Publ. Astron. Obs. Belgrade No. 54 (1996), 77 – 80 *Contributed paper*

ON p-ADIC AND ADELIC WAVE FUNCTION OF THE UNIVERSE

B. DRAGOVICH[1] and Lj. NEŠIĆ[2]

[1] *Institute of Physics, P.O.Box 57, 11001 Belgrade, Yugoslavia*
E-mail dragovic@shiva.phy.bg.ac.yu.

[2] *Department of Physics, Faculty of Philosophy, P.O.Box 92, 18000 Niš*
E-mail nesiclj@ban.junis.ni.ac.yu.

Abstract. In the Hartle-Hawking approach the wave function for the ground state of a spatially closed universe obtains by Feynman's path integral method. It corresponds to an amplitude of transition from "nothing" to 3-dimensional hypersurface over all compact 4-metrics connecting these two 3-space states. This article represents further development of one of the authors' (B.D.) idea that integration over compact real metrics has to be extrapolated to all p-adic compact metrics. Adelic wave function of the Universe, unifies the wave functions calculated over real and p-adic metrics, and yields the space-time discreteness.

1. INTRODUCTION

While all experimental and observational data always belong to the field of rational numbers Q, theoretical models mainly use the field of real $R=Q_\infty$, and field of complex numbers. Completion of Q with respect to the absolute value ($||_\infty$) gives the field of real numbers. According to the Ostrowski theorem (Vladimirov V. S. *et al.* 1994) any non-trivial norm on the field of rational numbers Q is equivalent to the usual absolute value $||_\infty$ or to the p-adic norm $||_p$, p-a prime number. Completions of Q with respect to the p-adic norms give fields of p-adic numbers Q_p. p-adic norm is ultrametric and for a rational number, $x \in Q$, $x = p^\gamma \frac{m}{n}$, $0 \neq n, \gamma, m \in Z$, has a value $|x|_p = p^{-\gamma}$. Any p-adic number $x \in Q_p$ (in a canonic form) is an infinite expansion $x = p^\gamma(x_0 + x_1 p + x_2 p^2 + ...), 0 \leq x_i \leq p-1$.

Real and p-adic numbers can be unified by means of adeles. Infinite sequences in form $a = (a_\infty, a_2, ..., a_p, ...)$, where $a_\infty \in Q_\infty$, $a_p \in Q_p$ with restriction that $a_p \in Z_p$ ($|a_p|_p \leq 1$), for all but a finite number of p, is adele (Gel'fand I. M. *et al.* 1966). An important function on A is additive character

$$\chi(x) = \prod_v \chi_v(x_v) = exp(-2\pi i x_\infty) \prod_p exp(2\pi i \{x_p\}_p). \quad (1)$$

A motivation for application of adeles in quantum cosmology comes from the fact that an adelic spacetime contains archimedean (usual) and nonarchimedean (ultrametric) geometries (Dragovich B. 1995a,b), which might be present at the very beginning of the Universe evolution. According to quantum gravity an uncertainty of measuring distances is $\Delta l \geq l_{pl} = \sqrt{\hbar G/c^3} \approx 10^{-35} m$, l_{pl} is the Planck lenght. For these very small distances the Archimedean axiom of the Euclidean geometry is no more valid. It seems that l_{pl} separates possible geometries into: (i) real (archimedean) if $l \geq l_{pl}$, (ii) p -adic (non-archimedean) if $l \leq l_{pl}$.

2. ADELIC QUANTUM MECHANICS

Let S be a finite set of p-adic norms and let $A(S) = Q_\infty \times \prod_{p\in S} Q_p \times \prod_{p\notin S} Z_p$. Space of all adeles is then $A = \bigcup_S A(S)$ and is a topological ring. Adelic quantum mechanics (Dragovich B. 1994, Dragovich B. 1995c) is a triple $(L_2(A), W(z), U(t))$, where: A-additive group of adeles, z-adelic point of a classical phase space, t-adelic time, $L_2(A)$-Hilbert space of complex square integrable functions with respect to the Haar measure on A, $W(z)$-unitary representation of the Heisenberg-Weyl group on $L_2(A)$ and $U(t)$-unitary representation of the evolution operator on $L_2(A)$.

