheta\cos\vartheta+l^{-2}e^{2(\Phi+Mr^{-1})}r^2\sin^4\vartheta\frac{\partial\Phi}{\partial\vartheta}=0 \tag{9}$$

$$t''-2r^{-1}(1-2Mr^{-1})r't'-2\text{ctg}\vartheta\ \vartheta't'-c^{-2}l^{-2}r^4\sin^4\vartheta e^{2(\Phi+3Mr^{-1})}\frac{\partial\Phi}{\partial t}=0\ . \tag{10}$$

If we denote by t_s ("static") the solution of (10) in the case $\Phi=0$ and assume that all the variables and their first derivatives have the same values in the static and the nonstatic case at some point of the orbit, we shall obtain, as the condition of accelerated motion

$$t_s''>t''\Leftrightarrow\frac{\partial\Phi}{\partial t}<0 \tag{11}$$

i.e. the decrease of Φ with respect to the coordinate time results in accelerated orbital motion in comparison to the static case. The first condition (11) proceeds from

$$t'=\frac{dt}{d\lambda}=\omega^{-1}\Rightarrow t''=-\frac{1}{\omega^3}\frac{d^2\lambda}{dt^2}\ .$$

Let us make now a similar assumption concerning the radial coordinate r of the body. If the considered celestial body moves in cycles having limited maximal distances from the gravitational source, we shall assume that, under the action of the additional field Φ, the radial distance of the moving body decreases. Then, applying to (10) the same way of reasoning as for (9) and (11), we shall obtain

$$r_s''>r''\Leftrightarrow\frac{\partial\Phi}{\partial r}>Mr^{-2}(1-e^{-2\Phi})\ . \tag{12}$$

It is obvious, from the second of the above inequalities, that $\Phi>0\Rightarrow\partial_r\Phi>0$, but $\Phi\leq 0$ is also compatible with $\partial_r\Phi>0$. The sign of Φ can not be determined from (10), and its decrease with time is necessary only in the domain considered. But, as we have just seen from (12), some minimal conclusions concerning its sign in that domain are related to the condition $r_s''>r''$. The function Φ, being the solution of the wave equation, must vanish at the space-like infinity, so that we also have, in accordance with (12), $\partial_r \Phi_\infty = 0$.

In contrast to (8) and (10), Eq. (9), corresponding to the coordinate ϑ, cannot be directly connected with the acceleration of the orbital motion considered. Nevertheless, its cumulative effect exists and we shall therefore assume that

$$\vartheta''_s > \vartheta'' \Rightarrow \frac{\partial \Phi}{\partial \vartheta} > 0 \tag{13}$$

which is obtained by the same way of reasoning as for (11) and (12). In particular, if we assume the symmetry $\Phi(\vartheta) = \Phi(\pi - \vartheta)$, the amplitude of the orbit with respect to the equatorial plane ($\vartheta = \frac{1}{2}\pi$) will tend to decrease with time.

Assuming that, after some period of activity, ϑ' and r' have decreased together with t', we see that the action of all terms in (10) leads to the further decrease of t'' with respect to t''_s.

All these conclusions are simplified in the case of plane motion with $\vartheta = \frac{\pi}{2}$, but the general conditions of accelerated motion remain unmodified for the variables r and t.

We point out the fact that the existence of a conformal field defined by a function Φ has no effect on the velocity of light, as we have obviously

$$d\tilde{s}^2 = e^{2\Phi} ds^2 = 0 \Leftrightarrow ds^2 = 0 \ . \tag{14}$$

Let us add, in the end, that the preceding conclusions are not incompatible with the phenomena of the secular acceleration of planets in the Solar system and with the very small radial decrease of their orbits. These questions have been explained from a quite different standpoint (Schroeder and Treder 2008), but the nonstatic character of conformally equivalent gravitational fields in Rosen's bimetric gravitation theory does not present difficulty to the explanation of such phenomena.

References

Lukačević, I.: 1986, *Gen. Relativ. Gravit.*, **18**, 923.
Lukačević, I. and Ćatović, Z.: 1992, *Gen. Relativ. Gravit.*, **24**, 827.
Rosen, N.: 1979, *Gen. Relativ. Gravit.*, **10**, 639.
Schroeder, W. and Treder, H. J. private communication.