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SOME ASPECTS OF INCLUDING NON-ZERO INITIAL MOMENTA INTO IONIZATION OF ATOMS BY STRONG LOW-FREQUENCY LASER FIELDS

V. M. RISTIĆ, T. B. MILADINOVIĆ and M. M. RADULOVIĆ

Department of Physics, Faculty of Science, Kragujevac University,
R. Domanovića 12, Kragujevac 34000, Serbia
E–mail: ristic@kg.ac.yu

Abstract. Including non-zero initial momenta for ejected electrons in strong infrared laser fields is further developed [compare Ristić et al. 2007]. It has been stressed that, apart from being natural, the including non-zero initial momenta enables one to go into deeper analysis of the process of tunnel ionization of atoms in strong laser fields (intensity up to 10^{16} W/cm^2). It is indicated that all electrons that could be ejected, under the circumstances, are ejected at field intensity $\sim 10^{13}$ W/cm^2, and that the effect of ionization after that is strongly diminished, which can be seen from the slope of the plates on Fig. 2. This, also, explains the saturation effect for the fields up to 10^{16} W/cm^2 (Ristić et al. 2006, Ristić and Stefanović 2007, Ristić et al. 1998, Milosevic et al. 2002), and probably this saturation goes on until the fields raising relativistic effects $\sim 10^{18}$ W/cm^2 (Milosevic et al. 2002). Opposite to what was believed earlier (Milosevic et al. 2002), that atomic field intensities could be raised to values over 10^{17}W/cm^2 only when more than 10 electrons are ejected from the atom, it is shown that, properly calculated, ionization of 9 electrons raises the atomic field intensity to $\sim 10^{18}$W/cm^2.

1. INTRODUCTION

There are several approaches to the problem of multiphoton ionization, and especially to the tunneling regime, when the low frequency lasers are involved (Ristić et al. 2006). But, to our opinion, the closest to the phenomenological picture which underlies theoretical model is the one that leans on assumptions based on the Keldysh approximation (Keldysh 1965). First, that the internal potential of atoms does not affect the energy of the final state of ejected electron, when it leaves the atom, because the electron is far enough from the nucleus (the short-range potential). Second, that the potential of the external field does not influence the initial energy of the electron (for this the external laser field should be smaller than the atomic field intensity $\sim 10^{16}$ W/cm^2). Thus the main effect of the external field was the speeding up of ionized electrons. The next step was to treat the Coulomb potential of the electromagnetic field as a perturbation of the final state energy, which was the essential in the ADK-theory (Ammosov et al. 1986). Yet, when constructing ADK-theory, the Coulomb interaction was not included into calculations of the turning point τsg which, when revised, lead to the corrected ADK-theory, or cADK-theory (Ristić et al. 2006, Ristić and Stefanović 2007, Ristić et al. 2005, 1998, Milosevic et al. 2002). But it was always assumed that the ionized electrons are leaving the atom with zero initial momenta, which is not a natural assumption. In paper Ristić et al. (2007, see also references thereof) we were interested in how the non-zero initial momentum influences the transition probability of tunnel ionization. Now, using more precise expression for the momentum of ejected electrons (Bauer 2002), we are discussing results that emerged during this new research: the downshift of the probability maximum, its dependence on momenta of ejected electrons and above all the indicative result that gives one the insight into the process of tunnel ionization of atoms in the strong laser field (up to 10^{16} W/cm^2), see comment under Fig. 2.

2. CALCULATING NON-ZERO INITIAL MOMENTUM AND THE TRANSITION PROBABILITIES

Now we shall obtain the exact expression for the momentum that electron possesses when leaving the atom. In order to do this, we shall introduce parabolic coordinates $\xi = r + z$, $\eta = r - z$, $\phi = \text{arctg}(y/x)$. Atomic unit system $e = \hbar = m_e = 1$ will be used throughout this paper. So following (Bauer 2002), momentum of ejected electrons is given as

$$\mathrm{p}\left(\eta\right) = \frac{1}{2}\left(\sqrt{\mathrm{F}\eta - 1} + \frac{1}{\eta\sqrt{\mathrm{F}\eta - 1}} + ...\right) \quad \text{outside barrier } \eta > \frac{1}{\mathrm{F}}. \tag{1}$$

It is obvious that $\eta_L = 1/F$ is a certain limit depending on field intensity (atomic unit system): $\eta_L = 1/F[10^{12}] = 185.455$. We have chosen as the lowest field intensity of 10^{12} W/cm^2, at which we shall begin our evaluations of the transition rate for ejected electrons from potassium atoms in a strong field of a CO_2 − laser.

First we shall analyze the dependence of momenta of ejected electrons on the coordinate η. It can be seen from Fig. 1 that momentum of the ejected electrons is gaining in its strength as the intensity of the field is rising. Obviously the stronger the laser field is, the more energy is transferred to ejected electrons.

