Publ. Astron. Obs. Belgrade No. 100 (2021), 107 - 114 *Invited Lecture*

WIND MODELS OF MASSIVE STARS AND MASS-LOSS RATES DETERMINATION

J. KUBÁT and B. KUBÁTOVÁ
Astronomický ústav, Akademie věd České republiky, 251 65 Ondřejov, Czech Republic
E–mail: kubat@sunstel.asu.cas.cz

Abstract. Determination of mass-loss rates of massive stars is an important output of massive star analysis, which influences our understanding of stellar evolution. Stellar mass-loss rates are usually determined using wind models with a different level of sophistication. Commonly used models are based on an assumption of spherical symmetry and solve the NLTE radiative transfer consistently for a given density and velocity structure, which means that the hydrodynamic structure is held fixed. Usually, an approximate dependence of velocity on radius is being assumed (the so-called β-velocity law). Using a different approach, mass-loss rates can be predicted by hydrodynamic models, which do not solve the radiative transfer, but they describe the radiation force in a parametric way (using force multipliers). The most sophisticated wind models do not use the simplifications of the β-velocity law and the force multlipliers. Consistent NLTE wind models including both the wind dynamics and NLTE radiative transfer can be calculated.

1. INTRODUCTION

The large luminosity, short life, and final supernova explosions of massive stars make them an important source of heating and ionization of interstellar medium, as well as they provide kinetic energy via their outflows. Their importance is not diminished by the fact that they are rare, they form a small but important fraction of the stellar population visible even at large distances.

The main task of the quantitative spectroscopy of massive stars is a determination of the stellar mass M_*, stellar radius R_*, stellar luminosity L_*, chemical composition (abundances α_k for each element k) and of wind parameters, namely the terminal wind velocity v_∞ and mass-loss rate $\mathrm{d}M/\mathrm{d}t = \dot{M}$ for each studied star. The mass-loss rate is of a particular importance. Besides its direct effect of lowering the stellar mass, it is also an important input parameter for stellar evolution codes. It can not be measured directly, its determination relies on wind models, which are briefly discussed in this paper.

2. MASS-LOSS RATE DETERMINATION AND WIND MODELLING

Unlike terminal wind velocity (v_∞) measurement, which is relatively straightforward and can be done by direct measurement of the position of the blue edge of the P-Cygni line profile (see, e.g. Lamers & Cassinelli, 1999), direct mass-loss rate determination is not possible. Wind mass-loss rate is being determined indirectly, by comparison of model predicted spectra with observed ones. If the spectra match, then the mass-loss rate used in calculation of the theoretical spectrum is attributed to the star. However, since the problem is complicated, approximations have to be used in wind model calculations and theoretical spectra predictions.

Because of high complexity of the problem, simplifying assumptions considered as 'standard' ones are being used. The winds are assumed to be stationary and spherically symmetric consisting of homogeneous spherical shells, which means that 1-D models are being solved. In addition, a core-halo approximation is often being used. This means that the wind region is considered as a shell around a star with a photospheric radiation as a lower boundary condition and with no back influence of the wind region on the stellar photosphere. The inclusion of a quasi-hydrostatic lower part of the model atmosphere improved the situation.

Wind models are calculated assuming basic stellar parameters (M_*, R_*, L_*, and Z) with the final goal to predict emergent radiation and compare it with observations. A 1-D spherically symmetric wind model can be considered as a dependence of a set of wind describing structural quantities on the radial coordinate r. These involve temperature T, mass density ρ, radial velocity v, atomic level populations (number densities) n_i (i stands for the level index), and the radiation field for each frequency, represented here by its specific intensity $I(\nu)$. In some cases, some of the radius dependences or additional parameters (e.g., $\dot{M}$) are assumed in order to simplify and speed up the process of model calculations. The final step of modelling is comparison with observations. This can be done only by comparison of emergent radiation from the model with observed radiation from the stellar source. If both match, the parameters used for calculation of a model, and both assumed and calculated structural quantities are attributed to the stellar source.

Let us describe several options used in modelling winds and mass-loss rate determination. Note that the division presented here is rough and methods that do not exactly fit to our classification categories may exist.

