Publ. Astron. Obs. Belgrade No. 85 (2008), 25 - 30 *Contributed paper*

BASICAL FORMS OF REALISATION OF HEURISTIC FUNCTION OF MATHEMATICS IN PHYSICS AND ASTRONOMY

STOJAN OBRADOVIĆ and SLOBODAN NINKOVIĆ

Visoka škola za vaspitače strukovnih studija, Pivarska bb, 18220 Aleksinac, Serbia
E–mail: stojano@ptt.yu

Astronomical Observatory, Volgina 7, 11060 Belgrade 38, Serbia
E–mail: sninkovic@aob.bg.ac.yu

Abstract. The paper considers the role of mathematics in the process of acquiring new knowledge in physics and astronomy. The basic forms of representing the heuristic function of mathematics at theoretical and empirical levels of knowledge are studied: deducing consequences from the axiomatic system of theory, the method of mathematical hypothesis, "pure" proofs for existence of objects and processes, mathematical modelling, formation of mathematics on internal mathematical principles and the mathematical theory of experiment.

1. INTRODUCTION

The heuristic function of mathematics in physics and astronomy concerns its role in the process of acquiring new knowledges and/or informations on contingently new knowledges. A really new knowledge is defined as a given unit of scientific knowledge satisfying requirements of being scientific and to be, at the moment of its creating, not listed among the scientific knowledges established till that time. If a knowledge is declared as a contingently new, predicaments, systems of predicaments, theories, etc, though formally correct, must be verified, also concerning the contents of objective truth in them. Predicaments, systems of predicaments, mathematical theories, etc, must reflect real processes, states and physical objects of nature if we want the heuristic function of mathematics in physics and astronomy to be realized.

Though extensively present in many disciplines of modern science, mathematics has been developed as an independent scientific branch. Traditionally, it has been conceived as a part of science dealing with notions such as number, quantity and measure. Modern mathematics is characterised by its axiomatic form, abstract character (Bell 1951) and polyvalency. A theory is polyvalent if it "covers" or involves, as special cases, several particular anisomorphic theories. In addition to the abstract character of pure mathematics Kline (1985) also emphasizes the idealisation as its essential property. The idealisation does not appear substantially as a serious deviation from reality. Just because of this mathematics has an enormous importance in physics and astronomy. There are very early attempts aimed at explaining the physical reality by applying mathematics. Very often it was believed that nature was formed following a mathematical concept (Kline 1985). Even Newton followed such line of reasoning (Kline 1985). However, formation of physical theories is a complicated process involving various steps (Bunge 1973). Mathematics expresses the logical structure of physical laws and provides to physical theories a predictional character. It has become a method of examining given natural phenomena, a method of predicting unknown properties and relationships, even new undiscovered objects and processes of nature. A good example coluld be the prediction of positrons and creation and annihilation of positron-electron pairs (Beiser 1981) on the basis of a purely mathematical analysis of the corresponding Dirac equation. Finally, changes in the modern experimental basis of physics have contributed that the mathematical theory of experiment has acquired an important heuristic function.

2. BASICAL FORMS OF REALISATION OF HEURISTIC FUNCTION OF MATHEMATICS

i) Deductive Derivation of Consequences from the Axiom System of Theory

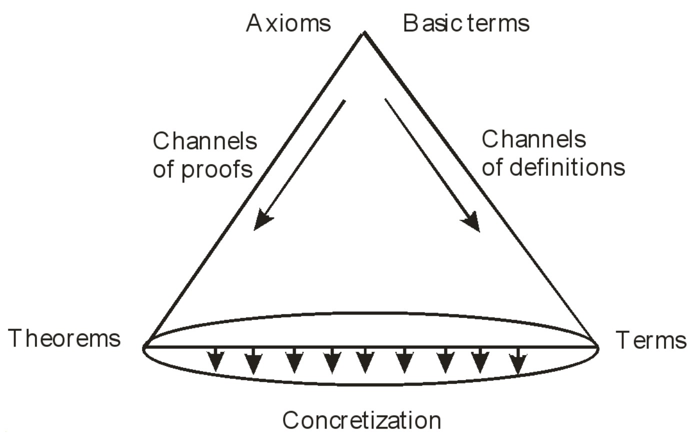

Figure 1: Pyramid of knowledge for deductive theories.

The scientific knowledge organised as a deductive theory can be represented in the form of a pyramid of knowledge as seen in Fig. 1.

