

THE FUNCTIONAL RELATION BETWEEN MEAN MOTION  
RESONANCES AND YARKOVSKY FORCE  
ON SMALL ECCENTRICITIES

I. MILIĆ ŽITNIK

*Astronomical Observatory Belgrade, Serbia*  
*E-mail: ivana@aob.rs*

**Abstract:** This work examines asteroid's motion with orbital eccentricity  $e$  in the range (0.1, 0.2) across the two-body mean motion resonance (MMR) with Jupiter due to the Yarkovsky effect. We calculated time delays  $dtr$  caused by the resonance on the mobility of an asteroid with the Yarkovsky drift speed. Our results considered only asteroids that successfully cross over the resonance without close encounters with planets. We found a functional relation that accurately describes dependence between the average time lead/lag  $dtr$ , the strength of the resonance (SR), and the semimajor axis drift speed  $da/dt$  with asteroids' orbital eccentricities in the range (0.1, 0.2). We analysed average values of  $dtr$  using this functional relation comparing with obtained values of  $dtr$  from the numerical integrations, which were performed in an *ORBIT9* integrator with a very large number of test asteroids. We checked the validity of our previous functional relation, derived for asteroids' orbital eccentricities in the range (0, 0.1) (Figure 1), on the present results for eccentricities in the range (0.1, 0.2) (Figure 2). Also, we tried to find a unique functional relation for the whole interested interval of asteroids' orbital eccentricities (0, 0.2) and discussed it.

**Keywords:** asteroids, asteroid's motion, eccentricity, numerical integrations

## 1. RESULTS

Results for  $e$  in (0, 0.1),  $i = 5^\circ$  and  $\omega = 60^\circ$  according to equation:

$$\log_{10}(\langle dtr \rangle) = \beta \log_{10}(SR) + \gamma \log_{10}\left(\frac{da}{dt}\right) + c_2 \quad (1)$$

For all 11 MMRs:

$$\beta = 0.44 \pm 0.03, \quad \gamma = -1.09 \pm 0.20, \quad c_2 = 4.35 \pm 0.66 .$$

For the strongest 6 MMRs:

$$\beta = 0.47 \pm 0.04, \quad \gamma = -0.97 \pm 0.15, \quad c_2 = 5.11 \pm 0.54 .$$

So, the two set of parameters are statistically the same.

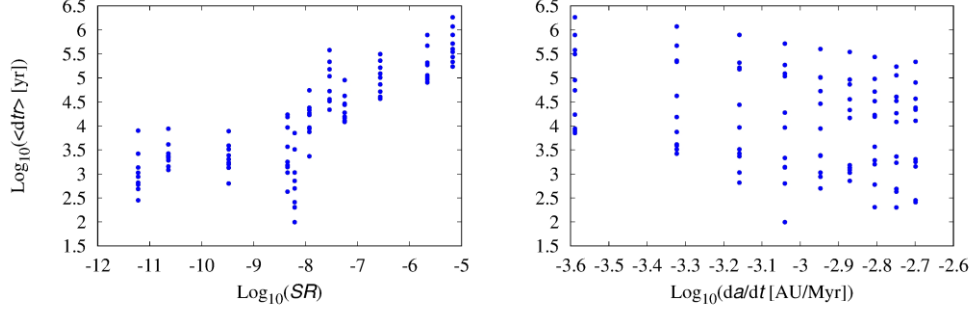


Figure 1: Results for  $e$  in (0, 0.1).

Results for  $e$  in (0.1, 0.2) according to equation:

$$\langle dtf \rangle = (0.5 + da) \log_{10}(SR) + (db - 1.0) \log_{10}\left(\frac{da}{dt}\right) + (dc + 5.0) \quad (2)$$

For the strongest 4 MMRs:

$$da = -1.62 \pm 0.31, \quad db = 2.46 \pm 0.5, \quad dc = -5.94 \pm 2.02;$$

For the weakest 7 MMRs:

$$da = -0.47 \pm 0.03, \quad db = 0.88 \pm 0.09, \quad dc = -5.14 \pm 0.32.$$

Obviously, two sets of coefficients  $\{0.5 + da, db - 1.0\}$  are different in sign, because of different relations for two cases of the weakest and the strongest resonances.

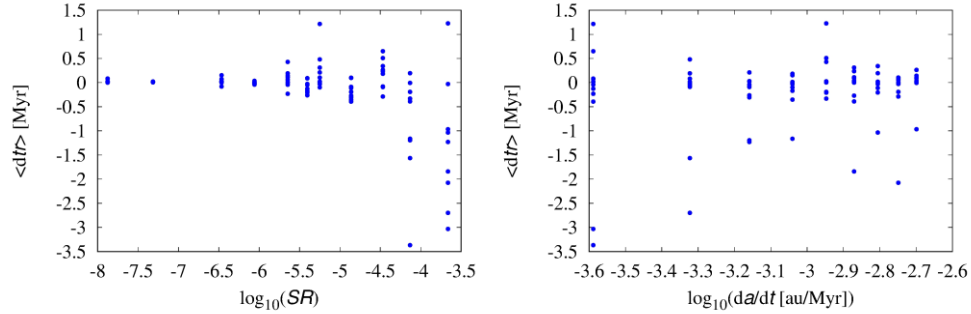


Figure 2: Results for  $e$  in (0.1, 0.2).

## 2. CONCLUSIONS

We presented a new description of the orbital behaviour of resonant asteroids under the influence of the Yarkovsky effect for eccentricity in the range (0, 0.2).

We derived and established Equation (1) that enables easy and fast calculation of the average time that an asteroid spent in a two-body MMR with known resonance's strength in  $[6 \times 10^{-12}, 6.7 \times 10^{-6}]$  interval, with the Yarkovsky drift speed in  $[2.6 \times 10^{-4}, 2 \times 10^{-3}]$  au/Myr interval and with an asteroid's orbital eccentricity in the range (0, 0.1).

We derived and established Equation (2) that enables easy and fast calculation of the average time that an asteroid spent in a two-body MMR with known resonance's strength in  $[1.3 \times 10^{-8}, 2.2 \times 10^{-4}]$  interval, with the Yarkovsky drift speed in  $[2.6 \times 10^{-4}, 2 \times 10^{-3}]$  au/Myr interval and with an asteroid's orbital eccentricity in the range (0.1, 0.2).

Moreover, Equation (1) and Equation (2) can be easily included in any Monte Carlo methods in order to follow the motion of asteroids across the mean motion resonances in the Main Belt.

Our future studies on the interaction between the mean motion resonances and the Yarkovsky drift speeds will include two-body MMRs with other planets, as well as three-body MMRs. Moreover, in future work, we plan to include asteroid's orbital inclination.

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### References

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