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ADELIC GENERALIZATION OF WAVE FUNCTION OF THE UNIVERSE

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Abstract. Recently adelic quantum mechanics has been formulated as p-adic and adelic generalization of ordinary quantum mechanics. Here, we propose adelic quantum mechanics to investigate quantum dynamics of the universe. We show that the Hartle-Hawking approach to quantum cosmology may be generalized in an adelic way. Adelic wave function of the universe is a product of the standard Hartle-Hawking wave function and the corresponding p-adic ones.

1. ADELIC QUANTUM MECHANICS

It is well known that experimental and observational data always belong to the field of rational numbers Q, and theoretical models traditionally use real R and complex numbers C. However, as completion of Q with respect to the usual absolute value gives R, in the same way completion of Q with respect to p-adic norms (p = a prime number) creates the fields of p-adic numbers Q_p (p = 2,3,5,...).

Since 1987, p-adic numbers have been applied in various branches of theoretical and mathematical physics (for a review, see Brekke and Freund 1993, and Vladimirov, Volovich and Zelenov 1994). One of the significant achievements is a formulation of p-adic quantum mechanics, whose complex-valued wave functions depend on p-adic variables. Ordinary and p-adic quantum mechanics may be naturally unified in a form of adelic quantum mechanics (Dragovich 1995).

A mathematical concept to unify real and p-adic numbers is an adele, which is an infinite sequence, $a = (a_\infty, a_2, ..., a_p, ...)$, where $a_\infty \in R$, $a_p \in Q_p$ with the restriction that $a_p \in Z_p = \{x \in Q_p \,||\, x\,|_p \leq 1\}$ for all but a finite number of p. The set of all adeles A is a ring under componentwise addition and multiplication. An additive character on A is $\chi(xy) = \chi_\infty(x_\infty y_\infty)\prod_p \chi_p(x_p y_p) = exp(-2\pi i x_\infty y_\infty)\prod_p exp 2\pi i\{x_p y_p\}_p$, where $x, y \in A$ and $\{a_p\}_p$ denotes the fractional part of the p-adic expansion of a_p.

On an adelic space A one can define a Hilbert space $L_2(A)$, which is a space of complex-valued and square integrable functions with respect to the Haar measure on A. An orthonormal basis of the adelic Hilbert space consists of infinite products of the orthonormal states $\Psi_\infty \in L_2(R)$ and $\Psi_p \in L_2(Q_p)$, where $\Psi_p(x_p) = \Omega(|\, x_p\,|_p)$ for all but a finite number of p. Here, $\Omega(a) = 1$ if $0 \leq a \leq 1$ and $\Omega(a) = 0$ if $a > 1$.

2. ADELIC QUANTUM COSMOLOGY

The main motivation for application of adeles in quantum cosmology comes from the fact that an adelic spacetime contains archimedean (usual) and nonarchimedean (ultrametric) geometries, which may be present at the very beginning of the universe evolution. Adelic quantum cosmology is the application of adelic quantum theory to the universe as a whole, and it is adelic generalization of the usual quantum cosmology.

The Wheeler-DeWitt equation, as well as the Schrödinger equation, is not appropriate to generalize to p-adic and adelic cases. However, the path integral approach for the fundamental quantum-mechanical amplitudes can be generalized to p-adic and adelic dynamics.

To obtain the Hartle-Hawking (Hartle and Hawking 1983) wave function for the ground state of the universe in adelic quantum cosmology one has to enlarge functional integration over all compact 4-geometries of the real case by adding the corresponding ones of p-adic cases. It means that adelic ground- state of the universe can be presented as

$$\Psi[h_{ij}] = \int \chi_\infty(-S_\infty) D(g_{\mu\nu})_\infty \prod_p \int \chi_p(-S_p) D(g_{\mu\nu})_p,$$

where h_{ij} is an adelic 3-metric, and $\chi(-S_v)$, $(v = \infty, 2, ...)$, are additive characters of the Einstein-Hilbert gravitational actions (without matter fields). Note that $S_v \in Q_v$ and must have number field invariant form. In a simple minisuperspace model (with scale factors a_v) the ground-state wave function should be of the form $\Omega(| a_p |_p)$ for all but a finite number of p. This is a rather strong restriction which may be helpful in further investigation of quantum cosmology.

According to this approach the universe at very small distances (about the Planck length) consists of real and p-adic spaces in the form of an adelic space.

The adelic de Sitter minisuperspace model is under consideration and will be presented elsewhere.

To better understand many notions contained in this article, see Aref'eva et al. 1991.

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