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THE SMOOTHING OF THE POLAR MOTION DATA BY THE LEAST – SQUARE COLLOCATION METHOD

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Abstract. Three methods of the data smoothing: the least – square collocation (LSC), the Vondrák (WRV) and the cubic spline (SPL) are applied to the polar motion data. Some advantages of the smoothing by LSC method and the comparison with other two methods (WRV and SPL) of filtering and smoothing are shown.

1. INTRODUCTION

The astronomical data, as the polar motion data, represented by the observed values l_i, can be separated into the signal s_i (or the systematic changes of the measured values) and the random errors n_i (the errors of observations). We want to remove n_i from the raw data l_i before the analysis. Let the values n_i represent the white noise. The ideal method of smoothing is the one which removes only the random part from the raw data and does not change the systematic part (the signal).

In the astronomical practice (like it was in the practice of BIH and now of IERS) the WRV method is commonly accepted and the alternative method is SPL, but the SPL is not superior to WRV in any sense. The LSC method, first applied in geodesy, gave good results also in the astronomical practice for the last few years. We want to show here some advantages of the smoothing by LSC.

The LSC is a linear transform $s_i = Fl_i$. It is a method of stochastic filtering. With good knowledge of the autocovariances of the signal it is possible to filter the noise most optimally and to estimate $\hat{s}_i$ (close to s_i as much as possible). The theoretical base was established by Moritz (1980). To make the algorithm we used the paper by Gubanov and Petrov (1994) wherein some formulae were corrected for some mistakes, and also the paper Titov (1995).

The series l_i and s_i (where $i = 1, 2, ..., N$) are centred and equidistant. We use here the raw values of x and y – component of the polar motion (IERS, 1993) over the interval [JD 2437669 (04.I 1962) – JD 2440664 (18.III 1970)] and the input data are 5 days spaced ($N = 600$).

Let us consider the series l_i and s_i as vectors **l** and **s**. Then, the problem of filtering the noise is to determine the operator F which satisfies the condition $||\hat{\mathbf{s}} - \mathbf{s}||^2 = min$

(Gubanov and Petrov, 1994). It is only necessary to know some of statistic characteristics of the signal (which can be known a priori), but not its mathematical model. In LSC we presume that the covariance function of the signal is known, the vector **n** does not have correlation, the signal and the noise are not mutually correlated.

By solving the problem of filtering by the LSC method, the autocovariance function of the signal (q_{ss}) should be obtained. There are several different ways to do it (Titov, 1995): using of the variance of the white noise σ_n^2 (obtained by approximation of raw data covariance function in vicinity of zero), using of the descending exponential model of covariance function or using of the covariance function obtained from independent observations.

We had already performed the filtering by LSC method after estimation the variance σ_n^2 (Damljanović, P.Jovanović and B.Jovanović, 1996). The autocovariance functions $q_{ll}(j)$ of the vector **l** and $q_{ss}(j)$ of the signal s can be estimated by the formulae presented by Gubanov and Petrov (1994). Then, $\sigma_n^2 = q_{ll}(0) - q_{ss}(0)$. The basic formula of LSC for filtering the noise is: $\hat{\mathbf{s}} = Q_{ss}Q_{ll}^{-1}\mathbf{l}$, where Q_{ss} and Q_{ll} are covariance matrices (with symmetric form) of the signal and raw data. A form of Q_{ss} and Q_{ll} was given by Damljanović *et al.* (1997), wherein the descending exponential model of covariance function was used.

Neither of algorithms, in our opinion, were sufficiently satisfactory. Therefore, in this paper we use q_{ll} and its approximation q_{ss}, but now the values $q_{ll}(j)$ (where $j = 0, 1, ..., N-1$) are fitted by elementary trigonometric functions using the B.Jovanović (BJ) method for approximation of arbitrary numerical data set as a sum of linear function, harmonics and exponential functions (B.Jovanović 1987, 1989, 1997).

2. RESULTS

The BJ method belongs to a class of algebraic harmonic analysis methods. The analytical representation of q_{ss} is developed here in the form

$$q_{ss}(j-1) = \sum_{i=1}^{m} A_i \cos(\omega_i t_j + \phi_i), \qquad j = \overline{1, N}$$

by use of BJ method and, thereupon, the LSC procedure is applied. In Table 1. are presented the calculated values for A_i, ω_i and ϕ_i (for the epoch JD 2437669); $m = 22$.