Formal solution of the radiative transfer equation. This is a solution of the radiative transfer equation for given opacity end emissivity (see Hubeny & Mihalas 2015). In the case of wind modelling it means that all structural quantities except the radiation field are given (i.e. assumed). This implies that also the mass-loss rate is given. The level populations are also given, either they are determined using a simple nebular model (e.g. Kraus et al. 2000) or the local thermodynamic equilibrium (LTE) is assumed. The latter assumption is used for calculation of the emergent radio flux from the outermost parts stellar winds (Panagia & Felli 1975, Wright & Barlow 1975), which can be then compared with radio observations to determine $\dot{M}$.

In optically thin media, absorption is usually considered in a simplified manner or is it neglected.

NLTE wind models. If the structure is given and LTE is not an acceptable assumption (which is always in massive star winds), we have to solve for the level populations, which have to be determined consistently with the radiation field. In this case we assume $T(r)$, $\rho(r)$, and $v(r)$ as given and seek solution for $I(\nu, r)$ and $n_i(r)$ by simultaneous solution of the radiative transfer equation and kinetic equilibrium equations. This type of model is usually referred to as the NLTE line formation problem.

In a generalized NLTE line formation problem the mass density is not assumed arbitrarily, but it is calculated from given $v(r)$ and $\dot{M}$ using the continuity equation. The velocity field is usually assumed in the form of a β-velocity law, which can be written as (β is a free parameter)

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^\beta . \tag{1}$$

This implies that v_∞ has to be assumed as well. Then the equations of kinetic equilibrium together with the radiative transfer equation (the above mentioned NLTE line formation problem) have to be solved to determine the emergent radiation.

Alternatively also $T(r)$ can be determined together with the solution of the NLTE line formation problem by adding the equation of radiative equilibrium or the thermal balance equation to the set of simultaneously solved equations.

Full radiation hydrodynamics (stationary) models. The ideal case for modelling is to solve for all structural variables mentioned at the beginning of this section. This means to solve the continuity equation, equation of motion, radiative transfer equation, the kinetic (statistical) equilibrium equations, and the energy equation to obtain the structure of the mass density $\rho(r)$, velocity $v(r)$, radiation field $I(\nu, r)$, level populations $n_i(r)$, and temperature $T(r)$, ideally simultaneously. To our knowledge, there is currently no code which does such full solution without fixing any of the structural variables. However, several codes use some simplifications or iterative schemes to obtain the full stationary RHD solution, and are on their way towards the full solution. A more detailed description of these models is in the Section 6.

Hydrodynamic wind models with a parametric radiation force. The last type of models we mention here does not allow a direct comparison with observations, as the radiation field is not an output of such models, rather it is used as an input to calculate the radiation force necessary to drive the line driven wind. An option to determine the radiation force in a parametric way is frequently used. This option implicitly means that both the radiation field and level populations are assumed, and the hydrodynamic equations (continuity equation, equation of motion, and sometimes also the energy equation) are solved to detemine mass density and velocity (and sometimes the temperature structure). These models are described in the Section 3.

3. HYDRODYNAMIC WIND MODELS

The task of hydrodynamic wind modelling is to obtain a solution of stationary hydrodynamic equations for given basic stellar parameters (M_*, R_*, L_*, and α_k). Hydrodynamic models seek solution of the continuity equation (which determines the mass-density ρ)

$$\frac{\mathrm{d}\left(r^2 \rho v\right)}{\mathrm{d}r} = 0, \tag{2}$$

equation of motion (which determines the radial velocity v)

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{G M_* \rho}{r^2} + g_{\mathrm{rad}} \tag{3}$$

(p is the gas pressure, G is the gravitational constant, and g_{rad} is the radiation force density), and sometimes also the energy equation (which determines the temperature T). Alternatively, a prescribed $T(r)$ or its constant value is used.

