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A SIMPLE METHOD TO TRACE THE MOTION IN 1/r FIELDS

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Abstract. We discuss a simple algorithm to trace the motions in 1/r fields which is based on the peculiar properties of the vectors of two hodograph velocities in configuration space. Since the hodograph velocities are uniquely determined by energy and angular momentum of the particle it follows that in this particular case, contrary to common belief, full analysis of motion is possible without the use of dynamics.

Two recent treatments of the ancient problem of planetary motion (Krpić and Aničin 1993; Chernikov 1998) suggest an interesting and at the same time perhaps the simplest of algorithms to trace the motion of bodies in $1/r$ fields. It makes use of the vectors of hodograph velocities which were shown in Krpić and Aničin 1993 to be uniquely defined by energy and angular momentum of the particle. It thus turns out that, contrary to the common belief, it is possible in this particular case to trace the motion completely without invoking the full dynamical analysis. In what follows we restrict ourselves to the case of the gravitational field. Throughout we follow the definitions and notation introduced in Krpić and Aničin 1993 and we refer the reader to this work for details.

For the particle of mass m moving with energy E and angular momentum L in the field of mass M the two hodograph velocities u and w (whose intensities are constants of motion) are introduced by means of expressions:

$$u^2 = \frac{2E}{m} + w^2 = const$$

and

$$w = \gamma \frac{mM}{L} = const.$$

In our previous work (Krpić and Aničin 1993) we took advantage of the properties of these quantities in the velocity space, in the first place of the fact that they satisfy the equation of a circle:

$$v_r^2 + (v_\phi - w)^2 = u^2,$$

where the radial and angular components of velocity and are

$$v_r = u \sin\phi, \quad v_\phi = u\cos\phi + w.$$

Now, in what follows, we make use of the advantageous properties of these quantities in the configuration space. The most useful in this respect is the vectorial relation stating that everywhere on the trajectory velocity is equal to the sum of the two hodograph velocities:

$$\vec{v} = \vec{u} + \vec{w}.$$

Moreover, the angle between the vectors of hodograph velocities $\vec{u}$ and $\vec{w}$ is seen to equal the polar angle ϕ. As seen from relations (4), and demonstrated by Chernikov (1998), as the particle moves along the trajectory vector $\vec{w}$ remains of constant intensity and is normal to the radius vector $\vec{r}$ while vector $\vec{u}$ is not only of constant intensity but also keeps constant orientation in the configuration space, always normal to the direction of polar radius at the point of shortest distance from the center of force (perihelion, for planetary motions). All this is illustrated in Fig.1.

