Publ. Astron. Obs. Belgrade No. 54 (1996), 143 – 147 *Contributed paper*

STABILITY VS. CHAOS IN ASTEROID MOTION

Z. KNEŽEVIĆ
Astronomical Observatory, Volgina 7, 11050 Belgrade, Yugoslavia
E–mail zoran@aob.aob.bg.ac.yu

Abstract. Various types of asteroid motions are briefly described and placed in an overall scheme of the motion of solar system bodies. Examples of stable and chaotic motions are mentioned, and a recently recognized class of behaviors called "stable chaos" introduced. A number of applications of chaotic behavior as a diagnostic tools for a study of dynamical phenomena and interpretation of the asteroid collisional evolution are presented and shortly discussed.

1. TYPES OF ASTEROID MOTION

It is nowadays well known that motions of all the bodies in the planetary system are basically chaotic, only the dynamical mechanisms giving rise to chaos, the time scales involved and the extent of the resulting macroscopic instabilities are different. Chaotic behavior can be recognized, for example, by looking in the outcomes of the long term numerical integrations, like in the time series of the semimajor axis; semimajor axes of chaotic orbits, namely, exhibit a typical irregular behavior, random walk, sudden jumps and temporary captures into resonances, etc., which can be easily recognized even at the first glance. To measure the chaos is somewhat more complicated, and what one usually does is to compute the so called maximum Lyapunov characteristic exponents (LCE); these, in a way, represent a measure of the rate at which two initially nearby orbits diverge with respect to each other in terms of some conveniently chosen metrics in the phase space of orbital elements. The inverse of the LCE is the Lyapunov time (T_L); being simpler to comprehend and interpret then LCE itself, it is often used in practice to discuss the implications and compare the characteristics of the chaotic motions of different bodies (Lyapunov time is in fact the time span needed to increase the distance between diverging orbits $exp(1)$ times).

Stable orbits are, in this sense, such that the time variations of their elements are generally regular, LCE is small (Lyapunov time long), and two initially nearby orbits stay close to each other or diverge at a very slow pace. Opposite obviously holds for the chaotic orbits. The examples of stable orbits among the small bodies in the solar system are those of the majority of main belt and Trojan asteroids, some trans-neptunian objects, etc., while chaotic are orbits of bodies inside the main resonances or those with large eccentricity, like near Earth asteroids (NEA), meteoroids, extinct comets, etc. Lyapunov times for the asteroids range from a few tens of years (NEA; see Whipple 1995) to several million years (Milani *et al.* 1996); stable among them typically have T_L's between 10^5 and 10^6 yr, while those with T_L less then, say, 10^5 yr can be considered as ostensibly chaotic. It should be mentioned that asteroids "are not allowed" to be less chaotic than the perturbing (major) planets, which, in turn, have Lyapunov times of $\simeq$ 5 Myr (inner planets; Laskar 1989), or even more (outer planets; Nobili 1989). This "forced" chaos is, however, an intrinsic feature of the motion of all asteroids, while we are interested in the "proper" chaos which is due to the resonant and irregular behavior of a particular asteroid itself. Obviously, proper chaos cannot be detected unless it is significantly faster than the forced one.

Only relatively recently (Milani and Nobili 1992) it has been recognized that chaos does not necessarily imply macroscopic instability; it was found, namely, that there is a third, intermediate class of orbits, characterized by a short Lyapunov time (hence chaotic by definition), but exhibiting essentially regular variations of proper elements, which undergo changes in a chaotic, but bounded way, with excursions not exceeding a given small amount. This phenomenon is called "stable chaos". (see Milani *et al.* 1996, for a list of such asteroids). The overall picture was later enriched by a variety of chaotic behaviors that in a multitude of ways did not fit in the standard schemes either.

Chaotic behavior is often used to study the dynamical mechanisms responsible, to establish the timing of events like interasteroidal collisions, or, more specifically, to estimate the age of the asteroid families, to constrain the collisional evolution in general or within the families, etc. Orbits, which belong to the stable chaos class, are particularly suitable for the purpose, and we shall discuss in the following a number of examples.

2. CHAOS AS DIAGNOSTIC TOOLS

Understanding of the relationship between chaos and stability and the recognition of the resonant dynamical mechanisms at work in particular cases are very much improved in the last few years, and can be summarized as follows. The behavior of a chaotic orbit over timespan orders of magnitude longer than the Lyapunov time depends upon the resonance responsible for the chaos. If the resonance is a secular one, instability can be very great over a timespan of the order of the Lyapunov time, typically, say, $\simeq 10^4$ yr (Milani and Knežević 1994); if the resonance is in the mean motion and of low order (even if it is only close to) macroscopic instabilities can also occur in a comparatively short time of, say, $\simeq 10^6 - 10^7$ yr (Milani and Farinella 1995, Knežević *et al.* 1996); if there are only high order mean motion resonances in the vicinity of the initial orbit, this orbit can be chaotic, but the instabilities –if any– can occur only over a timespan many orders of magnitude longer than T_L (Milani *et al.* 1996).