Raw and filtered data (by LSC, WRV and SPL) of x and y coordinates of the polar motion are shown in Fig. 1. and Fig. 2. The values smoothed by LSC follow better the raw data and differ remarkably from the ones obtained by other two methods (WRV and SPL). We determined the value of ε (the smoothing parameter of WRV) as explained in Vondrák (1977) and as we did it in Damljanović *et al.* (1997); $\varepsilon = 10^{-8}$ for x - component and $\varepsilon = 1.5 * 10^{-9}$ for y. The value of $\frac{\sigma_A}{\sqrt{2}}$ is $0''.019$ for both (x and y) components, where $\sigma_A = \sqrt{\frac{\sum_{i=1}^{N-1}(l_{i+1}-l_i)^2}{N-2}}$. The smoothing parameter S of SPL method (Reinsch, 1967) is the mean value of the confidence interval $[N - \sqrt{2N}, N + \sqrt{2N}]$ (for a normal distribution of errors). Hence, $S = 600$.

The values of standard deviations (σ) of the residuals, after smoothing by LSC, WRV and SPL, are: $0''.018$, $0''.019$ and $0''.020$ respectively, for x - coordinate ($0''.021$,

Table 1.

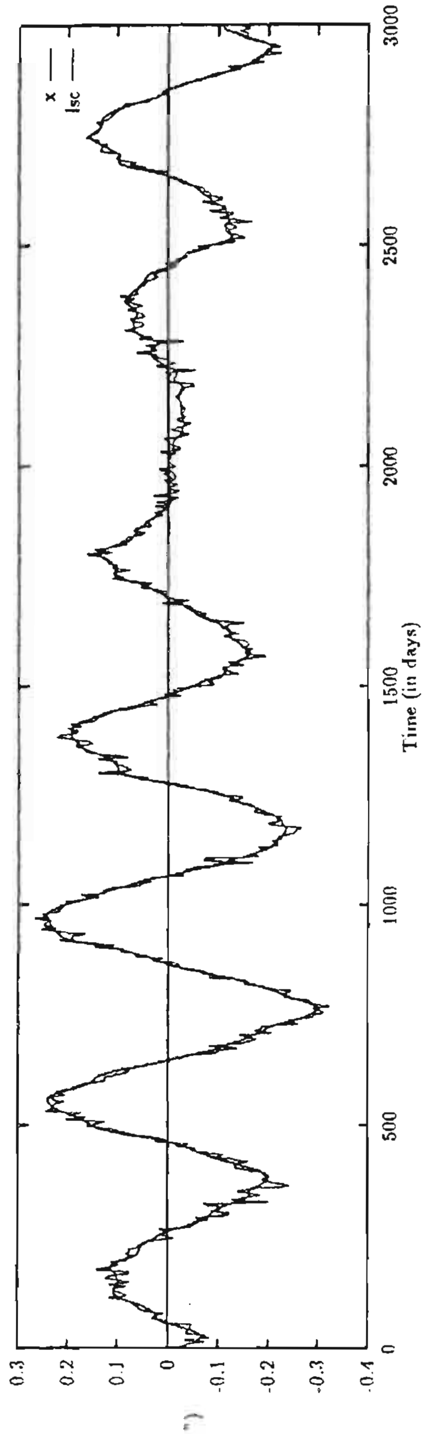
	x - coordinate			y - coordinate		
i	$A_i('')$	$\omega_i(rad/day)$	$\phi_i(°)$	$A_i('')$	$\omega_i(rad/day)$	$\phi_i(°)$
1	0.63971E-02	0.14518E-01	29.85	0.61126E-02	0.14518E-01	32.53
2	0.48210E-02	0.13865E-01	-40.26	0.48488E-02	0.13865E-01	-42.64