The key quantity in hydrodynamic equations of a radiatively driven stellar wind is the radiation force density. It is caused by radiation-matter interaction and it accelerates the stellar wind. It is given by the integral (Hubeny & Mihalas, 2015, Eq. 11.50)

$$\vec{g}_{\mathrm{rad}} = \frac{1}{c} \int_0^\infty \mathrm{d}\nu \oint \mathrm{d}\omega\, \vec{n} \left[\chi(\nu, \vec{n}) I(\nu, \vec{n}) - \eta(\nu, \vec{n}) \right]. \tag{4}$$

which clearly describes the fact that $\vec{g}_{\mathrm{rad}}$ may be large if both opacity and radiation flux are large. It can be split to the radiative force caused by radiation-matter interaction in continuum transitions (ionization, free-free transitions, and continuum scattering) $\vec{g}_{\mathrm{rad,C}}$, and to the radiative force caused by absorption and scattering in spectral lines $\vec{g}_{\mathrm{rad,L}}$. The latter is calculated as a sum of the radiation force caused by individual line transitions, $\vec{g}_{\mathrm{rad,L}} = \sum_{\mathrm{lines}} \vec{g}_{\mathrm{rad},l}$. Then

$$\vec{g}_{\mathrm{rad}} = \vec{g}_{\mathrm{rad,C}} + \sum_{\mathrm{lines}} \vec{g}_{\mathrm{rad},l}. \tag{5}$$

The detailed calculation of the line radiation force is quite time-consuming and complicated, since the radiation force in each line depends on corresponding atomic level populations, which have to be obtained by solution of kinetic equilibrium equations for each element considered. This is why approximations are used. The most common approximation of the line radiation force (the so-called CAK approximation) was introduced in first hydrodynamic wind models (Castor, Abbott, Klein 1975; Abbott 1982) and later modified by Gayley (1995). The radiation force in this approximation is expressed with the help of parameters (also referred to as force multipliers) k (or Q), α, and δ (for a more detailed description, see, e.g., Hubeny & Mihalas 2015, Section 20.3). These parameters offer a simple approximation of the radiation force. They can be determined by detailed calculation of the line opacity (e.g., Abbott 1982). This is usually done only for a restricted set of stellar parameters.

Using this simplified description of the influence of radiation on stellar wind, hydrodynamic wind codes calculate the model hydrodynamical structure (depth dependence of structural variables on radius). As a part of the output, they also predict values of v_∞ and $\dot{M}$. The latter quantity is usually referred to as the *predicted mass-loss rate*. Hydrodynamic wind models do not offer emergent spectrum.

Hydrodynamic wind modelling was initiated by seminal works of Castor et al. (1975), Abbott (1980), and Pauldrach et al. (1986). Later the stationary hydrodynamic codes gradually incorporated the radiative transfer solution to improve the parametric treatment of the radiative force. However, parametric description of the radiative force is still used in time dependent hydrodynamic simulations of stellar winds (e.g. Owocki 2011, and references therein).

4. NLTE WIND MODELS

NLTE wind models assume the hydrodynamic wind velocity structure, which usually has the form of the β-velocity law (1) for the radial velocity component. Note that this law with $\beta = 0.5$ was derived by Chandrasekhar (1934) assuming that the wind driving force was proportional to gravity. Knowing $v(r)$, the mass density $\rho(r)$ is calculated from the continuity equation (2) using the assumed value of the mass-loss

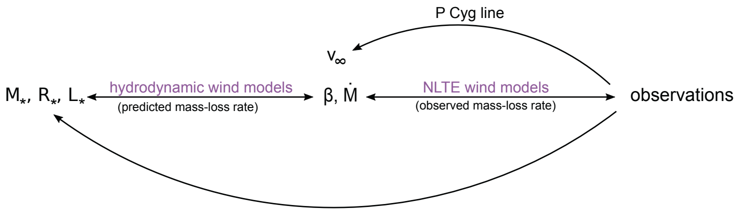

Figure 1: The scheme of stellar (M_*, R_*, L_*) and wind (v_∞, $\dot{M}$) parameters determination using independent NLTE wind models and hydrodynamic wind models.

rate $\dot{M}$. Then the equations of radiative transfer and kinetic equilibrium are solved simultaneously to obtain consistent values of level populations and radiation filed. This solution can be accompanied with a solution of an energy equation (radiative equilibrium or thermal balance) to determine temperature.

