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ELECTRON TRANSPORT, STREAMER PROPAGATION AND LIGHTNING IN THE ATMOSPHERE OF TITAN

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Abstract. In this work, we study the transport of electrons, the inception and propagation of streamers and the possibility for lightning in Titan's atmosphere. A multi term theory for solving the Boltzmann equation is used to calculate electron transport coefficients in a N_2-CH_4 mixture that mimics the atmosphere of Titan, over a range of the reduced electric fields. The calculated transport coefficients are then used as input into the classical fluid model to study the transition from an electron avalanche into a negative streamer. We also apply a 2.5 dimensional Particle-in-Cell Monte Carlo Collision (PIC/MCC) code with cylindrical symmetry and photoionization to simulate the inception, propagation and branching of both positive and negative streamers in the ambient electric field between $1.5E_k$ and $3E_k$, where E_k is the breakdown electric field. We found that a successful inception of streamer in Titan's clouds at altitudes of about 20 km, requires a large electric field of 4.2 MV/m, which corresponds to $3E_k$.

1. INTRODUCTION

Lightning is one of the most dramatic phenomena in the Earth's atmosphere, releasing an enormous amount of electromagnetic energy across the wide range of frequencies. Lightning is a signature of atmospheric dynamics and cloud structure, and it plays a vital role in atmospheric chemistry, through the production of numerous chemical compounds. Some of these chemicals are organic molecules and therefore of great significance to biological processes.

Lightning is a phenomenon that is not confined solely to the Earth's atmosphere. Since the time of the Voyager missions in the 1980s, we have clear evidence of lightning in the atmospheres of gas giants, Jupiter and Saturn, and ice giants, Uranus and Neptune (Desch et al. 2002, Yair 2012, Aplin et al. 2020). On the other side, one of the unresolved issues after the Cassini-Huygens mission to the Saturnian system is whether or not lightning occurs in Titan's atmosphere. Titan is the largest of Saturn's satellites and has a massive atmosphere with a surface pressure higher than Earth's by approximately 50%. Titan's atmosphere is made up primarily of N_2 and CH_4 and traces of H_2, HCN and a wide range of hydrocarbons and nitriles (Nixon et al. 2018). Titan's atmospheric chemistry modeling studies suggest the existence of lightning, as the quantity of HCN and C_2H_2 in the atmosphere cannot be explained in terms of photochemical models (Vuitton et al. 2019).

In this work, we discuss the basic properties of streamer discharges, pre-cursors of lightning, in Titan's atmosphere. Streamers are thin channels of non-equilibrium plasma whose dynamics is controlled by the highly-localized non-linear regions of space charge and steep gradients of the electron number density. Depending on the conditions, they can form complex structures resembling trees or other forms, and can spread over a few centimeters in laboratory experiments or a few kilometers in planetary atmospheres. The first step in studies of streamer discharges is to consider the transport of electrons in a mixture of gases that mimics the atmosphere of Titan. Using the cross section sets for electron scattering in N_2 and CH_4 as input, we solve the non-conservative Boltzmann equation in order to calculate electron transport coefficients. In addition, from the E/N-profiles of the ionization and attachment rates (where E is the electric field while N is the gas number density), we determined the breakdown electric field, which is an important quantity in streamer studies. While electron transport coefficients are required as input data in fluid equation-based models of streamer discharges, sets of cross sections for electron scattering in N_2 and CH_4 are required as input data in the PIC/MCC modeling.

In Section 2, we briefly review the methodology of the present work. First, we present the basic elements of a multi term theory for solving the non-conservative Boltzmann equation which has been used to calculate electron transport coefficients as a function of the electric field. The classical fluid and PIC/MCC models are also briefly outlined and used to study the transition from an electron avalanche into a negative streamer and propagation of streamers in various electric fields and various gas mixtures. The variation of electron transport coefficients with E/N and properties of both negative and positive streamers, are reported in Section 3.

