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TURING FORMATIONS IN ACCRETION DISC - AS A REACTION-DIFFUSION SYSTEM

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Abstract. The necessary conditions which have to be realized for structures formation are presented. We have given the description of Turing formations and their corresponding equations, related to the reaction-diffusion processes. As a result, we have shown that in accretion discs the solutions for appearing of these structures exist.

1. Introduction

We present the following statements and results on the base of the theory of regular spatial structures and new tendency in nonlinear sciences in some hydrodynamical flows.

Below are shown the principles and necessary conditions for arizing of such structures (Stratonovich 1985, Ebeling 1979):

- Dissipative structures can be formed in open systems only, where the energy exchange is possible, which provide the existence of order states.
- Dissipative structures may arize in macroscopic systems. Necessary are states far from equilibrium.
- The evolution laws must operate in these systems.
- Dissipative structures can be formed in systems, described only by nonlinear equations for macroscopic functions.

When given conditions for thermodynamical equilibrium exist, then this corresponds to the largest rate of disorder. Due to this, higher organized states must be in nonequilibrium.

The states, obtained from equilibrium by continuous disturbance, Prigozin (Glensdorf & Prigozin 1973) terms thermodynamical branching. If the deviation from equilibrium exceeds the critical value, these states would become unstable. After that the system turns into new regime and establishes as a dissipative structure.

This qualitative reconstruction of the system, when the parameter crosses the critical value, is known as bifurcation.

To make this clear, we show one process (Nicolis & Prigozin 1979), which causes systematical deviations from equilibrium by increasing some parameters, describing states or outer influence - λ .

According to the minimum entropy theorem - the closest equilibrium stationary states are asymptotically stable /Fig. 1/ (a).

When the parameter of the system reaches the critical value $\lambda_{\tilde{n}}$, it is very possible that the thermodynamical branching becomes unstable (b).

In this case the smallest perturbation is sufficient to change the system. The new stability regime in the system probably corresponds to the orderly state (c). So, with $\lambda = \lambda_{\tilde{n}}$ the bifurcation acts and as a result the new branching of solution has occurred.

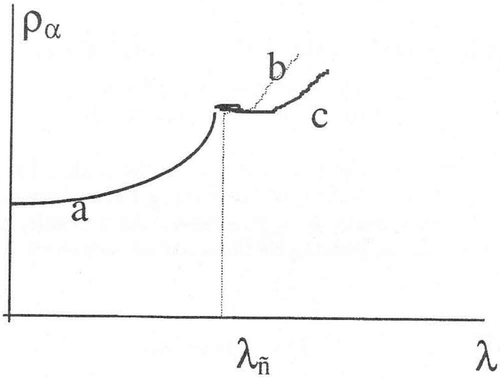

Fig. 1:

In our discussion of accretion disc, there is a similar transition. The arising of some hydrodynamical instability transfers the system from one state to another and therefore the structures are formed in disc. We have shown in the previous work, that the accretion discs are typical dissipative structures. Our aim here is to consider in particular their mode.

2. Reaction - diffusion equations and Turing instability

The reaction - diffusion systems are the exibition of spatial or temporal patterns if they are far from equilibrium (Engelhardt 1994), which is one important condition for formation of the dissipative structures.

First we apply one standard equation of reaction - diffusion (Engelhardt 1994):

$$\frac{\partial c}{\partial t} = F(C) + D\nabla^2 C \tag{1}$$

where the first term on the right side is the reaction and the second is the diffusion. D is the diffusion coefficient (or matrix of the transport coefficient), C- concentration of matter.

In some systems the coupling between two transport procceses provides the engine of instability. Then the growth of this instability is defined by the difference of the diffusion coefficients in the different direction of the transport acting there (Borckmans et all. 2001).

Then the reaction - difusion equation could be written as:

$$\frac{\partial C}{\partial t} = F(C) + D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} \tag{2}$$

That difference between two components of diffusion coefficients is the necessary restriction for the Turing instability to appear (Engelhardt 1994).

3. The accretion disc equations and the appearing of Turing structures

The accretion disc is considered as a hydrodynamical systems and it is described by the hydrodynamical equations (Frank et all. 1991).

