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TESTS OF GRAVITY THEORIES WITH BLACK HOLE OBSERVATIONS

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Abstract. Black holes with different masses are observed in a wide range of electromagnetic radiation frequencies. Astronomers believe that they have detected neutrinos whose sources are associated with black holes. The first detection of gravitational radiation from merging binary black holes occurred using the LIGO–Virgo gravitational wave detectors just a few years ago. After that, researchers began to talk about the fruitfulness of multi-messenger astronomy. At present, we can say that the general relativity is the best theory of gravity, however, in recent years, many alternative theories of gravity have emerged and the emergence of at least part of these theories has been associated with attempts to explain the problems of dark matter and dark energy with changes in the gravity law. We discuss the possibilities of using black hole observations to test the predictions of general relativity and obtain constraints on the parameters of alternative theories of gravity. Earlier, we discuss constraints on the theory of gravity from observations of the supermassive black holes at the Galactic Center and at the center of the galaxy M87.

1. INTRODUCTION

In 2002 I was invited to present a lecture on gravitational microlensing and dark matter problem at the XIII National Astronomical Conference in Belgrade. In 2003 my contribution for the conference proceedings was published in Publications of the Astronomical Observatory of Belgrade (Zakharov 2003) and it is a pleasure to note that this paper was cited by Weinberg (2008). Since these times I established a fruitful and pleasant collaboration with Serbian astrophysicists.

In the paper I mention prizes and their recipients working in BH physics which closely connected with our studies in the subject because it means a recognition of an importance and an interest to these fields among the wide world scientific community.

1. 1. INCREASING IMPACT OF ASTROPHYSICS AND GENERAL RELATIVITY

There are not many Nobel prize recipients who were awarded the prize for their studies in astrophysics and general relativity earlier because for many years astrophysics was not treated as a branch of contemporary physics since astrophysics was not precise enough (a famous Russian physicist V. L. Ginzburg ironically told about the issue that "in astronomy one is equal to ten") meanwhile general relativity was checked only in a weak gravitational field limit. However, I would like to note that works of three Nobel prize winners (in 2017, 2019 and 2020) in last four years are devoted to astrophysical applications of general relativity.

Word combination "black hole" is extremely popular not only in physics and astronomy (Google counts around 924 million cases of its usage) and three Nobel prizes are connected with this astrophysical object. In 1983 Subrahmanyan Chandrasekhar[1] was awarded the Nobel prize for the discovery of limiting mass of white dwarfs (Chandrasekhar 1931, 1934) (see, an interesting discussion of the issue by Yakovlev (1994) where contributions of other authors (Frenkel 1928; Stoner 1930, 1932; Landau 1932) was discussed). Later, the conclusion about a limiting mass of neutron star was found (Gamow 1937; Oppenheimer & Volkoff 1939). I would like to mention that some time ago Yakovlev et al. (2013) revisited the Landau's contribution in a creation of the neutron star concept. I would like to note that the result about mass limits for neutron stars and white dwarfs was interpreted by Chandrasekhar as a physical reality in contrast to many other famous scientists like A. Eddington and L. Landau who noted that the conclusion about the limiting mass of white dwarfs should be incorrect if laws of quantum mechanics and quantum statistics are violated (Landau 1932). These assumptions were inspired probably by N. Bohr (who claimed that energy conservation law could be violated) since in twenties and thirties of XX century it was a period of a formation of new non-classical physics and basic physical concepts were changed and a process of trial and error to find new physical theory was in action.

In 2017 Rainer Weiss, Kip S. Thorne and Barry Barish[2] were awarded the Nobel prize in physics "for decisive contributions to the LIGO detector and the observation of gravitational waves" from binary black holes (Abbott et al. 2016). In the paper the authors discovered gravitational waves and binary black holes and in addition, they found a constraint on graviton mass. We will discuss graviton mass constraints from different type of observations below.

[1] At the same year in his fundamental book on black holes Chandrasekhar noted that "the black holes of nature are the most perfect macroscopic objects, there are in the Universe: the only elements of their construction are our concepts of space and time. And since a general theory of relativity provides a only single unique family of solutions for their descriptions are the simplest objects as well". This family of solutions is determined by only three parameters, mass, charge and spin of a black hole. Initially, this statement was formulated by J. Wheeler as the theorem on an absence of hairs for black holes. Later, the theorem was proven assuming rather natural conditions.