2. METHODS OF CALCULATIONS

2. 1. BOLTZMANN EQUATION ANALYSIS

The behavior of electrons in Titan's atmosphere under the influence of the spatially homogeneous electric field $\mathbf{E}$, is described by the phase-space distribution function $f(\mathbf{r}, \mathbf{c}, t)$ representing the solution of the Boltzmann equation (Dujko et al. 2010, 2011):

$$\frac{\partial f}{\partial t} + \mathbf{c} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{e_0}{m} \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{c}} = -J\left(f, f_0\right), \tag{1}$$

where $\mathbf{r}$ and $\mathbf{c}$ denote the position and velocity coordinates, e_0 and m are the charge and mass of the electrons and t is time. Swarm conditions are assumed to apply and $J\left(f, f_0\right)$ denotes the rate of change of f due to binary collisions between electrons and neutral molecules. The neutral molecules are assumed to remain in thermal equilibrium, characterized by a spatially homogeneous phase-space distribution function f_0.

The angular dependence of $f(\mathbf{r}, \mathbf{c}, t)$ in velocity space, is resolved by expanding the distribution function in terms of spherical harmonics $Y_l^{(m)}\left(\widehat{\mathbf{c}}\right)$, where $\widehat{\mathbf{c}}$ denotes the angles of $\mathbf{c}$. Under hydrodynamic conditions, $f(\mathbf{r}, \mathbf{c}, t)$ is further expanded in terms of the gradients of the electron number density $n\left(\mathbf{r}, t\right)$. Thus, $f(\mathbf{r}, \mathbf{c}, t)$ can be expanded as

$$f(\mathbf{r},\mathbf{c},t)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{s=0}^{\infty}\sum_{\lambda=0}^{s}\sum_{\mu=-\lambda}^{\lambda} f(lm|s\lambda\mu)Y_l^{(m)}(\widehat{\mathbf{c}})G_\mu^{(s\lambda)}n\left(\mathbf{r},t\right), \tag{2}$$

where $G_\mu^{(s\lambda)}$ is the irreducible gradient tensor operator. The coefficients $f(lm|s\lambda\mu)$ are functions of the speed c, obtained by the expansion

$$f(lm|s\lambda\mu)=\omega\left(T_b,c\right)\sum_{\nu=0}^{\infty}F\left(\nu lm|s\lambda\mu\right)R_{\nu l}\left(T_b,c\right), \tag{3}$$

where $\omega\left(T_b,c\right)$ is a Maxwellian distribution at an adjustable base temperature T_b, and $R_{\nu l}$ is related to a Sonnine polynomial of order (ν,l). The coefficients $F\left(\nu lm|s\lambda\mu\right)$ are called "moments"and they are related to the electron transport quantities.

Substitution of expansions (2)-(3) into the Boltzmann equation (1) leads to a hierarchy of coupled differential equations for the moments $F\left(\nu lm|s\lambda\mu\right)$. These equations are numerically solved and all transport coefficients of interest, including the mean energy, drift velocity and diffusion tensor as well as rate coefficients are expressed in terms of these moments (Dujko et al. 2010, 2011).

2. 2. THE FLUID MODEL

Transition from an electron avalanche into a negative streamer is studied by using the classical fluid model (Dujko et al. 2013). In this model the electron flux is obtained by assuming a steady-state of the momentum balance equation, and that the electron energy of the field-directed motion is much greater than the thermal contribution. The generalized one-dimensional continuity equation for the electron number density is

$$\frac{\partial n_e}{\partial t}=\frac{\partial}{\partial x}\left(W\mathrm{sgn}\left(E\right)n_e+D_L\frac{\partial n_e}{\partial x}\right)+\left(\nu_i-\nu_a\right)n_e, \tag{4}$$

where W and D_L are the electron drift velocity and longitudinal diffusion coefficient, respectively, electric field E is oriented along the x-axis, while ν_i and ν_a are the ionization and electron attachment coefficients, respectively. The drift and diffusion of both positive and negative ions are neglected on the time scales of interest in the present work. Thus,

$$\frac{\partial n_p}{\partial t}=\nu_i n_e,\quad \frac{\partial n_n}{\partial t}=\nu_a n_e, \tag{5}$$

where n_p and n_n are positive and negative ion densities.

