Publ. Astron. Obs. Belgrade No. 80 (2006), 245 - 249 *Contributed paper*

MUTUAL RELATIONSHIP BETWEEN MATHEMATICS AND ASTRONOMY IN THE ANCIENT GREECE

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Abstract. In the paper we consider the foundations of mathematics in the ancient Greece as a deductive system, especially the Euclidean geometry. We investigate the concepts of continuum and discreteness in mathematics and nature. A special attention is given to the mathematics applied to the foundation of the Pythagorean concept of the universe and adoption of Aristotle's and Ptolemy's worldviews.

1. INTRODUCTION

Based on many ancient texts mentioning travels to the spaces of the Babylonian and Egyptian civilizations, especially those of Thales and Pythagoras (Dadić, 1975), one finds a doubtless influence of these civilizations on the foundation of the Greek science. In these lands Thales and Pythagoras (Dadić, 1975) learnt of some mathematical facts and accepted them giving a new interpretation. Thales is said to be the founder of the deductive method (Obradović, 2002), although his method was axiomatic in some cases and empirical in other ones. His proof was based on no rigorous principles of logic.

A further step in formulating mathematics as a deductive system was done by Euclid (Obradović, 2002). In his opinion mathematics is a part of logic, hence we find his inclination towards rigorous proofs (Obradović, 2004). Axioms and postulates were sharply detached by him. To him axioms were, in themselves, clear and general truths to any research. Postulates were thought as less obvious and they concerned the subject under study only. In the ancient Greece the continuous and atomistic pictures of nature got their fundaments. The continuity and discreteness were defined , not only in nature, but also in mathematics. Here one should specially emphasize Aristotle's continuum theory and the atomistic one of Leucippus and Democritus (Obradović, 1997 for both). According to the ancient Greeks everything in the world was strictly determined by mathematical laws. Due to this arose the Pythagorean vision of a mathematical universe and the naturo-philosophical universe concept of Aristotle and Ptolemy as a rounded system.

2. CONTINUOUS AND ATOMISTIC PICTURES OF NATURE

According to Aristotle's (206 a 20) theory of continuum, what is continuous is actually not divided, but is divisible towards infinity. This infinity is not something which can be realized, but something in a permanent state of arising. The continuum contains no actual parts, but potential parts are possible. The continuum is, first of all, an ideal, thought and mathematical construction. According to Aristotle (Dadić, 1975) continuous quantities are geometrical objects: lines, surfaces and bodies. He assured that mathematical abstractions were taken from the material world and that the whole nature was continuous. Following this he defined time as a continuous one-dimensional continuum. Time consisted of potential or actual intervals (Arsenijević, 2003). Since nature was continuous, according to Aristotle, light propagated as a vibration of transparent objects. The general means of light propagating was ether. It filled the whole interstellar space.

The mathematical atomism was proposed by Leucippus and Democritus (Dadić, 1975). They thought that a line consisted of an infinite number of points. A mathematical atom was an infinitely small part of a line or a kind of infinitesimals, contained in the diagonal or side of a square. This idea was applicable to other geometric forms, especially to the volume determination for a cylinder. Democritus thought that the world consisted of a full part and of an empty one. The full part was composed of small, indivisible particles called atoms. Their number was infinite, they were eternal and absolutely simple. They were of similar quality, but their shapes, orders and positions were different. They could have specific, cubic, pyramidal and a variety of other shapes also including those with a hook for the purpose of easier interconnecting. Any single object was made of atoms in an infinite number of different combinations and in an infinite number of different ways. Democritus (Obradović, 1997) was the first to claim that there was an empty space, for which he did not strictly say to be infinite. However, since the number of atoms in space was infinite, we can adopt that in his opinion the space has the same property. The atomists interpreted all physical phenomena in the framework of their theory. Light was explained by them as a stream of atoms coming into the eye, producing the sense of shining.

It can be said that discreteness is a possible approach in mathematics followed by all theories based on the atomism concept. History of science shows that it is impossible to give a full description of nature by using continuity. Establishing an analogy between the mathematical continuum and continuity of nature Aristotle gave rise to one of the conditions for a successful realization of the heuristic function of mathematics. Due to the notion of mathematical continuum one can consider some natural phenomena. This offers possibility to perform corrections within the limits of empirical measuring and to give exact predictions within the same limits. This is also true for the mathematical discreteness model.

3. THE FOUNDATION OF THE PYTHAGOREAN CONCEPT OF UNIVERSE AND OF THAT OF ARISTOTLE AND PTOLEMY

According to the Pythagoreans the world followed principles of mathematics. In that world the relations among the numbers formed an essential base and an instrument how to learn the order in it. They realized a complete abstraction for geometric objects and number where the latter one, according to them, comprised positive integers only. Number was the essence creator for the matter and world of phenomena. By them geometric objects and numbers were brought in some kind of correspondence. A point became "one", a line "two", a planar surface "three" and a body "four" depending on the minimum number of points necessary to determine each of these geometric objects. Geometric objects were viewed as things though they were complete abstractions. A point had its size, a line its thickness, a surface its depth. Points were added to lines, lines to surfaces and surfaces to bodies. Accordingly a line consisted of points of their own size, but also of units because a point was identified to a unit number. Since abstract geometric objects were viewed as "things", then "things" and numbers were identified. A "thing" could be built of numbers. A "thing", in itself, was not the world of phenomena, but lay in its essence. The numbers through the essence of physical bodies became elements of the physical reality and nature. In a specific sense, it can be said, that numbers one, two, three and four were world constituents. According to the Pythagoreans a circumference and a sphere were the most perfect shapes. For this reason the Earth and other celestial bodies had to be spherical and their trajectories circular.

