Publ. Astron. Obs. Belgrade No. 84 (2008), 95 - 98 *Contributed Paper*

TEMPORAL RELAXATION OF ELECTRON SWARMS: THE EFFECTS OF NON-CONSERVATIVE COLLISIONS AND ANGLE BETWEEN THE ELECTRIC AND MAGNETIC FIELDS

S. DUJKO[1,2], R. D. WHITE[2], K. F. NESS[2] and Z. Lj. PETROVIĆ[1]

[1]*Institute of Physics, University of Belgrade, Pregrevica 118, Zemun, Serbia*
E–mail: sasa.dujko@jcu.edu.au

[2]*ARC Centre for Antimatter-Matter Studies, School of Mathematics, Physics and IT, James Cook University, Townsville 4811, QLD, Australia*

Abstract. A multi-term solution of the Boltzmann equation has been developed and used to investigate the temporal relaxation of charged-particle swarms under the influence of electric and magnetic fields crossed at arbitrary angles. We present results for the ionization model of Lucas and Saelee highlighting the explicit influence of the electric and magnetic field strengths, angle between the fields and non-conservative collisions on temporal relaxation characteristics, including the existence of transiently negative electron diffusivity.

1. INTRODUCTION

Studies of transport processes of a swarm of charged particles in neutral gases under the influence of electric and magnetic fields crossed at arbitrary angle is a topic of great interest both as a problem in basic physics and for its potential for application to modern technology. One of the major challenges in these investigations is an accurate representation of temporal relaxation of charged-particle swarms. Relaxation processes of a swarm of charged particles are related to various problems of gaseous electronics such as modeling of non-equilibrium plasma discharges, high-speed switching technique, swarm physics and physics of gas lasers (see Winkler et al. 2002). The knowledge of temporal relaxation is essential for a better understanding of electron-molecule interaction as well as for a better understanding of transient transport phenomena in gases such as transient negative electron mobility (Warman et al. 1985) or transient negative electron diffusivity (White et al. 2008). In addition, to fully appreciate the complex structure of the transport properties in radio-frequency (rf) electric and magnetic fields, a systematic investigation of the temporal relaxation of a swarm of charged particles in dc electric and magnetic fields is required.

We begin this paper with a brief review of multi term theory for solving the Boltzmann equation valid for both electrons and ions in time-dependent electric and magnetic fields. We investigate the temporal relaxation of electron swarms and focus on the effects of non-conservative collisions and angle between the electric and magnetic fields on temporal relaxation profiles of both the *bulk* and *flux* electron transport properties.

2. THORETICAL METHOD

The behavior of charged particles in neutral gases under the influence of electric and magnetic fields is described by the phase-space distribution function $f(\vec{r}, \vec{c}, t)$ representing the solution of the Boltzmann equation

$$\frac{\partial f}{\partial t} + \vec{c} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{q}{m}\left[\vec{E}(t) + \vec{c} \times \vec{B}(t)\right]\frac{\partial f}{\partial \vec{c}} = -J(f, f_o) \; , \tag{1}$$

where $\vec{r}$ and $\vec{c}$ denote the position and velocity co-ordinates while q and m are the charge and mass of the swarm particle and t is the time. The right-hand side of $J(f, f_o)$, denotes the linear charged particle-neutral molecule collision operator, accounting for elastic, inelastic, and non-conservative (e.g. ionizing or attaching) collisions. The electric and magnetic fields are assumed to be spatially homogeneous and time-dependent. In what follows, we employ a co-ordinate system in which $\vec{E}$ defines the z-axis while $\vec{B}$ lies in the $y - z$ plane, making an angle ψ with respect to the $\vec{E}$.