According to adelic quantum mechanics the ground state of a quantum-mechanical system is of the form

$$\Psi(x) = \Psi_\infty(x_\infty) \prod_{p\in S} \Psi_p(x_p) \prod_{p\notin S} \Omega(|x_p|_p) \tag{2}$$

where $\Omega(|x_p|_p)$ is a characteristic function of a unit circle, defined by $\Omega(u) = 1$ if $0 \leq u \leq 1$ and $\Omega(u) = 0$ if $u > 1$.

3. ADELIC QUANTUM COSMOLOGY

Adelic quantum cosmology is the application of adelic quantum theory on the Universe as a whole. In other words, it is adelic generalization of ordinary quantum cosmology. A necessary condition for cosmological model to be adelic is the existence of p-adic ground-state wave function in the form $\Omega(|x_p|_p)$, for almost all p. Adelic wave function for the ground state of the Universe can be expresed as (Dragovich B. 1995b) product of path integrals

$$\Psi[h_{ij}] = \int \chi_\infty(-S_\infty) D(g_{\mu\nu})_\infty \prod_p \int \chi_p(-S_p) D(g_{\mu\nu})_p \tag{3}$$

(h_{ij}-adelic compact 3-metric, $(g_{\mu\nu})_\infty$ and $(g_{\mu\nu})_p$-real and p-adic 4-metrics, respectively). $\chi(-S_v)$ is adelic character of the Einstein-Hilbert action $S = (S_\infty, S_2, ..., S_p, ...)$ for the gravitational field, the cosmological constant and the matter fields.

We consider two minisuperspace cosmological models: the de Sitter model in $D = 4$ and the model with the cosmological constant in $D = 3$ dimensions. The corresponding model in D dimensions is given by the Einstein-Hilbert action with cosmological term

$$S = \frac{1}{16\pi G} \int_M d^D x \sqrt{-g}(R - 2\Lambda) + \frac{1}{8\pi G} \int_{\partial M} d^{D-1} x \sqrt{h} K, \tag{4}$$

(R-scalar curvature of the D-manifold M, K-trace of the exstrinsic curvature K_{ij} at the boundary ∂M of the D-manifold M) and by the Robertson-Walker metric (Halliwel J. J. and Myers R. C. 1989)

$$ds^2 = \sigma^2 \left[-N^2 dt^2 + a^2(t) d\Omega^2_{D-1}\right]. \tag{5}$$

In this expression $d\Omega^2_{D-1}$ is the metric on the unit (D–1)-sphere, $\sigma^{D-2} = 8\pi G/[V^{D-1}(D-1)(D-2)]$, V^{D-1}-volume on the unit $(D-1)$-sfere, $N(t)$ is the lapse function and $a(t)$ is the scale factor. In the case $D = 4$, we shall use

$$ds^2 = \sigma^2 \left(-\frac{N^2(t)}{q(t)} dt^2 + q(t) d\Omega^2_3\right) \tag{6}$$

which was considered in the real case (Halliwel J. J. and Louko J. 1989) and leads to quadratic action. For this metric classical v-component of adelic action is

$$S^{cl}_v = \frac{\lambda^2 T^3}{24} - [\lambda(q_2 + q_1) - 2]\frac{T}{4} - \frac{(q_2 - q_1)^2}{8T} \tag{7}$$

($q_1 = q(0), q_2 = q(T)$). Minisuperspace propagator is

$$G_v(q_2, q_1) = \int dT K_v(q_2, T; q_1, 0) \tag{8}$$

where $K_v(q_2, T; q_1, 0)$ is the usual quantum-mechanical propagator

$$K_v(q_2, T; q_1, 0) = \int \chi_v(-S[q]) D(q)_v \tag{9}$$

which, owing to the fact that action is quadratic on q_1 and q_2, is

$$K_v(q_2, T; q_1, 0) = \frac{\lambda_v(-8T)}{\sqrt{|4T|_v}} \chi_v(-S_{cl}).$$

The function $\lambda_v(a)$ has the following properties: $|\lambda_v(a)|_\infty = 1$, $\lambda_v(b^2 a) = \lambda_v(a)$, $\lambda_v(a)\lambda_v(b) = \lambda_v(a+b)\lambda_v(a^{-1} + b^{-1})$ for any $v = \infty, 2, \ldots$ and $a \neq 0, b \neq 0$.