A specially important application of mathematics was to extend Aristotle's natural philosophy, i. e. to put in accordance mathematically the observations of the motions of the planets and of the Sun with Aristotle's philosophy. Apollonius of Perga (Dadić, 1975) gave the first satisfactory theory of motion of celestial bodies. Instead of heliocentric spheres he introduced epicycles. If the Earth (E) was situated at the centre of the world and on its centre a larger circle or deferent was centered, then the centre of the epicycle, along which a planet's motion took place, moved along the deferent, as seen in Fig. 1.

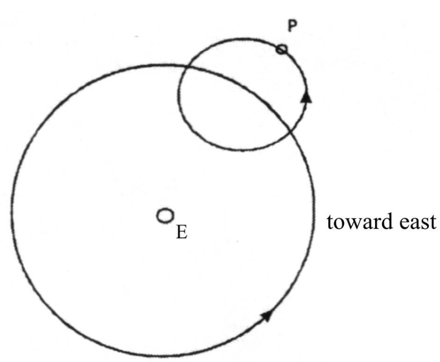

Figure 1: Planetary motions according to Appolonius of Perga.

If the ratio of the radii for the two circles and that of the velocities of motion for the epicycle centre and the planet, itself, were suitably chosen, the planet motion would be surely retrograde. The motions of the deferent and epicycle were uniform. The resulting motion of the planet was not uniform and did not take place along a circumference. Apollonius also introduced an eccentric circle along which the motion of the Sun took place as seen from Fig. 2. Its centre did not coincide with that of the Earth. If the Sun moved uniformly along it, it would seem from the Earth that the motion in one half $S_4S_1S_2$ was faster, i. e. slower in the other one $S_2S_3S_4$. The motion of the Sun along the circumference was uniform, but due to the eccentric position of its orbit with respect to the Earth's centre, the apparent motion would be non-uniform.

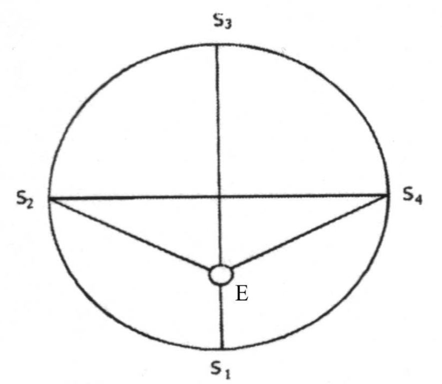

Figure 2: Solar motion according to Appolonius of Perga.

In the third quarter of the second century BC, Hipparchus (Dadić, 1975) introduced smaller epicycles. They resulted in no retrograde motion of planets, instead they changed the shape of the orbit only converting a circle into an elongated closed curve seen in Fig. 3. In this way he put in accordance the motion of the Sun and of the Moon with the theory.

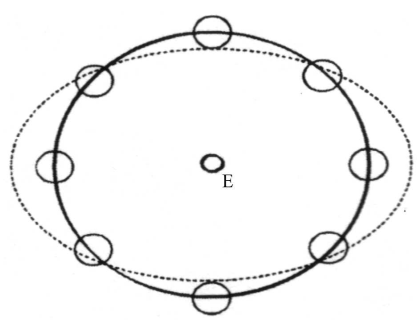

Figure 3: Solar motion according to Hiparchus.

The results of Apollonius and Hipparchus were completed by Ptolemy by introducing the notion of equant with which he wanted to explain the motion of the Sun. According to his idea the centre of the Sun's deferent was at the Earth's centre (E Fig. 4). The motion of the Sun along the deferent was not uniform and as such it was seen from the Earth. It was uniform with respect to a point within the equant.

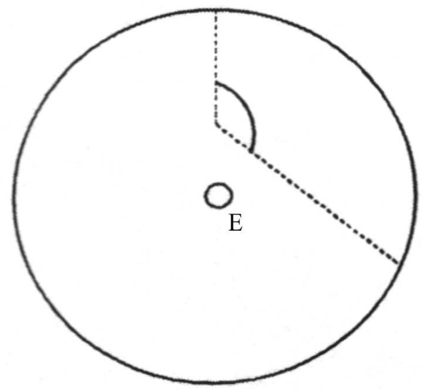

Figure 4: Solar motion according to Ptolemy.

Ptolemy systematised all the results of Apollonius and Hipparchus and extended the theory quantitatively. His Almagest became the first systemic mathematical discussion yielding complete, detailed and quantitative amounts of all the celestial motions. The principal weak point of Aristotle's natural philosophy was the impossibility of giving a satisfactory mathematical explanation for the planet motion. This drawback was overcome by Ptolemy's theory, often said to be the greatest scientific achievement of the ancient world. The geocentric system exists in the unity of Ptolemy's mathematical theory and Aristotle's natural philosophy.

4. CONCLUSION

In the ancient Greece mathematics was formulated as a deductive system. The notions of continuum and discreteness were formulated in mathematics and in natural sciences. The ancient Greeks projected mathematical knowledge onto nature. They thought that the world was founded on mathematical principles.

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