Typical representants of this modelling are codes `PoWR` (e.g. Hamann & Gräfener 2004), `FASTWIND` (e.g. Santolaya-Rey et al. 1997, Puls et al. 2005), and `CMFGEN` (e.g. Hillier & Miller 1998). In these codes, the dependence $v(r)$ is assumed, either in the form of the β-velocity law (1) or a more involved relation using two free β-like parameters (the double-β velocity law). Also the stellar mass-loss rate $\dot{M}$ is assumed. Then a full NLTE line formation problem is solved, which also predicts the emergent radiation spectrum. Note that the back influence of radiation changes on wind acceleration is typically not considered. The emergent spectrum is then compared with observations. If both spectra match with sufficient accuracy, the assumed value $\dot{M}$ is then referred to as the *observed mass-loss rate*.

5. OBSERVED AND PREDICTED MASS-LOSS RATES

The two ways of modelling mentioned in preceding sections (3 and 4) offer two values of mass-loss rates, namely the "predicted mass-loss rate" and the "observed mass-loss rate". If both values match, it is usually considered as the match of theory and observations. This situation is schematically depicted in the Fig. 1. Note that the observed mass-loss rate is in fact based on a hydrodynamically simplified model, so instead of match of observations and theory it is rather a match of the approximate hydrodynamic model with exact radiative transfer and exact hydrodynamic model with approximate radiative transfer.

6. FULL RADIATION HYDRODYNAMICS NLTE WIND MODELS

The drawbacks of the mass-loss rate determination mentioned in the preceding section are removed when full radiative hydrodynamic NLTE models are used. This means that for given M_*, L_*, R_*, Z a solution of the continuity equation, equation of motion, energy equation, radiative transfer equation, and kinetic equilibrium equations is performed to obtain $I(\nu, r)$, $n_i(r)$, $\rho(r)$, $v(r)$, and $T(r)$. This full solution gives also the values of $\dot{M}$ as the integral of the continuity equation and v_∞ as the velocity at the outermost point of the model.

In the modelling process several common additional assumptions used in hydrodynamic (Section 3) and NLTE (Section 4) wind models can be released, mainly the parametric representation of the radiative force and the division of the atmosphere model to a photosphere and wind (i.e., the core-halo approximaton).

Line radiation force. The parametric description using force multipliers (the CAK approximation) is not used any more. Instead, a detailed full description of the line radiative force (5) is used, where contributions of all transitions are taken into account separately. The need to calculate opacities for each transition causes the necessity to calculate NLTE level populatons, since LTE is not applicable in stellar winds. This is done using the kinetic equilibrium equations, which (neglecting time and advection terms) can be expressed for the level i as

$$n_i \sum_j \left[R_{ij}(I) + C_{ij}\right] = \sum_j n_j \left[R_{ji}(I) + C_{ji}\right]$$

where the dependence of radiative rates on radiation I is emphasized. Knowing n_i, the radiation force can be calculated. An equation (4) simplified for isotropic emissivity (in the comoving frame) can be used. The pioneer in construction of this type of models was A. Pauldrach, first steps were summarized in Pauldrach et al. (1994). This work was extended in Pauldrach et al. (2001) and applied to wind modeling of ζ Pup in Pauldrach et al. (2012). An independent modeling method that includes consistent calculation of radiation force was developed by Krtička & Kubát (2004), who used Sobolev approximation for line force calculation, later replaced by CMF radiation transfer (Krtička & Kubát, 2010).

Photosphere. The core-halo approximation splits the expanding stellar atmospheres to two parts, namely the hydrostatic photopshere and the wind above it. Using this assumption the photospheric radiation is considered as a lower boundary condition for the wind model. The influence of the wind part on the photosphere (wind blanketing, Abbott & Hummer, 1985) is not taken into account. As a drawback, there is usually no smooth transition from the photosphere to the wind.

Relaxing the core-halo approximation means that the almost static photosphere becomes an integral part of the model. The wind model starts already at large optical depths where the diffusion approximation is valid and the transition from the photosphere to the wind is smooth. First attempts to consistently remove the artificial splitting were done in an approximate way by Gabler at al. (1989).