In the present approach equation (1) is solved by decomposing $f(\vec{r}, \vec{c}, t)$ in terms of spherical harmonics in velocity space and powers of the gradient operator acting on the number density $n(\vec{r}, t)$ in real space, i.e

$$f(\vec{r}, \vec{c}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{s=0}^{\infty} \sum_{\lambda=0}^{s} \sum_{\mu=-\lambda}^{\lambda} f(lm|s\lambda\mu; c, t) Y_m^{[l]}(\hat{\mathbf{c}}) G_\mu^{(s\lambda)} n(\vec{r}, t) \; , \tag{2}$$

where $Y_m^{[l]}(\hat{\mathbf{c}})$ denotes the spherical harmonics and $G_\mu^{(s\lambda)}$ denotes the sth application of the gradient operator in irreducible tensor notation. The speed dependence is treated as follows:

$$f(lm|s\lambda\mu; c, t) = \omega(\alpha(t), c) \sum_{\nu=0}^{\infty} F(\nu lm|s\lambda\mu; \alpha(t), t) R_{\nu l}(\alpha(t)c) \; , \tag{3}$$

where $\alpha(t)^2 = m/kT_b(t)$ and $\omega(\alpha(t), c)$ is a Maxwellian,

$$R_{\nu l}(\alpha(t)c) = N_{\nu l}\left[\frac{\alpha c}{\sqrt{(2)}}\right]^l S_{l+\frac{1}{2}}^{(\nu)}(\alpha(t)^2 c^2/2) \tag{4}$$

$$N_{\nu l}^2 = \frac{2\pi^{3/2}\nu!}{\Gamma(\nu + l + 3/2)} \; , \tag{5}$$

and $S_{l+\frac{1}{2}}^{(\nu)}$ is a Sonine polynomial. Using the above decompositions of f and implicit finite difference evaluation of time derivatives, the Boltzmann equation is transformed into a hierarchy of doubly infinite coupled inhomogeneous matrix equations for the time-dependent moments. Finite truncation of both the Sonine polynomial and spherical harmonic expansions permits solution of this hierarchy by direct numerical inversion. Having obtained the moments, the transport coefficients and other transport properties may be calculated.

3. RESULTS AND DISCUSSION

In this section we consider the effect of the angle between the electric and magnetic fields on temporal relaxation of the electron transport properties under hydrodynamic conditions. Similar studies have been published previously using the two term approximation for solving the Boltzmann equation (see Loffhagen et al. 1999) and multi term theory for solving the conservative Boltzmann equation (White et al. 2008). We extend these studies by: (i) overcoming the inherent inaccuracies of the two term approximation; (ii) addressing the temporal relaxation of spatial inhomogeneities through a study of the diffusion tensor and (iii) highlighting the explicit effects of non- conservative collisions on temporal relaxation profiles of various transport properties. We consider the temporal relaxation processes of the electrons for the ionization model of Lucas and Saelee (see Lucas and Saelee (1975)). The initial conditions represent the steady state magnetic field free case where the electron swarm is acted on solely by a dc electric field. At time $t = 0$, a magnetic field is switched on while the electric field is left unaltered. The relaxation properties of the electron swarm is monitored as a function of normalized time (Nt). The relaxation process is followed until the steady- state is reached. We consider the electric field strength of 10 Td (1 Td = 10^{-21} Vm^2) and the reduced magnetic field strength of 500 Hx (1 Hx = 10^{-27} Tm^{-3}).

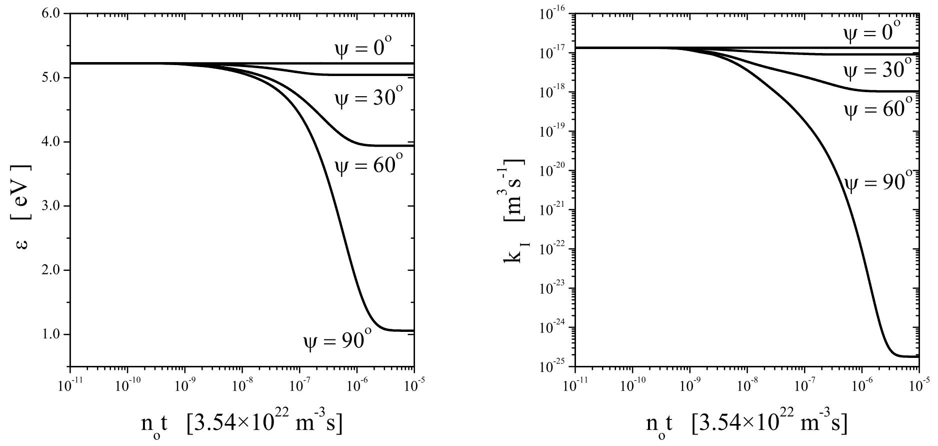

Figure 1: Temporal relaxation of the mean energy and ionization rate as a function of ψ.