As aforementioned, chaos can also be used as a clock, to infer the age of the asteroid families. Families are clusters of objects resulting from disruptive collisions, identified in the space of the so-called proper elements. By definition, proper elements are the invariants of motion, while in practice, they are very nearly constant in time for most asteroids. The "chaotic diffusion" they undergo is so slow that families can still be recognized and reliably identified billions of years after formation. Some asteroids, on the other hand, wander in the proper elements space at a higher pace; if they are members of a family, they may exit from the region occupied by their family after a characteristic time shorter than the lifetime of the solar system (say, in $\approx 10^6$ to 10^9 years, which is the range of likely family ages). In such a case, this characteristic time provides an approximate upper bound to the age of the family, which is, of course, reliable only in a statistical sense, because a chaotic diffusion is an essentially random process.

This "chaotic timing" method has been recently applied to the case of a relatively small and rather compact Veritas family (Milani and Farinella 1994). This family is located in the vicinity of a couple of high order mean motion resonances (Milani *et al.* 1996), and two of its major members, including the largest body in the family 490 *Veritas*, exhibit a typical stable chaos behavior. Proper elements of these two members undergo chaotic diffusion at a rate that brings them outside of the borders of the family in less then 50 Myr (unless the family is much larger than presently known, in which case this figure might become accordingly higher). Thus, if the clustering procedures used to identify the family were applied on the set of proper elements computed at an instant more than 50 Myr away from the initial epoch, the two asteroids would be considered as background objects. As both asteroids undoubtedly belong to the family (as inferred from the evidence on similar physical characteristics of the family members; Tedesco *et al.* 1992), it follows that the family must be younger then that age. The order of magnitude of the estimate also appears to be fairly reliable, as confirmed by the remarkably similar behavior of both objects.

Another case in which chaos and its consequences have been used to study certain aspects of asteroidal collisional evolution, but this time within an asteroid family, is due to Milani and Farinella (1995). They, namely, studied a Koronis family member 2953 *Vysheslavia*, an asteroid about 15 km in diameter, located at the very edge of the Kirkwood gap associated with the 5/2 mean motion resonance. Numerically integrating orbit of this asteroid , Milani and Farinella found that the orbit, apparently another example of the stable chaos, in a relatively short time (< 10 Myr) suddenly becomes highly unstable. Asteroid subsequently falls into the resonance and, after experiencing close approach(es) to Jupiter, ends up into a hyperbolic orbit and escapes from the solar system. Since the outcome of any particular integration of chaotic orbit cannot be considered as necessarily predictive for its real behavior, they extended their analysis by integrating orbits of a number of clones and fictitious neighbours of *Vysheslavia* (obtained by changing its initial orbital elements by an amount, either smaller or only slightly larger than their errors). 13 out of 19 test bodies fell into the resonance in about the same or somewhat longer timespan, and Milani and Farinella concluded that this result is consistent with a dynamical half-life of $\simeq 10$ Myr, and with a very small probability of "survival" in the zone in which asteroid is presently located for more than $\simeq 100$ Myr. Knežević *et al.* (1996) have later demonstrated that there is another asteroid (as yet unnumbered one, 1991 UA_2), located only slightly farther away from the 5/2 resonance than 2953 *Vysheslavia*, which exhibits a similar behavior, with only the time scales involved being somewhat longer. They also showed that the "dangerous" zone at the edge of 5/2 is bounded by a narrow high order mean motion resonance (all the orbits outside that resonance being essentially regular), while another nearby such resonance appears to be responsible for injecting the bodies into the major resonance, providing a dynamical route to it.

Physical implications of the fact that at least two Koronis family members must be dynamically very young objects represent a strong constraint for the plausible interpretations of the formation and possible later collisional evolution of this large family. It is obvious that two objects can neither be simple chance interloopers into the family, nor family members moved to their present location by some peculiar dynamical mechanism (pushed, for example, by a (series of) small non-disruptive collision(s), or by close encounters with Ceres, etc.). They must be fragments of a recent disruptive event, and the question which remains to be answered is whether this was the event in which the family as a whole was formed (in which case it would be the most recent collision involving such a big parent body), or perhaps some secondary break-up of a large primary fragment, originated in the family-forming event and located in the vicinity of the resonance area (which would, then, imply that a number of these secondary fragments, presumably injected deeper into the 5/2, must have already escaped through this "gateway"). At present, there are only indications and hints favouring the latter explanation, but the whole issue is still to be considered as not yet fully settled.