Global NLTE wind models. Krtička & Kubát (2010, 2017, 2018) presented a method to solve all the above mentioned equations while relaxing the core-halo and parameterized line force assumptions. The chemical composition of the wind is arbitrary and no force multipliers are used. In their code the radiation force is calculated using actual level populations obtained by solution of kinetic equilibrium equations. The radiation field, which enters these equations is calculated in a slightly simplified way, the continuum radiative transfer is solved exactly (as it is practically the same as in the static case) and the line transfer is solved using the Sobolev approximation. The line radiation force is calculated using the radiation field obtained by a CMF solution of the radiative transfer, which improves the previously used Sobolev radiative transfer solution. The photosphere-wind transition is smooth. Application of the global wind models to stars from our Galaxy (Krtička & Kubát, 2017) and Magellanic Clouds (Krtička & Kubát, 2018) showed that the mass-loss rates obtained using global models are lower than the commonly used values of Vink et al. (2001). The latter were obtained using a detailed Monte Carlo evaluation of the driving force for given $v(r)$.

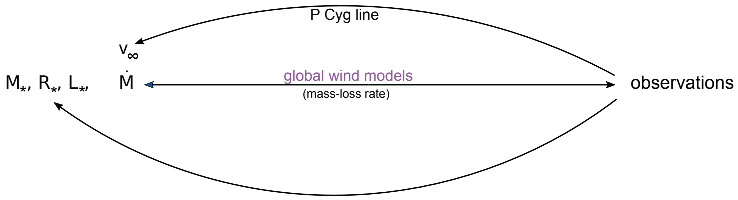

Figure 2: The scheme of stellar (M_*, R_*, L_*) and wind (v_∞, $\dot{M}$) parameters determination using full radiative hydrodynamic NLTE wind models.

Recently, NLTE wind modeling codes mentioned in the section 4 were also improved to include hydrodynamic solutions as a part of modelling (e.g. `PoWR`, Gräfener & Hamann, 2005; Sander et al., 2017; `FASTWIND`, Sundqvist et al., 2019).

Improved velocity law. An important aspect of the velocity structure obtained from the full radiation hydrodynamic NLTE wind model is that it differs from the commonly used β-velocity law (1). A more reliable analytical description of the velocity structure can be obtained using the polynomial expansion, $v(r) = \sum_{i=0}^{3} \tilde{v}_i \tilde{P}_i(1 - R_*/r)$, using Legendre polynomials $\tilde{P}_i$ and fitting coefficients $\tilde{v}_i$ (see Krtička & Kubát, 2011). A similar improved formula

$$v(r) = \sum_{i=1}^{2} \tilde{v}_i \left(1 - \gamma \frac{R_*}{r}\right)^i, \tag{6}$$

($\tilde{v}_i$ and γ are fitting parameters) was used to fit the calculated model velocity structure obtained by Krtička et al. (2021) for B supergiants.

Clumping. The big challenge in current methods for $\dot{M}$ determination are wind inhomogeneities (clumping), which is a 3-D phenomenon. However, currently existing unified (global) models are 1-D, and the only way to implement clumping in them is to use simplifying assumptions or parameterized methods (Sundqvist & Puls 2018, and references therein). The 3-D modelling of optically thick clumping was done only for limited spectral regions, but its effect on mass-loss rate determination was clearly shown (Šurlan et al. 2013).

7. SUMMARY AND CONCLUSIONS

The current commonly used method for mass-loss rate determination consists of two steps. First the NLTE wind models with a prescribed (β) velocity law are calculated and the “observed mass-loss rates” are determined. Then these values are verified by hydrodynamic calculations to match the “predicted mass-loss rates”. This two-step determination should be replaced by full radiation hydrodynamic NLTE wind models, which offer the consistent mass-loss rate values directly (see Fig. 2). These models represent an improvement in massive star analysis. They offer a tool for more reliable modeling of atmospheres and winds of massive stars.