The model is realized in a 1.5 dimensional (1.5D) setup. In this model, we assume that the space charge is contained inside a cylinder with radius R_0 and that the charge density varies along the axial direction only. Thus, the total electric field in the system is evaluated as the sum of the uniform external electric field and the electric field due to space charge:

$$E\left(x,t\right)=E_0+\frac{e}{2\varepsilon_0}\int_0^d\left(n_p-n_n-n_e\right)\left(\mathrm{sgn}\left(x-x'\right)-\frac{x-x'}{\sqrt{\left(x-x'\right)^2+R_0^2}}\right)dx', \tag{6}$$

where E_0 and ε_0 are the external (applied) electric field and vacuum permittivity, respectively, and d is the length of the system.

The above fluid equations are closed, assuming the local field approximation. According to this approximation, the input data, including W, D_L, ν_i and ν_a are assumed to be functions of the local instantaneous electric field. In the numerical implementation of our fluid model, the spatial discretization is performed by using the second order central finite difference, while the fourth order Runge–Kutta method is used for the integration in time.

2. 3. PARTICLE-IN-CELL/MONTE CARLO COLLISION SIMULATIONS

In addition to classical fluid model, we also apply a 2.5D PIC/MCC model with a cylindrical symmetry to simulate the development of both positive and negative streamers in the ambient electric field (Chanrion and Neubert 2008, Köhn et al. 2019). In order to simulate positive streamers, the photoionization model initially developed by Zheleznyak et al. (1982) is implemented into the code. In PIC/MCC simulations, we follow the motion of single electrons as they move through a background medium under the influence of the electric field. The Monte Carlo method is used to determine the exact time and the nature of a collision between a single electron and the background molecule. After particle positions have been updated, they are mapped onto the grid for the purpose of computation of the charge density distribution. The space charge profile is then used as a source term for the Poisson equation that determines the electric field for the next time step. As boundary conditions, we use Neumann conditions $\partial\phi/\partial r = 0$ for $r = 0$, L_r, and Dirichlet conditions for the electric potential $\phi(r, 0) = 0$ and $\phi(r, L_z) = \phi_{max} = E_{amb}\, L_z$, where L_r and L_z are dimensions of the simulation domain, while E_{amb} is the ambient electric field.

We initiate all simulations with a charge neutral electron-ion patch at the center of the simulation domain. The initial electron density is given by the Gaussian distribution with a a peak density of $n_{e,0} = 10^{20}$ m^{-3}. For more details on PIC/MCC simulations, the reader is referred to Köhn et al. (2019).

3. RESULTS AND DISCUSSION

On Titan clouds form between 20 and 35 km altitude with the pressure varying between approximately 0.1 bar and 0.6 bar, the ambient temperature varying between 70 and 75 K and the level of CH_4 varying between 1.6% and 2.0%. In our calculation we choose 1.6% of CH_4 while the pressure and temperature are set to 0.3 bar and 75K, respectively. Complete and self-consistent cross section sets for electron scattering in N_2 and CH_4, initially developed by Stojanović and Petrović (1998) and Šašić et al. (2004), are used as an input for solving the Boltzmann equation and performing PIC/MCC simulations.

In figure 1 we show the variation of the drift velocity (a), longitudinal diffusion coefficient (b) and rate coefficients for ionization and electron attachment (c) as a function of E/N. The properties and energy dependence of the cross sections are reflected in the E/N-profiles of the presented transport coefficients. From the E/N-profile of the drift velocity, we observe that there are no signs of a negative differential conductivity (NDC), i.e., the drift velocity is a monotonically increasing function of E/N. We also note that the influence of non-conservative collisions on the electron transport is not apparent until about 100 Td and increases with the electric field. This is consistent with an increase in the ionization rate above this value of E/N.