Let's finally mention perhaps the most intriguing implication of chaos in asteroid motions — the possibility of a collision of such an asteroid with the Earth. The orbits of planet-crossing objects, including NEA's and comets, are strongly chaotic due to the close planetary encounters. We know that some of them, wandering around in an essentially unpredictable way, approach the Earth and even impact against it. Current estimates say that several km-sized objects hit the Earth per 1 Myr (Bottke *et al.* 1994), most of them being just NEA's. Approximately 150 such bodies are known at present, some of which quite large in size, but many more are believed to exist (Rabinowitz *et al.* 1994). Even the asteroids whose orbits currently do not cross the orbit of the Earth may pose a threat, if they evolve into the Earth-crossing one. It is also widely believed nowadays that impacts on Earth of celestial bodies exceeding a few kilometers in size have been important factors in the evolution of terrestrial biosphere, and that they also represent one of the major hazards for the future of the human civilization. Many recent studies of NEA's are motivated just by the hazard they pose, but we shall here, for obvious reasons, describe only one.

Asteroid 433 *Eros* currently does not cross the Earth's orbit, but it undergoes frequent close encounters with Mars that give rise to its chaotic behavior. It has a 22 km diameter and thus is twice as large (an order of magnitude more massive) as the object that formed the Chicxulub crater some 65 Myr ago, and probably caused the catastrophic K/T boundary extinction of living species (Alvarez *et al.* 1980). Hence, an investigation of the evolution of Eros's orbit and estimation of the chances that it will become an Earth-crosser and hit our planet might provide important clue to the occurence of global impact catastrophes in the history of the Earth. Michel *et al.* (1996) performed a study of *Eros's* motion, by integrating numerically its nominal orbit and 7 additional clone orbits, obtained by varying slightly the initial conditions and integration circumstances. They demonstrated that there is a significant probability for this asteroid to become an Earth-crosser; it is located in a dynamically complex region, where mean motion and secular resonances act in such a way, as first to prevent, and then to enable, close encounters with the Earth. As a result of these encounters, 3 out of 8 test bodies are going to become Earth-crossers after some time, while for one of them a collision with the Earth was pedited 1.14 Myr after the beginning of the integration.

Althought the sample of the test bodies and the time span covered by integration (2 Myr) were both quite limited, some conclusions could be drawn on the basis of the obtained results. Firstly, it must be clearly stated that the described result does not mean that the real body will actually collide with the Earth in precisely 1.14 Myr time. Chaotic orbits being subject only to probabilistic considerations, the above result just means that we cannot exclude the chance that *Eros* will strike the Earth within the next few million years; at the same time, the results indicate that there is practically no chance of such an event over the next $\simeq 10^5$ yr. Statistically, we expect a much longer lifetime of this asteroid ($10^8 - 10^9$ yr, with respect to both an Earth impact and a disruptive collison with another asteroid), with most of the orbital evolution occuring in the Mars-crossing region. Secondly, it is also clear that impacts with massive asteroids are not unlikely to have occured in the history of our planet (or to occur in its future), and that large impactors, even if at present not found in the Earth's neighbourhood, can be delivered there through the peculiar dynamical mechanisms, which probably affect the dynamics of many other near-Earth asteroids.

References

Alvarez, L., Alvarez, W., Asaro, F., and Michel, H.V.: 1980, *Science*, **208**, 1095.
Bottke, W.F., Nolan, M.C., Greenberg, R., and Kolvoord, R.A.: 1994, in: *Hazards due to Comets and Asteroids*, p. 337 (T. Gehrels, Ed.), Univ. Arizona Press, Tucson.
Knežević, Z., Milani, A., and Farinella, P.: 1996, *Planet. Space. Sci.*, in preparation.
Laskar, J.: 1989, *Nature* **338**, 237.
Michel, P., Farinella, P., and Froeschlé, Ch.: 1996, *Nature* **380**, 689.
Milani, A., and Farinella, P.: 1994, *Nature* **370**, 40.
Milani, A., and Farinella, P.: 1995, *Icarus* **104**, 209.
Milani, A., and Knežević, Z.: 1994, *Icarus* **107**, 219.
Milani, A., and Nobili, A.M.: 1992, *Nature* **357**, 569.
Milani, A., Nobili, A.M., and Knežević, Z.: 1996, *Icarus*, in press.
Nobili, A.M.: 1989, *Celestial Mechanics* **45**, 293.
Rabinowitz, D., Bowell, E., Shoemaker, E., and Muinonen, K.: 1994, in:*Hazards due to Comets and Asteroids*, p. 285 (T. Gehrels, Ed.), Univ. Arizona Press, Tucson.
Tedesco, E.F., Veeder, G.J., Fowler, J.W., and Chillemi, J.R.: 1992, *IRAS Minor Planet Survey*, Phillips Laboratory, Hanscom AFB.
Whipple, A. L.: 1995, *Icarus* **115**, 347.