In Figures 1 and 2 we show the influence of ψ on the temporal relaxation process associated with the mean energy (ε), ionization rate (k_I), longitudinal drift velocity component (W_z) and diffusion coefficient along the y-direction ($n_0 D_{yy}$). We observe that all transport properties show high sensitivity with respect to ψ. Note that as ψ increases, ε and k_I decrease markedly as a consequence of the magnetic field cooling effects. For parallel fields ($\psi = 0°$), ε, k_I and W_z are not affected by the magnetic field in accordance with the symmetry properties (White et al. 1999). While ε and k_I

exhibit monotonic relaxation for all ψ considered, $n_0 D_{yy}$ shows the same monotonic relaxation profile only for perpendicular fields. For an orthogonal field configuration the Lorentz force does not act in this direction and hence there are no imprinted oscillations on the diffusion coefficient in this direction. On the other hand, for small angles between the fields, the electrons are under the action of Lorentz force producing the oscillatory relaxation profiles which can lead to the transiently negative diffusivity.

It is interesting to consider the differences between the bulk and flux components associated with W_z and $n_0 D_{yy}$. In the early and intermediate stages of the relaxation process, the significant deviations between the bulk and flux components associated with both W_z and $n_0 D_{yy}$ can be observed. This is a clear indication that the initial distribution function and its initial evolution is significantly affected by the ionization processes. However, for an increasing ψ, as the relaxation process proceeds in time, the distinction between the bulk and flux values for W_z diminishes until reaching the steady state where differences between the bulk and flux values are negligible. Similar but not identical behavior shows $n_0 D_{yy}$. These results suggest that due to the complex interplay of the action of the magnetic field and the energy and momentum dissipation of the electrons in collisions, a complicated redistribution of high energetic electrons occurs.

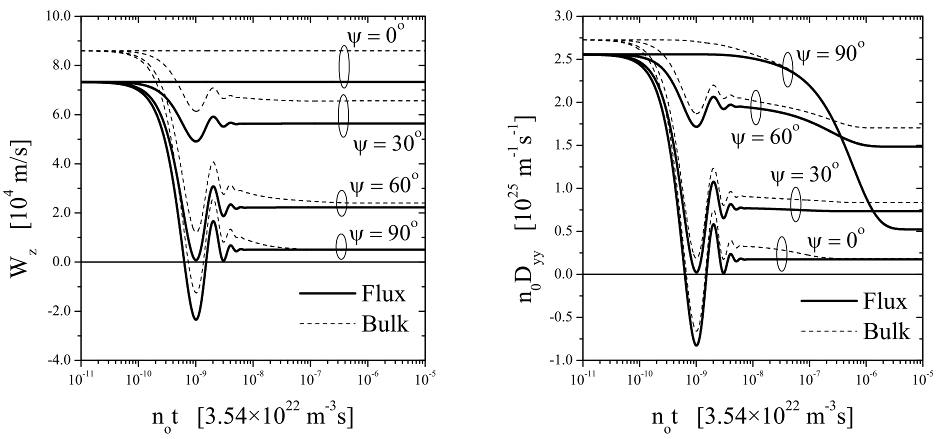

Figure 2: Temporal relaxation of the longitudinal drift velocity component and diagonal element D_{yy} of the diffusion tensor as a function of ψ.

References

Loffhagen, D., Winkler, R.: 1999, *IEEE Trans. Plasma Sci.*, **27**, 2231.
Lucas, J. and Saelee, H. T.: 1975, *J. Phys. D: Appl. Phys.*, **8**, 640.
Warman, J. M., Sowada, U., De Haas, M. P.: 1985, *Phys. Rev. A*, **31**, 1974.
White, R. D., Dujko, S., Ness, K. F., Robson, R. E., Raspopović, Z. M., Petrović, Z. Lj.: 2008, *J. Phys. D: Appl. Phys.*, **41**, 025206.
White, R. D., Ness, K. F., Robson, R. E., Li, B.: 1999, *Phys . Rev. E*, **60**, 2231.
Winkler, R., Loffhagen, D., Sigeneger, F.: 2002, *Appl. Surf. Sci.*, **192**, 50.