[2] One of the key founder LIGO project, Ronald Drever, in principle could get the Nobel prize since in 2016 he got the Breakthrough Prize, the Gruber Prize in Cosmology, the Shaw prize ("Nobel of the East"), the Kavli Prize in astrophysics, the Harvey prize in physics and technology and all these prizes he got together with Rainer Weiss and Kip S. Thorne for the development of LIGO detector and the discovery of gravitational waves from binary black holes. Unfortunately, outstanding experimentalist Ron Drever died on 7 March 2017 and cannot get the Nobel prize in 2017 but perhaps he could be a Nobel prize winner if he would still alive in fall 2017.

2. 2020 AS THE BLACK HOLE YEAR

In 2020 the black hole concept got the high recognition among a scientific community.

2. 1. ROGER BLANDFORD AND ENERGY RELEASE FROM BLACK HOLES

The Shaw Prize in Astronomy 2020 is awarded to Roger D. Blandford (USA) "for his foundational contributions to theoretical astrophysics, especially concerning the fundamental understanding of active galactic nuclei, the formation and collimation of relativistic jets, the energy extraction mechanism from black holes, and the acceleration of particles in shocks and their relevant radiation mechanisms".

R. Blandford contributed essentially in different topics of relativistic astrophysics including black hole physics and gravitational lensing. For instance, Blandford and Znajek (1977) proposed a way to extract energy from a rotating black hole in a presence of magnetic field with a special configuration. Frolov and Zelnikov (2011) provided a clear description of physical ideas which were used in the Blandford – Znajek process.

2. 2. SHAW PRIZE IN ASTRONOMY AS A PRECURSOR FOR NOBEL PRIZES

Many people got Nobel prizes after the Shaw prize, therefore, the Shaw prize recipients are highly likely pretender for the Nobel prize. We list a couple of examples. In 2004 P. James E. Peebles got the Shaw prize and in 2019 he got the Nobel prize. Geoffrey Marcy and Michel Mayor got the Shaw prize in 2005. G. Marcy was one of real candidates for searches of exoplanets with transit and Doppler shift measurements but in 2015 he was accused by the administration of his University (UC, Berkeley) in sexual harassment and after that he expressed his intention to step down from his professorship at UC Berkeley. In 2019 Michel Mayor and Didier Queloz got the Nobel prize. Saul Perlmutter, Adam Riess and Brian Schmidt got the Shaw prize in 2006 and they were awarded the Nobel prize in 2011. Reinhard Genzel got the Shaw prize in 2008 Ronald Drever (1931-2017); Kip S. Thorne and Rainer Weiss got the Shaw prize in 2016 and they were awarded the Nobel prize in 2017.

2. 3. THE CRAFOORD PRIZE IN ASTRONOMY

In 1980 Swedish economist and industrialist Holger Crafoord (1908 – 1982) and his wife Anna-Greta established the Holger Crafoord's Endowment, which was donated to the Royal Swedish Academy of Sciences for contributions in mathematics, astronomy, geosciences (particularly ecology) biosciences and rheumatology (the disciplines are complimentary to the Nobel prize ones). The Crafoord Prize has an excellent reputation among a scientific community and the Prize is awarded personally by the King of Sweden.[3]

In 2012 Reinhard Genzel and Andrea M. Ghez[4] were awarded the Crafoord Prize "for their observations of the stars orbiting the galactic centre, indicating the presence of a supermassive black hole". For the next Crafoord Prize in astronomy in 2016 Roger Blandford and Roy Patrick Kerr were selected "for fundamental work concerning rotating black holes and their astrophysical consequences". Really, in 1963 Kerr found the solution describing the metric of a rotating black hole but as I mentioned earlier

[3]https://www.crafoord.se/en/the-crafoord-foundation/the-founder-holger-crafoord/.

[4]The first woman to be awarded this prize.

Blandford (and Znajek) proposed a process where a part of a rotational energy of a black hole could be converted into energy of highly energetic jets. It means studies of black holes are the astronomical investigations with the highest priority for the the Crafoord Prize Committee.

