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ABEL INVERSION OF EXPERIMENTAL DATA USING JACOBI POLYNOMIALS WITH APPLICATIONS IN SPECTRAL LINE ANALYSIS

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1. INTRODUCTION

The experimental data fitting and Abel inversion procedure using orthogonal Jacobi polynomials are presented. Plasmas of cylindrical symmetry are frequently encountered in areas of plasma physics such as flames, sparks, arcs, plasma jets, radio frequency plasmas, etc. (Haraguchi *et al.*, 1977; Enloe *et al.*, 1990; Shumaker and Yokley, 1964; Blair, 1974; Elder *et al.*, 1965) If the assumption is made that the plasma is optically thin, i.e. that the self-absorption is negligible, then for a cylindrically symmetric plasma source the true emission distribution can be calculated from experimentally obtained data. In order to separate the contributions of the different radial plasma layers and to obtain local values of emission intensity it is necessary to solve Abel's integral equation. Here is presented a method based on orthogonal Jacobi polynomials.

2. THEORY

In general the Inverse Abel transform of

$$Q_n(x) = (1-x^2)^{p-q}\, G_n(p,q,x^2) \tag{1}$$

is

$$F_n(r) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(n+p-q+1)\,\Gamma(n+q+\frac{1}{2})\,\Gamma(q)}{\Gamma(n+p-q+\frac{1}{2})\,\Gamma(n+q)\,\Gamma(q+\frac{1}{2})} (1-r^2)^{p-q-\frac{1}{2}}\, G(p,q+\tfrac{1}{2},r^2) \tag{2}$$

where

$$G_n(p,q,y) = \Gamma(q) \sum_{k=0}^{n} \binom{n}{k} \frac{\Gamma(n+p+k)}{\Gamma(q+k)\,\Gamma(n+p)} (-y)^k \qquad (3)$$

are Jacobi orthogonal polynomials (Courant and Hilbert, 1953).

On base of this general equations the local plasma emissivity can be written as

$$\varepsilon(r) = \sum_{n=0}^{N} a_n \frac{\Gamma(q)}{\sqrt{\pi}\,\Gamma(q-\frac{1}{2})} (1-r^2)^{q-\frac{3}{2}}\, \Phi_n(r) = (1-r^2)^{q-\frac{3}{2}} \sum_{n=0}^{N} S_n\, D_n\, \Phi_n(r) \qquad (4)$$

where

$$S_n = \frac{(2n+2q-1)\,\Gamma(n+2q-1)}{2^{2q-1}\sqrt{\pi}\, n!\,\Gamma(q)\,\Gamma(q-\frac{1}{2})} = \frac{\Gamma(q)}{\Gamma(q-\frac{1}{2})\sqrt{\pi}} \frac{a_n}{D_n} \qquad (5)$$

$$a_n = \frac{(2n+2q-1)\,\Gamma(n+2q-1)}{[\Gamma(q)]^2\,\Gamma(n+1)\,2^{2q-1}} D_n \quad ; \qquad D_n = \int_{-1}^{1} Q(x)\, G_n(2q-1, q, \tfrac{1-x}{2})\, dx \qquad (6)$$

$$\Phi(r) = G_{\frac{n}{2}}(q-\tfrac{1}{2}, q-\tfrac{1}{2}, 1-r^2) \qquad \text{for even } n \qquad (7)$$

The $Q(x)$ represents experimental data points. For the fitting procedure and determination of the local plasma emissivity $\varepsilon(r)$, by Abel inversion procedure from equation (4), the recursion relation was used to calculate Jacobi polynomials

$$(n+p)(2n+p-1)(n+q)(n+q-1)\,G_{n+1} =$$
$$= (2n+p)(n+q-1)\,[2n(n+p)+pq-q-u(2n+p-1)(2n+p+1)]\,G_n -$$
$$-n(n+p-q)(2n+p+1)(n+q-1)\,G_{n-1}$$

with $\quad G_0(p,q,u) = 1; \quad G_1(p,q,u) = 1 - \frac{p+1}{q} u \quad$ where $u = (1+x)/2$.

The details of the theory calculations are given in Ref. Djurović (1998).

2. EXPERIMENTAL

The above Abel inversion technique was applied to a rf plasma to obtain radial intensity distribution of Ar I 415.86 nm line and also was applied to Ar I 425.9 nm line emitted from wall stabilized arc plasma.

For rf plasma source Gaseous Electronic Conference (GEC) Radio Frequency Reference Cell was used. The details of the design of the Cell are given in Ref. Hargis et al. (1994). The experimental setup is described elsewhere (Djurović *et al.*, 1993).

For arc plasma source a wall stabilized electric arc operating at atmospheric pressure is used. The experimental setup for spectroscopic measurements is described elsewhere (Djurović *et al.*, 1997).