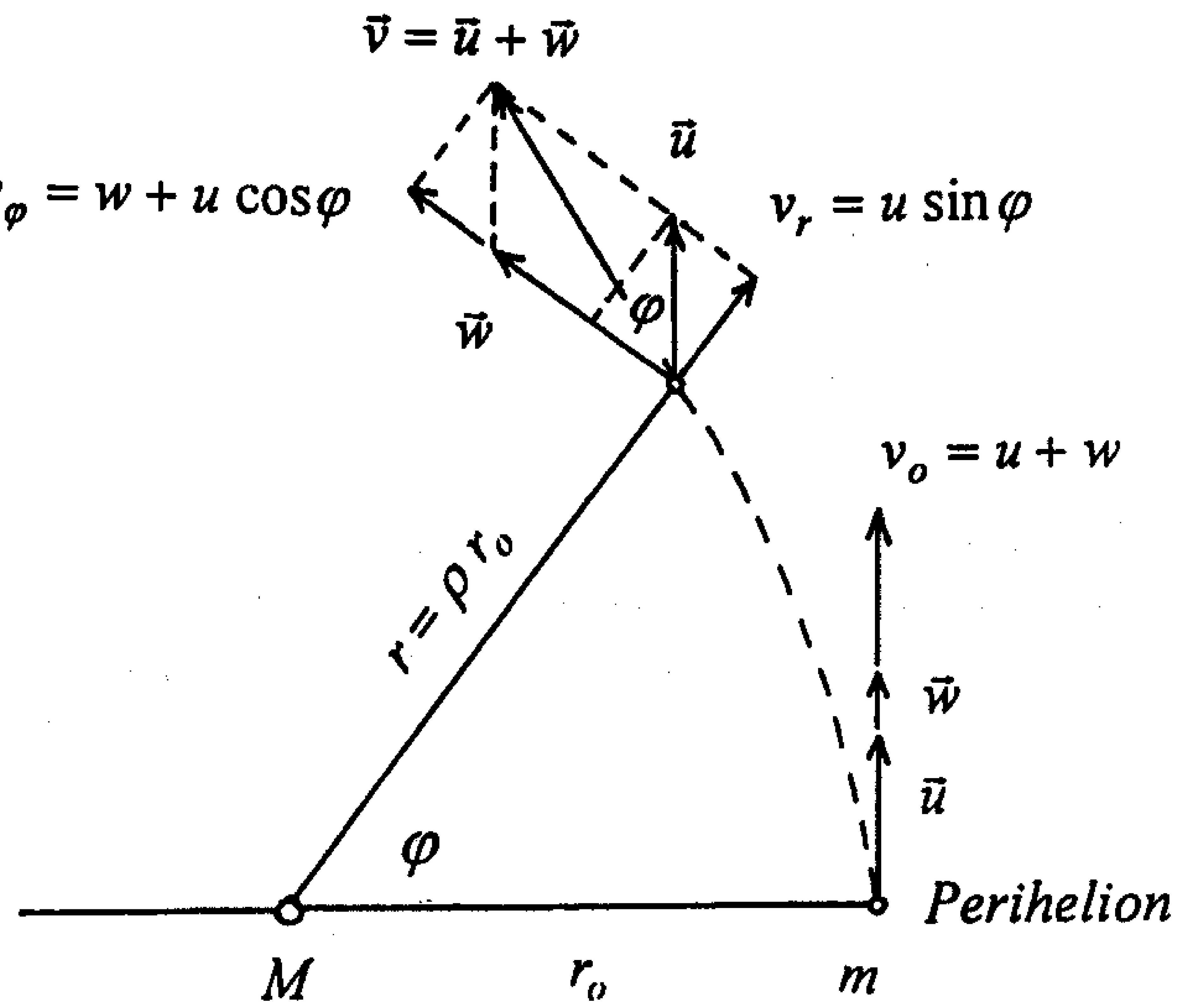

Fig. 1. Taking the point of the shortest distance r_0 from the center of force M as the initial position ($\phi = 0$) the vector of the initial velocity $\vec{v}_0$ is normal to the corresponding polar radius and its intensity equals the algebraic sum of the two hodograph velocities u and w, which are derived from the total energy and angular momentum of particle m. At any other point specified by angle ϕ the polar vector is again found from u and w by means of Eq.(6) while $\vec{v}$ is found as the vectorial sum of $\vec{u}$, which is of unchanged intensity and direction, and $\vec{w}$, which is of the same intensity but rotated by angle ϕ with respect to $\vec{u}$.

The equation of the trajectory in polar coordinates in terms of u and w is easily shown to read:

$$\rho = \frac{w+u}{w+\frac{\vec{w}\cdot\vec{u}}{w}} = \frac{1+e}{1+e\cos\phi}$$

where ρ is the polar radius expressed in units of the shortest distance from the center of force and e is the eccentricity of the orbit. Depending on the type of the conic section e can be anything from zero to infinity.

Finally, we sum up the relevant facts:

1. The distance of the arbitrary point from the center of force and the velocity at this point determine the total energy of the particle and its angular momentum and therefore the intensities of the two hodograph velocities. The polar angle of the particle, measured counterclockwise from the direction of the polar radius at the perihelion, is also found from here.

2. Vector of the first hodograph velocity $\vec{u}$ is of constant intensity and of constant orientation in the configuration space.

3. Vector of the second hodograph velocity $\vec{w}$ has constant intensity and is always normal to the polar radius.

4. Velocity at any point is equal to the vectorial sum of the two hodograph velocities.

5. The intensity of the polar radius is also found from the intensities of the two hodograph velocities.

The initial conditions given, all this now provides the algorithm for tracing the motion of the particle. It may run as follows:

From the arbitrary initial conditions (position and the vector of initial velocity) the intensities of the two hodograph velocities, the initial polar angle in respect to the direction of the perihelion and the polar radius at the perihelion are found. At every other point, defined by a given polar angle and the corresponding polar radius, velocity is found as the vector sum of the first hodograph velocity $\vec{u}$, which is always parallel to its initial orientation at the perihelion, when it was normal to the polar radius, and the second hodograph velocity $\vec{w}$, which is always normal to the polar radius (or which is rotated by angle with respect to the initial orientation at the perihelion, when it was collinear with the vector of the first hodograph velocity).

The short computer program written according to this algorithm in the C language for the PC is available at: **dkrpic@rudjer.ff.bg.ac.yu**.

As an example, the result of a simulated elliptical trajectory is presented in Fig.2.

