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A SIMPLE CLASSICAL MODEL OF AN ACCELERATING FRACTAL COSMOS

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Abstract. A simple model that provides a possible explanation of the recently observed cosmic acceleration is proposed. It is based on the hierarchical cosmological model, with inert material systems, like galactic clusters, superclusters *etc* immersed into the fluid-like dark-energy substratum. The resulting gravitational force drives then the repulsion of the large-separation subsystems, causing the observed global acceleration of galaxies.

1. INTRODUCTION

Two of the most exciting features of the modern cosmology that have been revealed in the last two decades (Taylor & Peacock 2001) are the phenomenon of the cosmic acceleration (Schwarzschild 2001) and the observational evidence that the universe might be of a (multi)fractal structure (Perlmutter *et al* 1997). A number of possible mechanisms responsible for driving this accelerating expansion have been proposed, the hypothetical quintessence being the most popular (Caldwell & Steinhardt 2000).

Though it is not generally recognized by the cosmological community, neoclassical (quasi-Newtonian) models provide realistic representations of the dynamical cosmos (Norbury 1998, Thatcher 1982). Here we consider a simple classical model which embodies both of the features mentioned above. Inert (material) cosmic objects, like galaxies and galactic clusters are supposed to be immersed into a cosmic fluid of higher density than average material one, and the Newtonian gravitational net interaction acting as a repulsive force, within the Charlier's hierarchical model (Grujić, unpublished).

2. THE CONTENT OF THE UNIVERSE

The Big Bang kinematics describes the early stages of the present day universe, but leaves the issue of the final faith of our Cosmos open. It is the inflatory paradigm (see, e.g. Collins *et al* 1989) which requires that the cosmic space should be flat (Euclidian), which implies that the overall matter-energy density ρ must be equal to the critical density $\rho_c = 3H_0/8\pi G$, where H_0 is the Hubble and G Newton's gravitational constants respectively. However, observational astronomy reveals that only 5 % of the total mass-energy density can be provided by the luminous (baryonic) matter.

The missing mass content has been called dark matter, but the present evidence estimates that this hypothetical exotic constituent can contribute at most 25% to the required critical density (see Taylor & Peacock 2001, and references therein). Hence, it has been estimated that $65-70\,\%$ of the necessary energy density must be provided by the so-called dark energy, whose existence is yet to be proved, if ever, by direct observation.

Since the discovery of the cosmic acceleration (see the very recent account, Schwarzschild 2001, and references therein) a number of mechanism that might drive this nondecelerated expansion have been invoked. The first choice was a field that might be associated with Einstein's cosmological constant Λ. This might be the energy of empty space, vacuum energy. On the other hand, it is a general result of the quantum field theory that every physical field has a zero-point energy, due to the uncertainty principle. The problem has arisen, however, with the magnitude of this vacuum energy content of the universe, since it turns out that it amounts to 120 orders of magnitude compared to the rest of (ordinary) matter. A possible solution has been proposed by assuming that the Λ-field and vacuum energy almost cancel, leaving a resulting dark matter just sufficient to drive the expanding cosmos. Another candidate has been proposed (Caldwell & Steinhardt 2000), which is called quintessence (Q) matter, and many candidates for the physical source of this exotic component of the material content of the universe have been explored (see, e.g. Banerjee & Pavon 2001).

Another dogma of the modern cosmology is that the matter spatial distribution is homogenous and that the cosmos is both isotropic and homogeneous (Cosmological principle). On the other hand observational evidence in the second half of the last century allows for a globally nonhomogeneous cosmos, with (multi)fractal structure. This finding has actualized the old ideas of the hierarchical Cosmos, that run back to Anaxagoras (Grujić 2001) and have been revived mostly by Charlier in the first quarter of the 20-ieth century (see, e.g. Grujić 2002).

Here we investigate a possibility that a combined effect of the inert matter of a hierarchical Cosmos inbedded into a dark energy field, treated as a classical fluid, might provide a mechanism for the accelerating cosmic expansion.