3	0.31260E-02	0.16396E-01	-27.96	0.27547E-02	0.16396E-01	-26.09
4	0.31050E-02	0.17238E-01	34.63	0.23006E-02	0.17238E-01	39.79
5	0.12951E-03	0.32444E-02	-50.28	0.99046E-04	0.38768E-02	44.99
6	0.10997E-03	0.20289E-01	-36.24	0.55450E-04	0.32444E-02	-44.70
7	0.86053E-04	0.38768E-02	45.76	0.34549E-04	0.77396E-02	-40.85
8	0.43833E-04	0.77396E-02	3.69	0.34367E-04	0.20289E-01	-111.39
9	0.31511E-04	0.29727E-01	39.41	0.15015E-04	0.29727E-01	94.10
10	0.16513E-04	0.26601E-01	-160.96	0.91291E-05	0.34666E-01	-28.39
11	0.11377E-04	0.43989E-01	-83.78	0.84060E-05	0.26601E-01	-14.56
12	0.72003E-05	0.51015E-01	77.95	0.51723E-05	0.10222E+00	-20.78
13	0.63945E-05	0.34666E-01	3.40	0.24350E-05	0.55799E-01	68.08
14	0.59154E-05	0.58496E-01	-44.56	0.23394E-05	0.69029E-01	-136.49
15	0.53794E-05	0.69029E-01	-161.07	0.18726E-05	0.84597E-01	42.04
16	0.42413E-05	0.55799E-01	-112.74	0.14712E-05	0.86671E-01	151.46
17	0.40723E-05	0.48301E-01	-128.11	0.14684E-05	0.43989E-01	-179.46
18	0.37985E-05	0.10222E+00	-128.60	0.13990E-05	0.80912E-01	-44.40
19	0.27058E-05	0.86671E-01	-39.40	0.13746E-05	0.91576E-01	153.43
20	0.21626E-05	0.91576E-01	174.63	0.11770E-05	0.51015E-01	169.33
21	0.17467E-05	0.84597E-01	65.41	0.10192E-05	0.48301E-01	127.80
22	0.12641E-05	0.80912E-01	167.22	0.78937E-06	0.58496E-01	-142.47