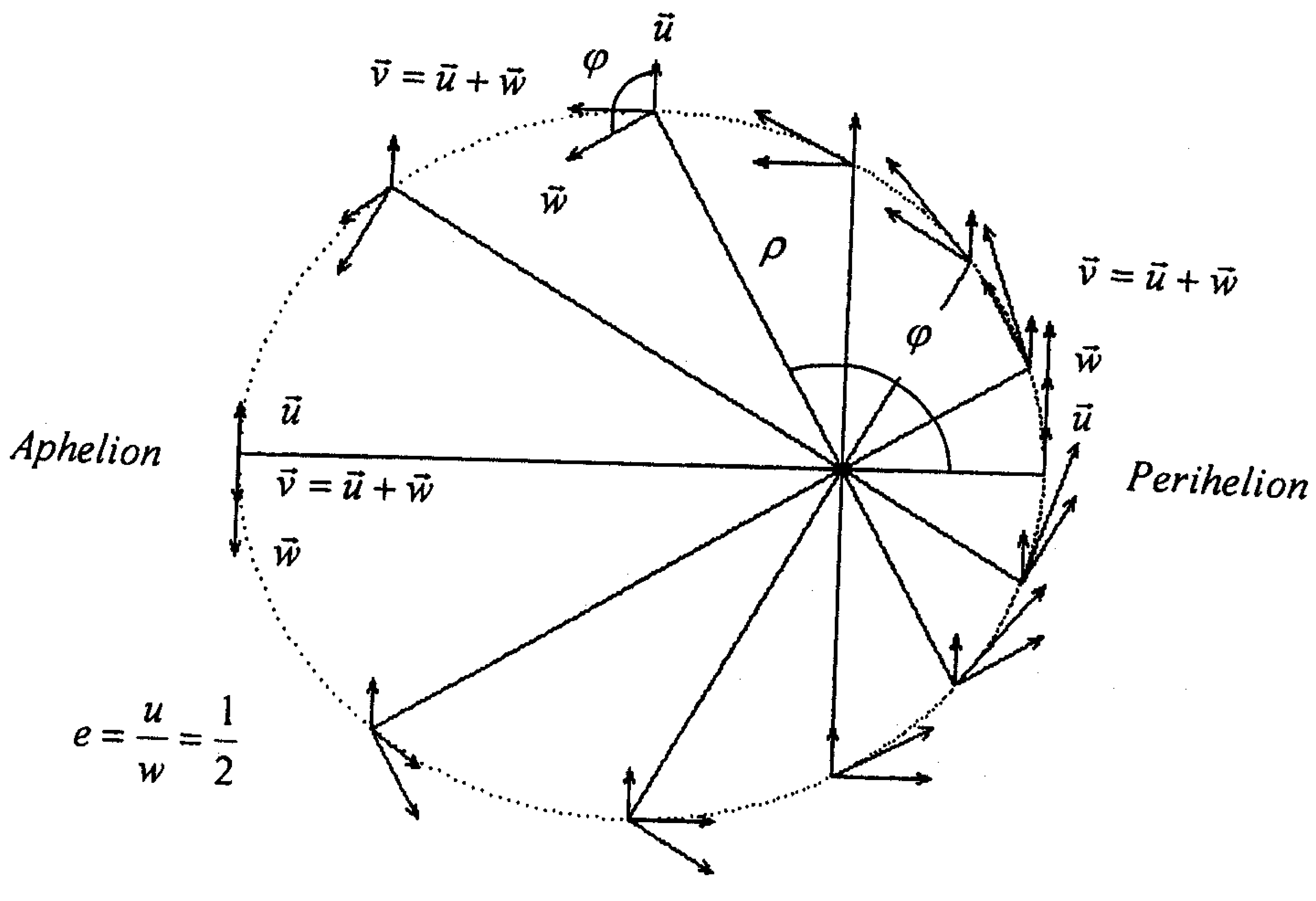

Fig. 2. An elliptical trajectory with $e = u/w = 0.5$ which exemplifies the use of the suggested algorithm. At each of the selected points separated by 30^0 the three vectors $\vec{v}$, $\vec{w}$ and $\vec{w}$ are presented to illustrate their mutual relationships.

References

Chernikov, N.A.: 1998, Dubna Preprint P2-98-247, DUBNA, "The figure, described by the velocity vector of a planet, in the Keplerian problem" (in Russian).

Krpić, D.K. and Aničin, I.V.: 1993, *Eur. J. Phys.* **14**, 255-258, "The planets, after all, may run only in perfect circles - but in the velocity space".