The attachment rate is negligible for all E/N studied and its explicit influence on the electron transport is minimal. The breakdown field E_k is defined as the electric field when the attachment rate equals the ionization rate. The transition from an avalanche of electrons to a streamer requires the electric field to be at least over the breakdown field. Assuming that Titan's atmosphere is composed of 98.4% of N_2 and 1.6% of CH_4, we find that the breakdown electric field is 50 Td.

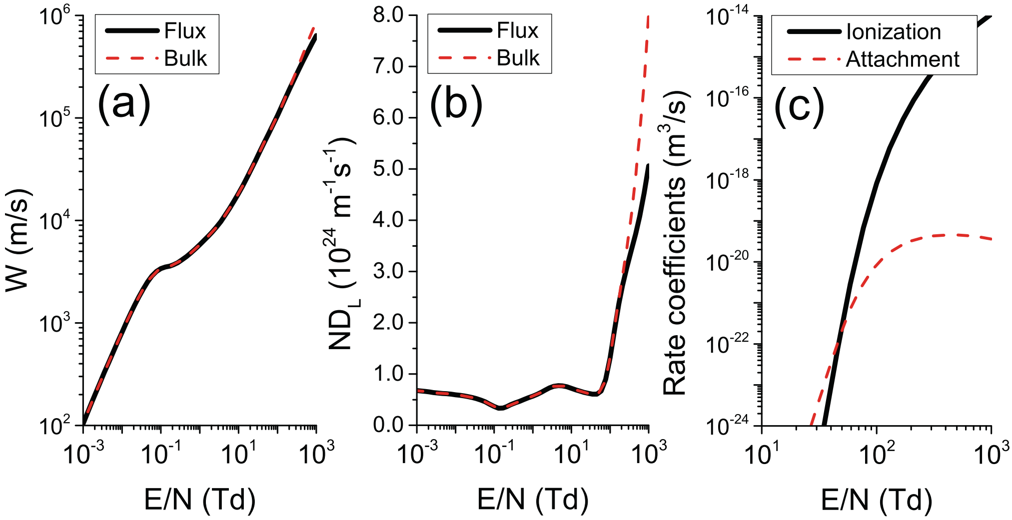

Figure 1: Variation of the drift velocity (a), longitudinal diffusion coefficient (b) and rate coefficients for ionization and attachment (c) with E/N.

Calculated with the 1.5D fluid code, in figure 2 we show the spatial and temporal development of the electron density and the electric field when the externally applied electric field is $4E_k$. The simulation begins with the same initial Gaussian-type distribution of electrons and positive ions reflecting the macroscopic plasma neutrality (panel a). Both flux and bulk transport data are used as an input to solve the system of fluid equations. At the beginning of the evolution, we see that the electron density grows as a result of the electron impact ionization (panel b). The electrons drift in the opposite direction to the electric field while the positive ions drift slowly in the opposite direction. The mobility of positive ions is much lower and therefore the space-charge effects develop (panel c). As the evolution continues, the electric field is enhanced at the front of the streamer, while within the streamer interior is almost entirely shielded. Consequently, ionization processes may not occur in the streamer channel (panel d).

We found that in electric fields $\leq 3E_k$ there is no transition from an avalanche into a streamer. For such electric fields, the initial Gaussian grows due to electron impact-ionization without distorting the external electric field. We also found that the velocity of the streamer considerably exceeds the bulk and flux drift velocities of the electrons. This is expected, as the velocity of a negative streamer is determined by