3. RESULTS

In order to deduce the radial distribution of the rf plasma emission from horizontally scanned raw data by the Abel inversion procedure, the Ar I 415.86 nm line in pure argon was used. Horizontal scanning is performed through the plane midway between the electrodes in the bulk plasma region. Fig. 1 shows the experimental emission profile. Different Jacobi polynomials were used for data fitting. In this case Chebyshev polynomial with 22nd polynomial order give the best result (see Fig. 1 full line). The Abel inversion of this data is given in Fig. 2 (full circles) together with Abel inversion by fast Fourier transform method (open circles) (Smith *et al.*, 1988). Present method is especially good in cases where the experimental data have a dip in the middle.

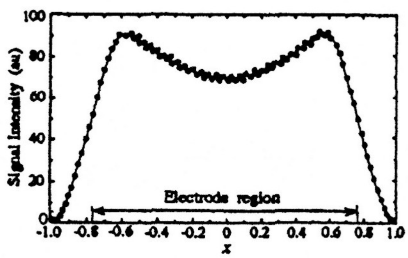

Fig. 1

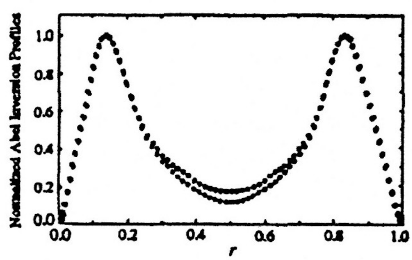

Fig. 2

Set of Ar I 425.9 nm line profiles, emitted from wall stabilized arc plasma, obtained after the inversion procedure is given in Fig. 3. The r is the plasma column radius.

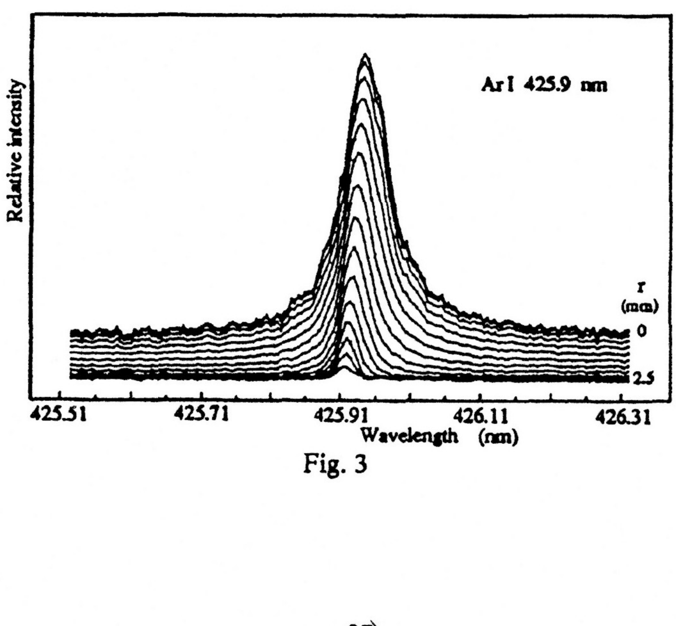

Fig. 3

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References

Blair, D. W. : 1974, *J. Quant. Spectrosc. Radiat. Transfer* 14, 325.

Courant R. and Hilbert, D. : 1953, *Method of Mathematical Physics*, Interscience Publishers, Inc., New York, London.

Djurović, S., Roberts, J. R., Sobolewski M. A. and J. K. Olthoff, 1993, J. *Res. Natl. Stand. Technol.* 98, 159.

Djurović, S., Mijatović, Z., Kobilarov, R. and Konjević, N. : 1997, J. Quant. Radiat. Transfer 57, 695.

Djurović. S. : to be published in 1998.

Elder, P., Jerrick T. and Birkeland, J. W. : 1965, *Appl. Optics* 4, 589.

Enloe, C. L., Brake M. L. and Repetti, T. E. : 1990, *Am. J. Phys.* 58, 400.

Haraguchi, H., Smith, B., Weeks, S., Johnson D. J. and Winefordner, J. D. : 1977, *Appl. Spectrosc.* 31, 156.

Hargis, P. J., Greenberg, K. E., Miller, P. A., Gerardo, J. B., Torcznski, J. R., Riley, M. E., Hebner, G. A., Roberts, J. R., Olthoff, J. K., Whetstone, J. R., Van Brunt, R. J., Sobolewski, M. A., Anderson, H. M., Splichal, M., Mock, J. L., Bletzinger, P., Garscadden, A., Gottscho, R. A., Selwyn, G., Dalvie, M., Hidenreich, J. E., Butterbaugh, J. W., Brake, M. L., Passow, M. L., Pender, J., Lujan, A., Elta, M. E., Graves, D. E. B., Sawin, H. H., Kushner, M. J., Verdeyen, J. T., Horwath, R. and Turner, T. R. : 1994, *Rev. Sci. Instrum.* 65, *140.*

Shumaker, J. B. Jr. and Yokley, C. R. : 1964, *Appl. Optics* 3, 83.