The basic terms and axioms of theory (Obradović 2002) are on the top of the pyramid, whereas the derived terms and theorems of theory are below. As seen from Fig. 1, there are two kinds of channels connecting the axioms and basic terms with concretization. In this case one speaks about the direct transfer of truth. If anything false is implanted in the system of axioms of theory, no stream through the deductive channels follows because from a false assumption an arbitrary conclusion can be inferred. Therefore, we may say that there is no direct transfer of false assumptions (Lakatos 1978). The chain of scientific investigations of deductive theories can be represented in the following form: abstraction-deduction-concretisation. In the axiomatico-deductive structure of theory the most important is the derivation of consequences or theorems and the search for their prototypes in objective reality. Assertions following from the system of axioms are subjected to a theoretical and experimental verifications. For instance, the transformations of coordinates and time, relativity of simultaneity, contraction of length and dilatation of time, etc (Beiser 1981) appear as consequences of the principle of relativity and that of constant velocity of light which form the system of axioms in Einstein's special theory of relativity.

ii) Method of Mathematical Hypothesis

The method of mathematical hypothesis is used in discovering a mathematical dependence of phenomena in the process of finding consequences from the conditional, assumed equations and their experimental verification. It appears as an application of the hypothetico-deductive method in physics. A hypothesis is confirmed or rejected, i. e. accepted or not, on the basis of experimental verification of the consequences following from it and other criteria (Popper 1962, Churchman 1956, Whewell 1847, Thagard 1978, Kuhn 1977). If the mathematical predictions and experimental results disagree, the initial equations are varied somewhat arbitrarily, but respecting general rules. Therefore, the initial equations are changed and the consequences are derived to be verified afterwards where all the consequences are required to be true, otherwise, the initial equations are varied again and the whole procedure is repeated.

Due to its heuristic role a mathematical hypothesis should satisfy a number of requirements, among others to be connected with earlier knowledge by involving it as a boundary case. For instance, in the fifties of the XIX century Maxwell (Davies 1984) committed very important discoveries using the principle of symmetry. He unified the the electric and magentic fields into a single, electromagnetic field. This time he introduced an additional term which can be interpreted in the way that a variable electric field begets the magnetic field confirmed by exepriments later on.

iii) "Pure" Proofs for Existence of Objects and Processes

By applying mathematical methods it is possible to predict the corresponding physical objects and their properties, as well as the processes in nature. For instance, by applying the group theory and principles of symmetry (Davies 1984) in physics of elementary particles Gell-Mann predicted the existence of Ω^- hyperon and its properties in 1962 (Burcham 1972). Futher on an important constructive extension of Ω^- hyperons detection was given by Barnes (1964). Later on, in 1964, Ω^- hyperon was detected in a bubble chamber. Also, Russian mathematician Friedmann (Grujić 2002) published in 1922 his dynamical solution of Einstein's equation of General Relativity where he predicted an expanding Universe afterwards confirmed observationally.

iv) Mathematical Modelling

In the process of mathematical modelling the distinction can be done among the following steps: to establish elements, properties and relationships in the modelled original of the primary and secondary importance: to form in thoughts the abstract image of the modelled original; to find the mathematical reflection of the abstract image; to reach conclusions formally by using methods of mathematics and/or formal logic; to interprete and evaluate the conclusions following from the formal derivations.

In this way the built mathematical models serve for the purpose of conquering new fields compared to the modelled extramathematical phenomena. Laws formulated mathematically have the character of a model. They also contain scientific extrapolations and interpolations. The interpolation and extrapolation indicate the possibility of individual relations involving quantities for which the mere existence should be proved only. An example may be the model of fractal cosmos (Mandelbrot 1982, Grujić 2001, Grujić 2002).

v) Formation of Mathematical Theories on Intramathematical Base

Mathematics can be built on the basis of practical problems or, fully on an intramathematical base. Mathematical theories formed in another way, for which at the moment of their formation there are no extramathematical, extralogical and, first of all, no objectively real prototypes, can find, many years later on, important extramathematical spheres of their application. Examples of special interest concern the application of group theory in physics of elementary particles (Kane 1987), application of Riemann's geometry in the general relativity (Kline 1985), application of the p-adic analysis in the Non-Archimedean theories (Vladimirov et al. 1994), etc.

3. HEURISTIC FUNCTION OF MATHEMATICAL THEORY OF EXPERIMENT

The experimental base of modern physics is followed both by improving measuring devices and by developing a new theoretical base where a special importance belongs to probabilistico-statistical methods.