$0''.019$ and $0''.019$ for y). The residuals are the differences between the smoothing curve (by LSC, WRV and SPL) and the raw data. The amplitude periodograms, by direct Fourier transforms (FT), of these residuals are shown in Fig. 3. for x – coordinate (in Fig. 4. for y). The residual systematic errors exist in the case of WRV and SPL smoothing, because it's evident that the peaks for Chandler and annual periods by FT are remarkably greater than ones in the case of LSC. It was not possible to separate the Chandler and annual wobbles by FT (see Figures 3. and 4.) because of the short interval (about 8 years).

3. DISCUSSION

The LSC method does not require any smoothing parameter, and that is not the case in WRV and SPL methods. With good approximation of signal covariance function, the LSC is very suitable method for filtering the errors of observations. All three methods are flexible.

The largest systematic errors of residuals (after smoothing by WRV and SPL) appear in the Chandler residual and annual oscillations (see Fig. 3. and Fig. 4.), but in the case of LSC they are negligible. The systematic discrepances remain in the residuals because the WRV and SPL are the smoothing methods using the third



Raw and smoothed data (WRV and SPL)

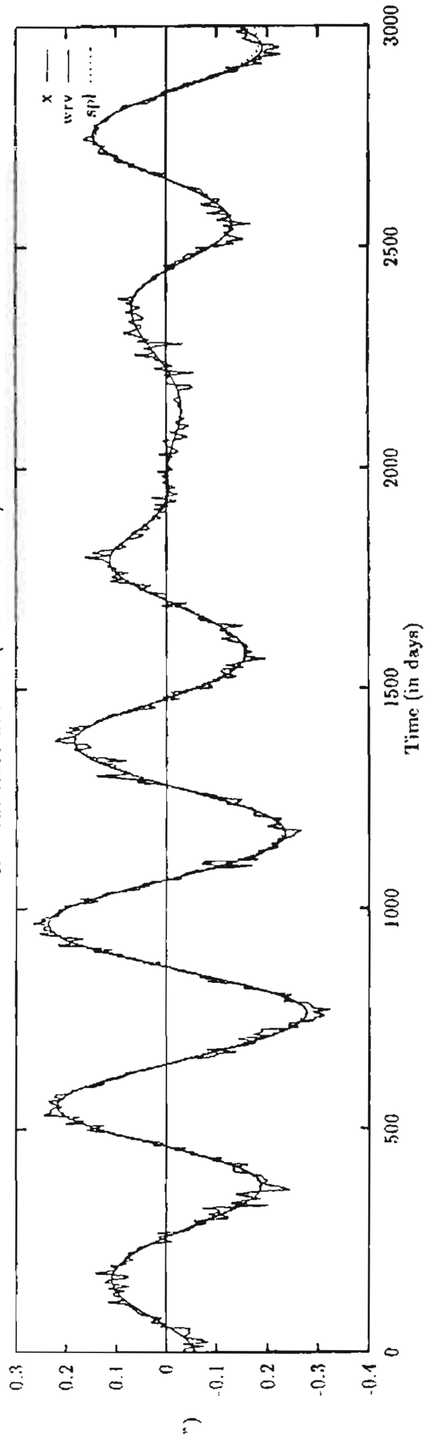
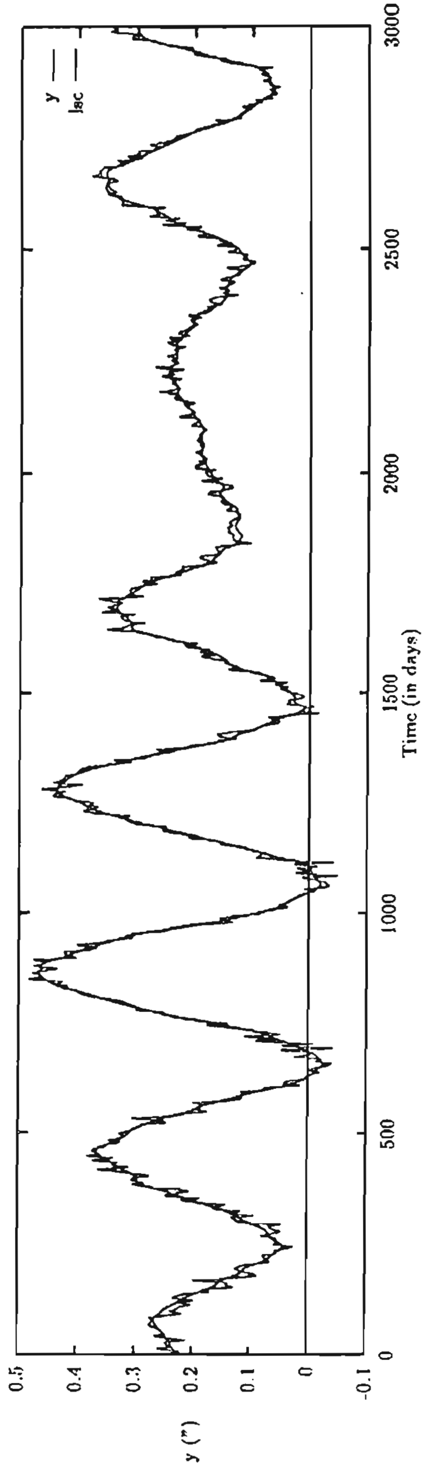


Fig. 1. Raw and smoothed data (LSC)



Raw and smoothed data (WRV and SPL)