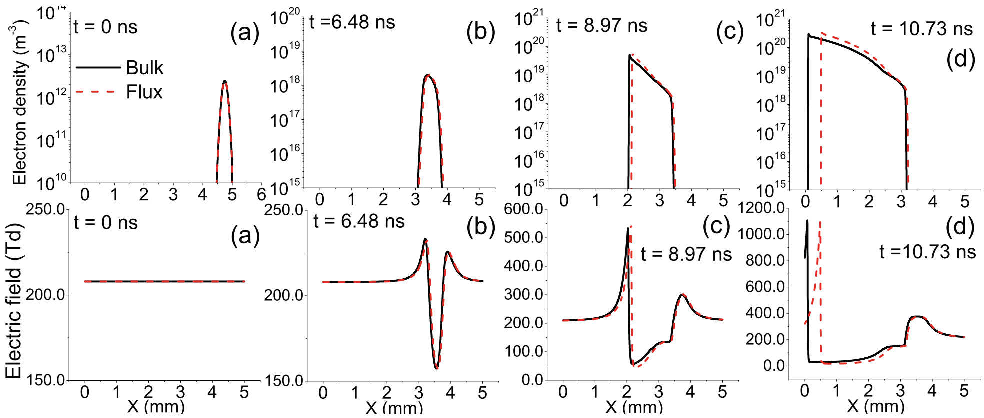

Figure 2: Electron density (first row) and the electric field (second row) in N_2:CH_4=(98.4%):(1.6%) after different time steps. The external electric field is $4E_k$ where E_k is the breakdown electric field.

the combination of the electron velocity and the rate of the electron impact ionization at the front of the streamer, where the electric field is significantly enhanced, as well as by the strong diffusive fluxes in the streamer front. Comparing streamers with bulk and flux input data, we observe that streamers with bulk data are faster.

In figure 3 we show the results of PIC/MCC simulations, including the electron density and the electric field in different spatially homogeneous ambient electric fields. For comparison, the last column shows the electron density and the electric field in N_2:O_2 mixture for the same number density of ambient gas molecules. The set of cross sections for electron scattering are detailed by Dujko et al. (2011). We observe the avalanche-to-streamer transition in N_2-CH_4 mixture mimicking the atmosphere of Titan only for fields above $3E_k$. For electric fields $\leq 3E_k$ there is a distinct motion of electrons towards the positive virtual electrode, while there is no movement of the positive front at all. However, the field enhancement is clearly visible on the positive streamer front as well as the field shielding on the negative upper forehead. This suggests that the avalanche-to-streamer-transition is unlikely to occur at later times.

For electric fields above $3E_k$ we observe an avalanche-to-streamer transition in both N_2-CH_4 and N_2-O_2 mixtures. We observe that the initial electron-ion patch evolves into a bidirectional streamer. The field enhancement is more apparent for the positive front as the positive streamer is narrower and therefore its field enhancement is larger than in the negative streamer. Comparing the evolution on Titan (third column) and in N_2-O_2 mixture (fourth column), we observe that both positive and negative streamer fronts propagate significantly slower in Titan's atmosphere. In addition, the electron density in N_2-CH_4 appears to be smaller and branching is favored.

Using a classical fluid model and PIC/MCC simulations, we have not observed any streamer inception for electric fields below approximately $3E_k$. In the modeling study of cloud dynamics in Titan's atmosphere, it was shown that the generated electrical field could reach 2 MV/m (Tokano et al. 2001). This suggests that the inception of streamers and the appearance of lightning is not possible in Titan's atmosphere.