Formation of the mathematical theory of experiment has taken place due to the complexity of scientifico-experimental studies of the microworld, submicroworld and megaworld (Grujić 2007). The application of the mathematical theory of experiment is aimed at reducing the duration of experiments and their complexity and at improving epistemological possibilities, economics and rentability of experiments. Examples are the experiments undertaken at CERN and astronomical observations large telescopes.

All the time by the first quarter of the XX century mathematics, in the form of mathematical statistics, had been mostly applied in the phase of treating the experimental facts. Other phases were mostly on the level of experimenter's intuition. The mathematical theory of experiment is a synthesis of experimental methods in science based on the ideas and methods of mathematical statistics and cybernetics, as well as on the possibilities of experiment facilities. The experimenting with mathematical models has given rise to the mathematical experiment which is essentially analogous to the traditional epistemological method "hypothesis-deduction-experiment". The mathematical experiment is applied in complex systems having a difusional type of organization and non-linear dependences among the basic parameters. Such systems are characterized by a large number of arguments, without possibility to indicate the factors of primary importance and to eliminate those of secondary one. Factors of primary and secondary importance are taken into account by means of the measure of scattering for random variables, i. e. dispersion.

4. CONCLUSION

Mathematics appears as an essential component of physical theory and that of physical experiment. It expresses the logical structure of laws of physics and provides the predicting character to theories in natural sciences (physics, astronomy, etc). Mathematics appears as an important tool in studying nature. In physics mathematics appears as a method of examining natural phenomena and of predicting unknown properties and relationships and new undiscovered objects and processes in nature. In the development of modern physics the role of mathematics becomes more significant. Thie best confirmation is the appearance of mathematical physics as a new scientific discipline.

Acknowledgements

For one of us (SN) this research has been supported by the Serbian Ministry of Science (Project No 146022 " History and Epistemology of Natural Sciences").

References

Barnes, V. E.: 1964, *Phys. Rev. Lett.*, **12**, 204.
Bell, E. T.: 1951, *Mathematics, Queen and Servant of Science*, New York: McGraw-Hill, ch. 2.
Beiser, A.: 1981, *Concepts of Modern Physics*, Thrid Editon, McGraw-Hill, International Book Company, pp. 64, 211, 476, 490-491.
Bunge, M.: 1973, *Philosophy of Physics*, Dordrecht: Kluwer, ch. 7.
Burch am, W. E.: 1972, *Nuclear Physics-An Introduction*, London: Longman Group Limited, ch. 21.
Churchman, C. W.: 1956, *Phi. Sci.*, **23**, 248.
Davies, P.: 1984, *Superforce*, New York: Simon & Schuster, ch. 4.
Grujic, P. V.: 2001, *Serb. Astron. J.*, **163**, 21.
Grujic P. V.: 2002, *Serb. Astron. J.*, **165**, 45.
Grujic P. V.: 2007, *Foundations of Science*, **12**, 39.
Kane, G.: 1987, *Modern Elementary Particle Physics*, Michigan: University of Michigan, Addison-Wesley.
Kline, M.: 1985, *Mathematics, and the Search for Knowledge*, New York: Oxford University Press, chs. 2, 5, 12.
Kuhn, T.: 1977, *Objectivity, Value Judgment, and Theory Choice, in Kuhn, The Essential Tension*, Chicago and London: University of Chicago Press, pp. 320-339.
Lakatos, I.: 1978, *Mathematics, Science and Epistemolgy (Lakatos Philosophical papers)*, Vol. 2, Cambridge University Press, pp. 3-24.

Mandelbrot, B. B.: 1982, *The Fractal Geometry of Nature*, (New York: W H Freeman and Co.).
Obradovic, S.: 2002, *Eur. J. Phys.*, **23**, 270.
Popper, K. R.: 1962, (1945), *The Open Society and its Enemies*, Vol. II (London: Routledge & Kegan Paul) 380.
Thagard, P. R.: 1978, *J. Phi.*, Vol. LXXV, No. 2: pp. 81 & 85.
Vladimirov, V. S., Volovich, I. V. and Zelenov, E. I.: 1994, *p-Adic Analysis and Mathematical Physics*, (Singapore: World Scientific).
Whewell, W.: 1847, *The Philosophy of the Inductive Sciences*, Vol. II (London), pp. 67-68.