References

Abbott, D. C.: 1980, *Astrophys. J.*, **242**, 1183.
Abbott, D. C.: 1982, *Astrophys. J.*, **259**, 282.
Abbott, D. C., Hummer, D. G. : 1985, *Astrophys. J.*, **294**, 286.
Castor, J. I., Abbott, D. C., Klein, R. I.: 1975, *Astrophys. J.* **195**, 157.
Chandrasekhar, S.: 1934, *Mon. Not. Roy. Astron. Soc.*, **94**, 522.
Gabler, R., Gabler, A., Kudritzki, R. P., Puls, J., Pauldrach, A.: 1989, *Astron. Astrophys.*, **226**, 162.
Gayley, K. G.: 1995, *Astrophys. J.*, **454**, 410.
Gräfener, G., Hamann, W.-R.: 2005, *Astron. Astrophys.*, **432**, 633.
Hamann, W.-R., Gräfener, G.: 2004, *Astron. Astrophys.*, **427**, 697.
Hillier, D. J., Miller, D. L.: 1998, *Astrophys. J.*, **496**, 407.
Hubeny, I., Mihalas, D.: 2015, *Theory of Stellar Atmospheres*, Princeton University Press, Princeton.
Kraus, M., Krügel, E., Thum, C., Geballe, T. R.: 2000, *Astron. Astrophys.*, **362**, 158.
Krtička, J., Kubát, J.: 2004, *Astron. Astrophys.*, **417**, 1003.
Krtička, J., Kubát, J.: 2010, *Astron. Astrophys.*, **519**, A50.
Krtička, J., Kubát, J.: 2011, *Astron. Astrophys.*, **534**, A97.
Krtička, J., Kubát, J.: 2017, *Astron. Astrophys.*, **606**, A31.
Krtička, J., Kubát, J.: 2018, *Astron. Astrophys.*, **612**, A20.
Krtička, J., Kubát, J., Krtičková, I.: 2021, *Astron. Astrophys.*, **647**, A28.
Lamers, H. J. G. L. M., Cassinelli, J. P.: 1999, *Introduction to Stellar Winds*, Cambridge Univ. Press.
Owocki, S. P.: 2011, *Bulletin de la Société Royale des Sciences de Liège*, **80**, 16.
Panagia, N., Felli, M.: 1975, *Astron. Astrophys.*, **39**, 1.
Pauldrach, A., Puls, J., Kudritzki, R. P.: 1986, *Astron. Astrophys.*, **164**, 86.
Pauldrach, A. W. A., Kudritzki, R. P., Puls, J., Butler, K., Hunsinger, J.: 1994, *Astron. Astrophys.*, **283**, 525.
Pauldrach, A. W. A., Hoffmann, T. L., Lennon, M.: 2001, *Astron. Astrophys.*, **375**, 161.
Pauldrach, A. W. A., Vanbeveren, D., Hoffmann, T. L.: 2012, *Astron. Astrophys.*, **538**, A75.
Puls, J., Urbaneja, M. A., Venero, R., Repolust, T., Springmann, U., Jokuthy, A., Mokiem, M. R.: 2005, *Astron. Astrophys.*, **435**, 669.
Sander, A. A. C., Hamann, W.-R., Todt, H., Hainich, R., Shenar, T.: 2017, *Astron. Astrophys.*, **603**, A86.
Santolaya-Rey, A. E., Puls, J., Herrero, A.: 1997, *Astron. Astrophys.*, **323**, 488.
Sundqvist, J. O., Puls, J.: 2018, *Astron. Astrophys.*, **619**, A59.
Sundqvist, J. O., Björklund, R., Puls, J., Najarro, F.: 2019, *Astron. Astrophys.*, **632**, A126.
Šurlan, B., Hamann, W.-R., Aret, A., Kubát, J., Oskinova, L. M., Torres, A. F.: *Astron. Astrophys.*, **559**, A130.
Vink, J. S., de Koter, A., Lamers, H. J. G. L. M.: 2001, *Astron. Astrophys.*, **369**, 574.
Wright, A. E., Barlow, M. J.: 1975, *Mon. Not. Roy. Astron. Soc.*, **170**, 41.