2. 4. ANDREW FABIAN: TESTS OF STRONG GRAVITY FROM X-RAY OBSERVATIONS OF BLACK HOLES

The Norwegian Academy of Science and Letters has decided to award the Kavli Prize in Astrophysics for 2020 to Andrew Fabian (University of Cambridge, UK) "for his ground breaking research in the field of observational X-ray astronomy, covering a wide range of topics from gas flows in clusters of galaxies to supermassive black holes at the heart of galaxies." Really, Fabian et al. (1989) proposed a way to recognize that emission region for X-ray radion is located near a black hole horizon and the authors outlined an opportunity to evaluate a black hole spin. Later, Tanaka et al. (1995) discovered a specific shape of the iron K_α with observations of the Seyfert Galaxy MCG-6-30-15 with the Japanese ASCA satellite and the authors founded that emission region for the iron K_α has to be located so closely to the black hole horizon and therefore they also concluded that spin has to be very close to extreme value. Recent reviews of the subject are presented by Fabian et al. (2000), Zakharov and Repin (2006), Jovanović and Popović (2009).

2. 5. APS EINSTEIN PRIZE FOR STUDIES OF COMPACT OBJECTS

The Albert Einstein Medal is an award established by the Albert Einstein Society in Bern. First given in 1979, the award is presented to people for "scientific findings, works, or publications related to Albert Einstein each year". In 2019 C. M. Will got the medal. American Physical Society Einstein prize selected Clifford Martin Will (University of Florida) and Saul Teukolsky (Cornell University and Caltech) as recipients of Albert Einstein prize in 2021 "for outstanding contributions to observational tests of general relativity with theories of gravitational waves, astrophysical black holes, and neutron stars.".

3. NOBEL PRIZE IN 2020: THEORETICAL AND OBSERVATIONAL STUDIES OF BLACK HOLES

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2020 to Roger Penrose[5] (University of Oxford, UK), Reinhard Genzel (Max Planck Institute for Extraterrestrial Physics, Garching, Germany and University of California, Berkeley, USA) and Andrea Ghez[6] (University of California, Los Angeles, USA). One half of the prize was given to Roger Penrose "for the discovery that black hole formation is a robust prediction of the general theory of relativity" and the other half was given jointly to Reinhard Genzel and Andrea Ghez "for the discovery of a supermassive compact object at the Centre of our Galaxy". Roger Penrose showed that the general theory of relativity leads to the formation of black holes. Reinhard

[5] In 2019 R. Penrose was awarded the Pomeranchuk prize established by the Institute of Theoretical and Experimental Physics (Moscow) and he is the only Pomeranchuk prize laureate who later got the Nobel prize.

[6] The forth woman to be awarded the Nobel prize in physics.

Genzel and Andrea Ghez discovered that an invisible and extremely heavy object governs the orbits of stars at the Galactic Center. A supermassive black hole is the most natural explanation for observations of Keck and VLT (GRAVITY) groups. “The discoveries of this year’s Laureates have broken new ground in the study of compact and supermassive objects. But these exotic objects still pose many questions that beg for answers and motivate future research. Not only questions about their inner structure, but also questions about how to test our theory of gravity under the extreme conditions in the immediate vicinity of a black hole”, says David Haviland, chair of the Nobel Committee for Physics. It means that a scientific community will wait for ground breaking discoveries in black hole investigations.