In (Ristić et al. 2006, Ristić and Stefanović 2007, Ristić et al. 2005) it was shown that the transition rate in cADK case is given by expression

$$W_{\text{cADK}} = \left[\frac{4Z^3 e}{Fn^{*4}}\frac{1}{1 + \frac{2ZF}{(p^2+2E_i)^2} + \frac{Z^2F^2}{2E_i(p^2+2E_i)^3}}\right]^{2n*-1} \exp\left(-\frac{2Z^3}{3Fn^*}\right); \tag{2}$$

in Ristić et al. (2007) also was obtained the transition rate for cADK with the correction for the non-zero initial momenta

$$W_{\text{pcADK}} = \left[\frac{4Z^3 e}{Fn^{*4}}\frac{1}{1 + \frac{2ZF}{(p^2+2E_i)^2} + \frac{Z^2F^2}{2E_i(p^2+2E_i)^3}}\right]^{2n^*-1} \exp\left(-\frac{2Z^3}{3Fn^*} - \frac{p^2\gamma^3}{3\omega}\right). \tag{3}$$

Thus the two transition rates, given by (2) and (3), when plotted together on a 3D graph for fields 10^{12}-10^{16} W/cm^2, and for η ranging from 185-585, arbitrary units for W, produce following drawing

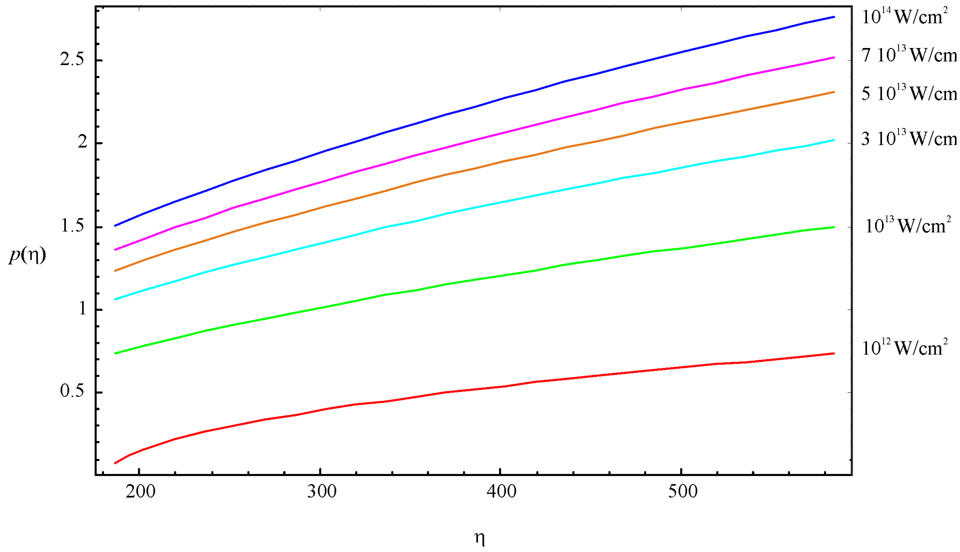

Figure 1: Momentum plotted against $\eta = 185 - 586$.

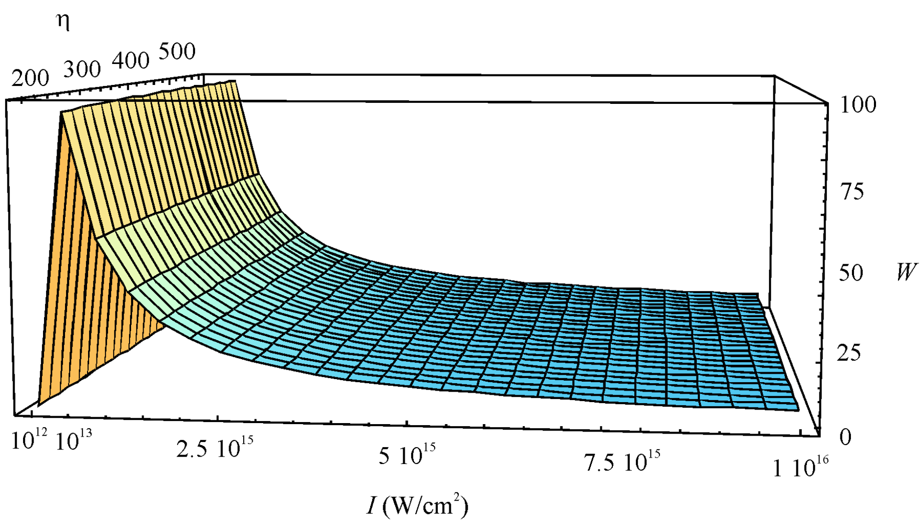

Figure 2: W_{cADK} and W_{pcADK} transition rates plotted together. The scale for the field intensities is not linear.