3. THE CLASSICAL NEWTONIAN MODEL

Despite the fact that the modern cosmology is based on Einstein's General relativity, Newtonian dynamics can provide a plausible picture of the cosmos and is still valuable tool both from practical and epistemological points of view (see, e.g. Norbury 1998). In particular Newtonian models may be indistinguishable from some Friedmannian solutions for quite general assumptions about the universe (Thatcher 1982).

We assume that the Newtonian law of the universal mutual attraction between (gravitating) masses holds at all distances, with the proviso that Einsteinian mass-energy equivalence holds too.

3. 1. LOCAL INTERACTION

For the sake of simplicity we consider the case of two identical spherical bodies of the mass density ρ_b, immersed into an infinitely expanded fluid of mass density ρ_f. The net force on each of the bodies, due to the gravitational interaction, depends on the mutual separation and the ratio of densities ρ_b/ρ_f. It is a matter of elementary calculation to show that the bodies will move to each other if $\rho_b/\rho_f > 1$ and recede from each other if $\rho_b/\rho_f < 1$. A straightforward calculation of the force on a body due to the presence of an identical body at the distance R is

$$F_1 = G_N \left[\frac{4r^3\pi}{3(R+r)} \right]^2 \rho_b(\rho_b - \rho_f), \tag{1}$$

where G_N is Newton's gravitational constant, R is the mutual bodies separation, r is the radius of the sphere. Hence, the force drives the body 1 towards body 2 if $\rho_b > \rho_f$ and *vice versa*. By the reason of symmetry the same holds for body 2. Obviously, since the system is axially symmetric, it is the mass distribution along the line through the centres that matters only. It is a matter of a straightforward demonstration that the same conclusion holds for any pair of bodies, as well as for a general system of N bodies immersed in a fluid.

If the zero-point field energy is taken as a fluid, we may imagine that all inertial bodies are immersed in an infinitely extended energy reservoir, which after the inflation has been encompassing both observable and dark matter of the Universe.

But this does not yet solve the problem. Bodies do attract each other at the macroscopic (celestial) level. This is easily explained by the fact that they have small dimensions relative to the cosmic scales and hence much larger mass densities than the surrounding energy field. The other extreme of large mutual cosmic separation is not so easily amenable to a convincing explanation. We propose here a model that might point to a plausible solution.

3. 2. LONG-DISTANCE BEHAVIOUR

The law of universal gravitation between material bodies explains easily lumping together inert masses, like celestial bodies, aggregations of stars like stellar clusters, galaxies, galaxy clusters etc. It can provide a plausible rationale for the formation of fractal structure of the observable cosmos. It can not explain, however, the observed nondecelerating flying apart of very distant cosmic objects, or alternatively, the accelerating overall expansion of the present-day universe. We propose a simple explanation of this phenomenon, within the present model of the ordinary matter immersed into cosmic fluid.

3. 3. FRACTAL COSMOS AND THE VACUUM ENERGY

Charlier conceived the cosmos as a collection of hierarchically arranged subunits, arranged in such a way that a collection of N_i (spherical) subunits at i-th level is a member of a higher unit at $i+1$ level etc, *ad infinitum.* The average density of luminous (and gravitating) objects decreases to zero with the distance from any observer and the luminosity of the sky is finite, provided the radii of the consecutive spheres satisfy the relation

$$\frac{R_i}{R_{i-1}} > \sqrt{N_i}, \tag{2}$$

Our principal problem is how to define an interaction between the vacuum field and a subunit of the supposed fractal distribution of the matter content. In the case of quintessence hypothesis an appeal was made to the quantal nature of the Q field, but the model still lacks a definite mechanism for treating large cosmic agglomerations as