4. ROGER PENROSE AND THEOREMS ON SINGULARITIES

One of the first vacuum solution of Einstein equation was the Schwarzschild solution (1916) in the form,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $r = (R^3 + \alpha^3)^{1/3}$, $\alpha = 2Gm/c^2$, while R is an ordinary radial polar coordinate. Soon after that Droste (1916) presented the solution in different coordinates, (see, Eq. (1)), but Droste (1916) interpreted r as an ordinary radial polar coordinate. These coordinates are now called Schwarzschild coordinates and such an object is called a black hole. A coordinate value $r_S = \alpha$ corresponds to a discontinuity of metric g_{rr} and it was called a singularity for many years. For instance, Regge and Wheeler (1957) investigated a stability of singularity (now we call it the event horizon). It was a long discussion about an opportunity to use the Schwarzschild solution for a real physical object. Oppenheimer and Snyder (1939) found that a black hole could be formed in a result of dust collapse. The authors used a spherically symmetric approximation to prove the claim. However, many people thought the Oppenheimer − Snyder model is too rough to describe a real astronomical object. For instance, Einstein (1939) considered an opportunity to mimic the Schwarzschild metric with moving objects and the Einstein conclusion was much more general than his analysis of his specific model and he noted that ”the essential result of this investigation is a clear understanding as to why the ”Schwarzschild singularities” do not exist in physical reality. Although the theory given here treats only clusters whose particles move along circular paths it does not seem to be subject to reasonable doubt that mote general cases will have analogous results. The ”Schwarzschild singularity” does not appear for the reason that matter cannot be concentrated arbitrarily.” It means that Einstein claimed the Schwarzschild metric has no chance to describe a real astronomical object. Einstein’s assistant P. Bergman (1942) expressed a similar opinion and wrote that ”in nature mass is never sufficiently concentrated to permit Schwarzschild singularity to occur in empty space. It means that Einstein and a lot of his followers thought that the Schwarzschild metric has only purely mathematical value and it is useless to model physical objects. A physical sense and a causal structure of the coordinate singularity at the event horizon started to be much more clear after studies of Finkelstein (1958) and Kruskal (1960).[7] There is a real singularity at the origin $r = 0$ for the Schwarzschild metric (and this

[7] A historical overview of the issue is given by Eisenstaedt (1982).

singularity cannot be removed with a coordinate transformation) since components of Riemann tensor for the metric tend to infinity for $r \to 0$. This circumstance leads to a publication of paper with title "Black holes 'do not exist'" (Ball, 2005). The author motivated this statement with the investigations by Chapline (2005) who claimed that at the end of stellar evolution a massive star could form so called "dark energy star" instead of black hole. It means that only around 15 years researchers discussed a useless of black hole concept for a description of a physical reality and practically they re-phrased old Einstein arguments formulated in 1939.

A formation of singularities in general relativity using natural assumptions about properties of stress-energy tensor was investigated by different researchers for many years. Using geometrical approach Roger Penrose investigated such a task without assumptions about symmetry conditions as it was done earlier. Penrose (1965) claimed that if actual physical singularities have to be occurred if inside of the collapsing object a) Positive local energy occurs; b) Einstein's equations are not violated; c) the space-time manifold is complete, d) concepts of space-time does not lose its meaning at high curvatures due to quantum phenomena. Recently, Penrose noted that the trapped surface is the key concept to prove existence of black holes.[8] Later, Hawking (1966) and Hawking and Penrose (1970) proposed different versions of the theorem. Now these statements are called Hawking – Penrose theorems on singularities (see, also Penrose (1968), Hawking and Ellis (1973), Misner, Thorne and Wheeler (1973)). One should keep in mind that we could be sure that the conclusions of the Penrose (1965) theorem are correct only in the case if its conditions occur, otherwise the theorem conclusions may be incorrect. For instance, equation of state for dark energy $p = w\rho$ and $w < -1$ and in this case a local energy may be negative (condition a) of the Penrose theorem is violated) and one could not claim for sure that physical singularities would be occurred. In principle, one could not exclude an opportunity that each of conditions (a) – d)) is violated and in the case one can not guarantee that physical singularities do exist. No doubt that the Penrose theorem (1965) played a fundamental role in a development of singularity studies in general relativity, however, we should note that conditions of the theorem (especially, a) and d)) can not be easily checked for real physical problem.

In addition to the trapped surface, Penrose introduced many important concepts in general relativity, for instance, the Hawking – Penrose theorems on singularities, the Penrose conformal diagrams, the Penrose twistors, the Newman – Penrose formalism (tetrads), the cosmic principle censorship, the Penrose process describing how to extract energy from a rotating black hole.

5. THE GALACTIC CENTER: OBSERVATIONS AND MODELS

It is generally accepted that there are supermassive black holes centers of galaxies, including our own Galaxy, see, for instance, paper by Kormendy and Ho (2013). In spite of that, theorists proposed many other alternative models (including exotic ones), for instance, a dense cluster of stars (Reid, 2009), fermion balls (Munyaneza and Viollier, 2002), boson stars (Jetzer, 1992; Torres, 2000), neutrino balls (de Paolis et al. 2001). Later, some of these models have been ruled out, or the range of parameters of these models are significantly limited with consequent observations (Reid, 2009).

[8] https://www.nobelprize.org/prizes/physics/2020/penrose/interview/.