Fig. 2 gives us opportunity to understand better the phenomenology of the process of tunnel ionization. Notice that the scale on Fig. 2 for the field intensities is not linear.

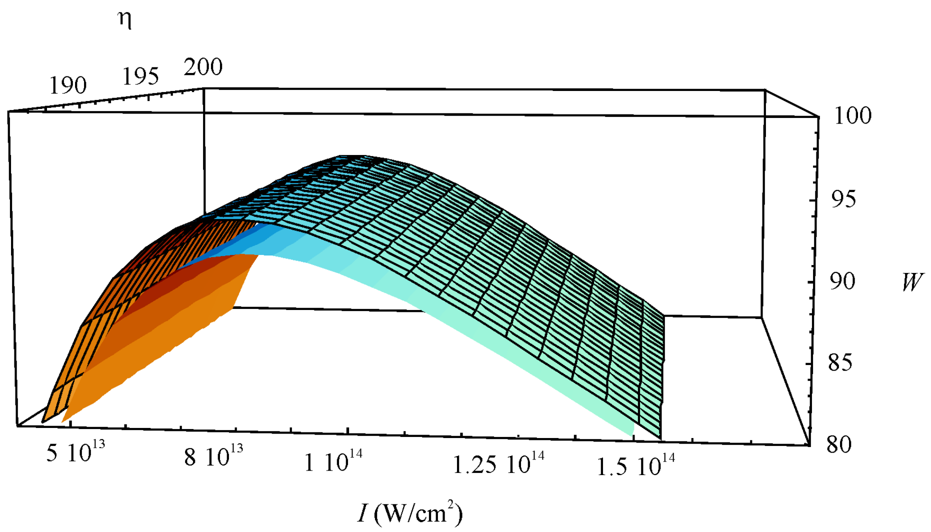

Figure 3: The peak of W_{cADK} and W_{pcADK} transition rates plotted together. Lower curved plate represents W_{pcADK}.

Fig. 3 is plotted for the same objects but in the range that shows only the peak of the graph on Fig. 2 and makes the differences obvious. So at the laser field intensity $\sim 10^{13} W/cm^2$, the transition rate has a maximum, which indicates that all electrons available are ejected. In the case of potassium, in the low frequency strong field of CO_2 laser, it is 1 valent electron assumingly the 8 electrons of the first closed shell, which makes 9 available electrons for ionization. with, i.e. for 10^{12}-10^{16}W/cm^2. Their depletion leads to freeing electrical charges of the atomic nuclei, making intensity of its electrical field of the order of magnitude 10^{18} W/cm^2. This enables us to use cADK-theory in the whole range of field intensities we are working

Close examination of Fig. 3 tells us that the transition rate in the W_{pcADK} case is a bit smaller then in the W_{cADK} case. This maximum was calculated to give values: $W^{\max}_{\mathrm{cADK}} = 7,9 \cdot 10^{13}\mathrm{W/cm}^2$, $W^{\max}_{pcADK} = 8,2 \cdot 10^{13}$ W/cm^2, and the differences of the two follow from the effect of transferring the energy of quanta from the laser beam to the momentum gain of ejected electrons.

It could be expected that this saturation goes further on, until the relativistic effects emerge at field intensities of $\sim 10^{18}$ W/cm^2 (Ristić et al. 1998).

3. FINAL REMARKS

We shall end this by reminding the reader of our analysis of Fig. 2, i.e. that at the laser field intensity $\sim 10^{13}$ W/cm^2, the transition rate has a maximum, which indicates that the most of available electrons are ejected. In the case of potassium in the low frequency strong field of CO_2 laser, which we have chosen as typical case, there are 9 electrons that, after being depleted, are releasing the electrical charges of the atomic nuclei, thus resulting in electrical field of the order of magnitude 10^{18} W/cm^2, opposite to what was believed earlier, that atomic fields could be raised to values over 10^{17} W/cm^2 only when more than 10 electrons are ejected from the atom, so we can use cADK-theory for the whole range of field intensities we are working with, i.e. for 10^{12}-10^{16} W/cm^2).

As mentioned earlier, it was always assumed that the ionized electrons are leaving the atom with zero initial momenta. This assumption, being unnatural, has forced us to examine how the non-zero momentum influences the transition probability of tunnel ionization, and we are discussing results that emerged in this new research: the downshift of the probability maximum, its dependence on momenta of ejected electrons. Also, we discuss the saturation effect during ionization of potassium atoms by a low frequency field of CO_2 laser [see comment after Figs. 3 and 4].

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