The Hartle-Hawking wave function obtains by integration of (8) for boundary conditions $q_1 = 0$, $q_2 = q$. To obtain wave function in p-adic case we have to perform the integration (8) in the range $|T|_p \leq 1$. Hence, in p-adic case, for the de Sitter model we have

$$\Psi_p(q, \lambda) = \int\limits_{|T|_p \leq 1} dT \frac{\lambda_p(-8T)}{\sqrt{|4T|_p}} \chi_p \left[-\frac{\lambda^2 T^3}{24} + (\lambda q - 2)\frac{T}{4} + \frac{q^2}{8T}\right]. \tag{10}$$

It was shown,(Dragovich B. 1995b), for $p \neq 2, 3$, that function Ψ_p becomes $\Omega(|q|_p)$ if $|\lambda|_p \leq 1$. Since the interpretation of adelic wave function performs only in rational points, if we use $\lambda = 3 \cdot 4 \cdot l$, $l \in Z$, then $\Psi_3(q, \lambda) = \Omega(|q|_3)$, while for $|q_2|_2 = 1, |q_2|_2 < 1, |q_2|_2 > 1$, we obtain $\Psi_2(q_2) = -1, 1, 0$, respectively.

The wave function for $D = 3$ in the real case is presented in (Halliwel and Myers, 1989). The corresponding p-adic wave function is obtained performing integration

$$\Psi_p(a,\lambda) = \int\limits_{|N|_p\le 1} dN \frac{\lambda_p(-2N)}{\sqrt{|N|_p}} \chi_p \left[\frac{\sqrt{\lambda} a^2}{2} cth \left(N\sqrt{\lambda} \right) \right]. \tag{11}$$

For $p \neq 2$ one obtains $\Psi_p(a,\lambda) = \Omega(|a|_p)$, when $p=2$ then $\Psi_2(a,\lambda) = \frac{1}{2}\Omega(|a|_2)$, with condition $\lambda = 16 \prod\limits_p p^2$.

4. CONCLUSION

Both models ($D = 4$, $D = 3$), are adelic, i.e. their adelic wave functions of the ground state have the form (2). The interpretation of the obtained wave functions we perform at rational points. The square adelic wave function of the de Sitter model, for $q \in Q$, and $\lambda = 3 \cdot 4 \cdot l$, is

$$|\Psi(q)|_\infty^2 = |\Psi_\infty(q)|_\infty^2 \prod_p \Omega(|q|_p).$$

For $q \in Z$ this expression becomes $|\Psi(q)|_\infty^2 = |\Psi_\infty(q)|_\infty^2$, while for $q \in Q \backslash Z$ is $|\Psi(q)|_\infty^2 = 0$. The wave function argument is the scale factor which is connected with radius Universe. If we express q in Planck lenght it means that there is nothing within the Plank distance (since the square of adelic wave function is equal to zero), i.e. the space is discrete. The similar conclusion holds for the $D = 3$ case.

References

Dragovich, B.: 1995a, Adelic generalization of wave function of the universe, *Publ. Obs. Astron. Belgrade*, 143.

Dragovich, B.: 1995b, Adelic wave function of the universe, *Proceedings of the Third Alexander Friedmann International Seminar on Gravitation and Cosmology*, S. Peterburg.

Dragovich, B.: 1994, *Theor. Math. Phys.* **101**, 349.

Dragovich B.: 1995c, *Int. J. Mod. Phys.* **A10**, 2349.

Gel'fand, I. M., Graev, I. M., Piatetskii-Shapiro, I. I.: 1966, *Representation Theory and Automorphic Functions*, Nauka, Moskva.

Halliwell, J. J., Louko J.: 1989, *Phys. Rev.* **D39**, 2206.

Halliwell, J. J., Myers, R. C.: 1989, *Phys. Rev.* **D40**, 4011.

Vladimirov, V. S., Volovich, I. V. and Zelenov, E. I.: 1994, *p-adic Analysis and Mathematical Physics*, World Scientific, Singapore.