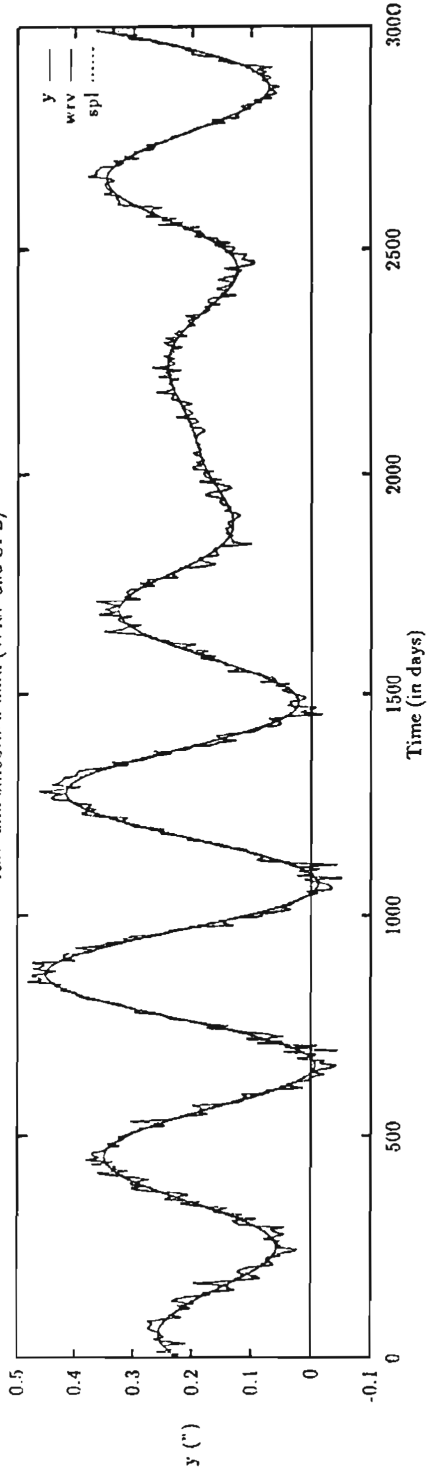


Fig. 2. Raw and smoothed data (LSC)

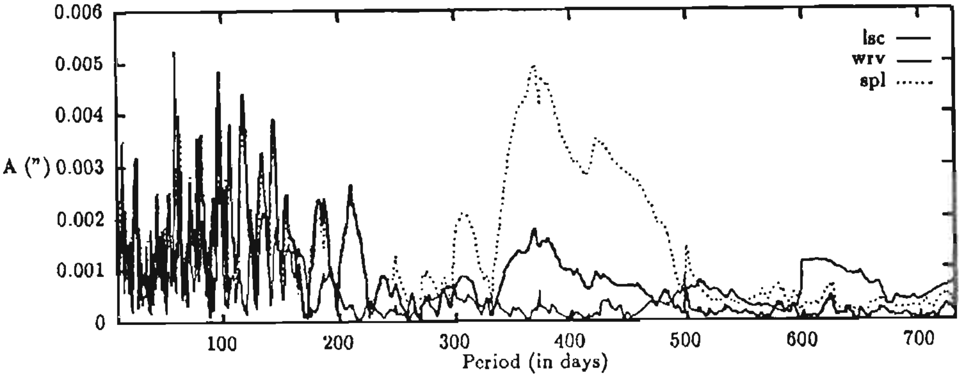


Fig. 3. FT of residuals (x - coordinate)

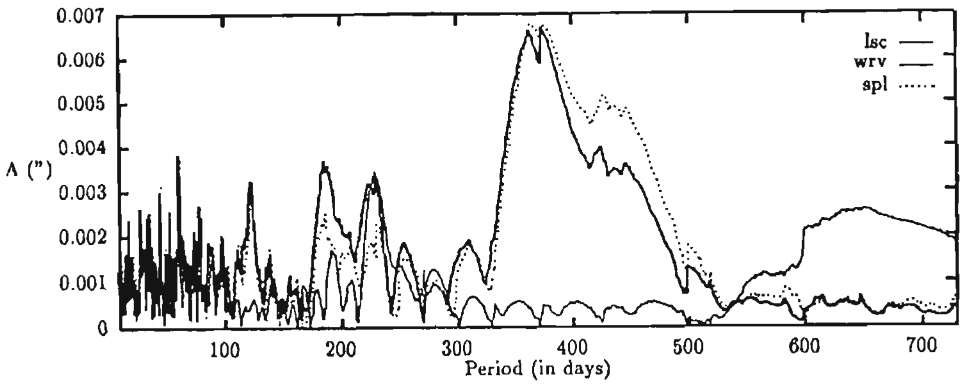


Fig. 4. FT of residuals (y - coordinate)

order polinomials, but the real data include a few harmonic oscillations. The smoothing curve by LSC better follows the raw data than WRV and SPL smoothing curves (especially at the beginning of the interval).

As it can be seen, the LSC method holds the indicated advantages and can be successfully used for the astronomical purposes.

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