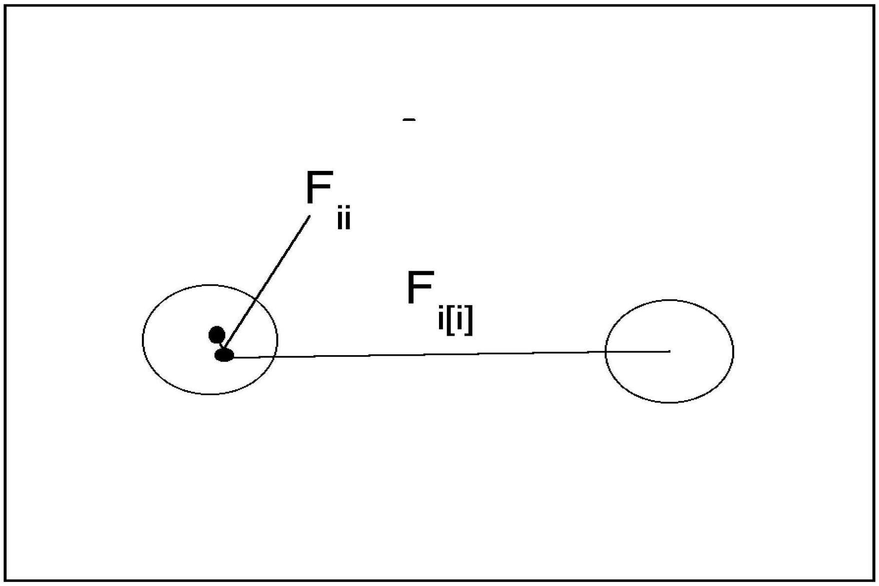

Figure 1: The same level subsystem interaction force F_{ii} and with the next higher one $F_{i[i]}$ (schematic).

compact physical systems. Within the Newtonian dynamics one may consider a sphere whose constituent subunits (say, galaxies) interact between themselves considerably more strongly than with subunits at the same hierarchical level, and even more so with regard to the higher level objects.

In Fig. 1 we sketch two neighbouring *i-th*-level units, with the dots representing their subunits. One may define a radius of an equivalent sphere around a subunit by the relation (Charlier 1922)

$$N_i \frac{4}{3} r_i^3 \pi = \frac{4}{3} R_i^3 \pi, \tag{3}$$

which gives

$$R_i = N_i^{1/3} r_i, \tag{4}$$

The Newtonian force between two neighbouring subunits of mass m_i can be written as

$$F_{ii} = G_N \frac{m_i^2 N_i^{2/3}}{4R_i^2}, \tag{5}$$

For the force between a subunit and the neighbouring unit (see Fig. 1) we have

$$F_{i,[i]} = G_N \frac{N_i m_i^2 N_{i+1}^{2/3}}{4R_{i+1}^2}, \tag{6}$$

From the condition $F_{ii} \gg F_{i,[i]}$, one has

$$\left(\frac{R_{i+1}}{R_i}\right)^2 \gg N_i^{1/3} N_{i+1}^{2/3}, \tag{7}$$

Accounting for (2) one arrives at

$$N_i^{1/3} > N_{i+1}^{1/3}, \tag{8}$$

Similarly, one obtains from the requirement $F_{i,i} \gg F_{i,i+1}$ the relationship between the second-neighbour populations

$$N_{i+2}^{1/6} < N_i^{1/6}, \tag{9}$$

With a reasonable assumption that $N_i > 1$, we arrive at the general condition

$$N_{i+1} < N_i, \tag{10}$$

for the Charlier's cosmos. If this is fulfilled, our subunits behave as the (quasi)rigid bodies, reacting to the external (gravitational) force as a whole. In that case one may consider the mass distribution arbitrary, including a fluid-like one. If the mass density

$$\rho_i = \frac{N_i m_i}{\frac{4}{3} r_i^3 \pi}, \tag{11}$$

appears smaller than the corresponding density of the vacuum dark energy, the net force between two subunits will be repulsive one, as shown above. The whole (fractal) system will undergo an overall expansion, as observed.

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