In the first place this is the continuity equation, which expresses the mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho v) = 0 \tag{3}$$

The conservation of momentum for each gas element gives with Euler equation:

$$\rho \frac{\partial v}{\partial t} + \rho v . \nabla v = -\nabla P + F \tag{4}$$

where for both equations the quantities ρ, v, P, F are respectively: the density, the velocity, the pressure and the selected force.

We present the equation of motion for viscous fluid (Navier Stokes eq.) in cylindrical coordinates. Due to averaging in z-direction we express all derivatives in terms of the coordinates (r, φ):

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{V_\varphi^2}{r} =$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{1}{\rho} F_r + \nu \left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varphi^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi} - \frac{V_r}{r^2} \right) \tag{5}$$

$$\frac{\partial V_\varphi}{\partial t} + V_r \frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi} - \frac{V_r V_\varphi}{r} =$$

$$-\frac{1}{\rho r}\frac{\partial P}{\partial \varphi} + \frac{1}{\rho} F_\varphi + \nu \left(\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_\varphi}{\partial \varphi^2} + \frac{1}{r}\frac{\partial V_\varphi}{\partial r} - \frac{2}{r^2}\frac{\partial V_r}{\partial \varphi} - \frac{V_\varphi}{r^2} \right) \tag{6}$$

Here ν is the kinematic viscousity, V_r and V_φ are the two component of the velocity.

The equation of energy transfer could be presented as follows:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\varepsilon\right) + \left[\left(\frac{1}{2}\rho v^2 + \rho\varepsilon + P\right) v\right] = f.v - \nabla . F_{rad} \tag{7}$$

where $\frac{1}{2}\rho v^2$ - the kinetic energy per unit volume, $\rho\varepsilon$ – internal or thermal energy per unit volume. The last term in the square brackets represents the so-called pressure work. On the righ hand side:

F_{rad} – the radiative flux vector;

$-\nabla.F_{rad}$ – gives the rate at which radiant energy is being lost by emission, or increased by absorbtion.

In accretion disc we consider the transport of "vortical" function or vorticity, which we may imply with Ψ. This term we take from the vortical equation (Nauta 2000):

$$\left(\frac{\partial}{\partial t} + \vec{v}.\nabla\right)\frac{\nabla \times \vec{v}}{\rho} = 0 \tag{8}$$

which is obtained, combining the rotation of momentum equation and the continuity equation. So, $\Psi = \nabla \times \vec{v}$ and for eq. (8) yelds:

$$\left[\frac{\partial \Psi}{\partial t} + \nabla.(\Psi\vec{v})\right] = f \tag{9}$$

We express that equation in cylindrical coordinates in terms of r, φ again:

$$[\frac{\partial \Psi_r}{\partial t} + V_r\frac{\partial \Psi_r}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{\Psi_\varphi^2}{r}] = f \tag{10}$$

$$[\frac{\partial \Psi_\varphi}{\partial t} + V_r\frac{\partial \Psi_\varphi}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi} - \frac{\Psi_r\Psi_\varphi}{r}] = f \tag{11}$$

Here with f we express the transport engine of the vortex or the diffusion from Eq. (1) and for our consideration the f is in the form: $D\nabla^2\Psi$.

Taking into account the vortical equation (9) and the expressions (10) and (11), the reaction - diffusion equation (1) becomes:

$$\frac{\partial \Psi_r}{\partial t} = h(r,\varphi) + D_r\nabla^2\Psi_r \tag{12}$$

$$\frac{\partial \Psi_\varphi}{\partial t} = g(r,\varphi) + D_\varphi\nabla^2\Psi_\varphi \tag{13}$$

h and g are the source functions and they take the form: $(\Psi.\nabla)v$

Thereby we obtained two equations with different diffusion coefficients, which is expanded in two components.

The evidence that in accretion discs the necessary condition is that the ratio between D_r and D_φ is not equal to unity and zero exists. This gives us the confirmation of possibility for the appearance of Turing instability in this reaction - diffusion system.

4. Conclusion

These instabilities being the expression of spatial pattern of the biffurcation area, the structures may arise in the disc, particularly: vortical structures and so called the Rossby solitons.

The essence in the accretion discs investigations is the finding out of connection between the kind of instability and the arising of the engine of structure formation, when all required above conditions are present.

The theory of Turing structures is one of the many of such modeling pattern formations and here we showed that it works in the reaction - diffusion system of the accretion disc, too.

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