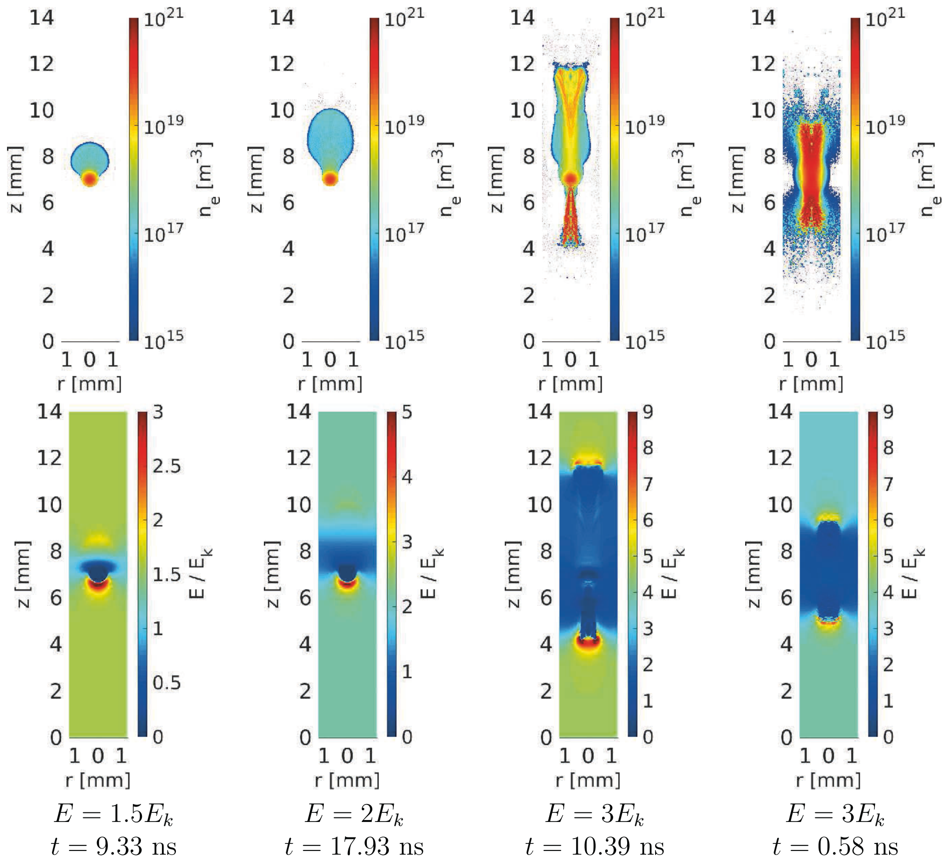

Figure 3: The electron density (first row) and the electric field (second row) in N_2:CH_4=(98.4%):(1.6%) (columns one to three) and in N_2:O_2=(98.4%):(1.6%) (last column) in different ambient electric fields after different time steps. In both mixtures the gas density $N = 2.9 \cdot 10^{25}$ m^{-3}, hence $E_k \approx 1.4$ MVm^{-1} for N_2:CH_4 mixture and $E_k \approx 3.0$ MVm^{-1} for N_2:O_2 mixture (Köhn et al. 2019).

However, this conclusion can be challenged by a comparison of cloud electric fields and breakdown electric fields in the Earth's atmosphere. In numerous independent studies, breakdown electric fields at altitudes between 4 and 20 km have been shown to be significantly higher than cloud electric fields. However, as discussed in the present work, the inception of streamers in Titan's atmosphere strongly depends on the ambient electric field, which is uncertain since it is only provided by models, but not by direct measurements (Köhn et al. 2019). Moreover, the existence of streamer discharges into Titan's atmosphere is still not conclusive proof of lightning. It should also be mentioned that during the 127 flybys of Cassini, and during the descent of a Huygens probe, the traces of lightning were not detected. However, although not observed, it may still mean that lightning exists, but is too weak or too rare for detection. In any case, the existence of lightning in Titan's atmosphere is still an open question, and requires more attention in the near future.

4. CONCLUSION

In this work, we have studied the transport of electrons, the transition from an electron avalanche into a streamer and its propagation, and the possibility for the appearance of lightning in Titan's atmosphere. We have presented the variation of the electron transport coefficients with E/N and we have determined the breakdown electric field in Titan's atmosphere. In this study, ionization collisions were observed to affect electron transport coefficients at high values of E/N, while the insignificant attachment rate had no observable effect. The inception and propagation of streamers were simulated using a fluid equation-based models and PIC/MCC simulations. We have found that the inception of streamers requires the ambient electric field above $3E_k$. We found that streamers that are modeled with bulk data are a little faster than those with flux data. Comparing N_2:CH_4 and N_2:O_2 mixtures, streamers in N_2:O_2 mixtures are much faster. The methodology and mathematical machinery presented in this work can be used as a basis for further studies of streamer discharges and lightning in the atmospheres of other planets in our Solar system, and in the atmospheres of exoplanets.

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