Since the Galactic Center is the closest galactic center, the object was observed in different spectral bands, from radio to X-ray and γ. Based on results of observations it was suggested that it has to a supermassive black hole with mass not more $5\times10^6\ M_\odot$ (Rees, 1982), while Genzel and Townes (1987) and Genzel, Hollenbach and Townes (1994) claimed that observations are consistent with a supermassive black hole with mass around $10^6\ M_\odot$, while a number of people thought that black hole mass should be in the range of $10\,M_\odot$ to $100\,M_\odot$ (Ozernoy, 1987). It is amazing that based on relatively simple theoretical models and on observational results which allowed different interpretations, Rees and Genzel, Townes et al. correctly found the mass interval for the black hole at the Galactic Center (recently, R. Genzel noted that it is impossible to overestimate Townes's contribution impact in Galactic Center studies).

Shen et al. (2005), Doeleman et al. (2008), Doeleman (2017) observed Sgr A* with mm-interferometric observations and these authors found that the apparent size of the source has to be < 1 AU.

6. OBSERVATIONS OF BRIGHT STARS AND GAS CLOUDS AT THE GALACTIC CENTER

Two groups of astronomers observed motions of bright stars near the Galactic Center for almost three decades. One group of American astronomers led by A. Ghez uses Keck telescopes at Hawaii, another German – French group led by R. Genzel uses Very Large Telescopes (VLT) in Chile. Now four VLT telescopes are combined in giant interferometer which is called GRAVITY. The PI of GRAVITY collaboration is Frank Eisenhauer. The collaboration got a number of excellent results including new confirmations of GR predictions. At the end of last century almost simultaneously Eckart and Genzel (1996, 1997) and Ghez et al. (1998) found motions of bright stars at the Galactic Center. Eckart and Genzel (1996, 1997) used special equipment at the the 3.5-m New Technology Telescope (NTT) which gave them opportunity to monitor the stars for four years, while Ghez et al. (1998) used speckle interferometry at the Keck telescope.

7. ADAPTIVE OPTICS FOR GALACTIC CENTER OBSERVATIONS

The possibility of compensating astronomical seeing which is called now adaptive optics was proposed by Babcock (1953). A creation of telescopes with adaptive optics is an excellent example of a wonderful application of mathematics for a development of new technologies, however, like in this case sometimes it needs a significant time. Many years ago, famous Russian mathematician A. N. Kolmogorov (1941) proposed his phenomenological model for turbulence. These ideas were developed by Kolmogorov's student Obukhov (1941) who later was an academician of Soviet Academician and a director of Institute which is called now for Obukhov Institute of Atmospheric Physics. V. I. Tatarski (1961, who was a head of the laboratory at the Institute under Obukhov) significantly developed the Kolmogorov – Obukhov turbulence theory for propagation of electro-magnetic radiation and this stochastic approach for turbulence laid theoretical foundations for adaptive optics, see paper by Becker (1993), where first astronomical results obtained with adaptive optics systems were discussed. An obscuration by interstellar dust at the Galactic Center is very strong at visible wavelength, however, observations at near IR band (or astronomical

K-band) observations are possible. Observations and experiments showed that the Kolmogorov – Obukhov turbulence theory describes rather well atmospheric turbulence at K-band. In observations with adaptive optics astronomers used natural or artificial guide star created by laser excitation. Based on observations of guide star movable secondary mirrors correct wavefront of electromagnetic radiation toward a selected object. The Keck's group is equipped with adaptive optics for more than 20 years (Wizinowich et al., 2000) and slightly later the MPE – ESO group got a similar facilities (Rousset et al., 2003).

8. TESTING GR PREDICTIONS WITH S2

8. 1. GRAVITATIONAL REDSHIFT NEAR PERICENTER PASSAGE OF S2 STAR

The S2 star passed the its pericenter on May 19, 2018 and it is natural to expect to find relativistic effect, relativistic redshift of S2. Really, soon after the passage that the GRAVITY collaboration: Abuter et al. (2018) reported the discovery of gravitational redshift for S2. The GRAVITY collaboration: Abuter et al. (2018) represented the total redshift obtained from spectroscopical observations in the following form

$$z_{\rm tot} = z_K + f \times (z_{GR} - z_K), \tag{2}$$

where z_K is the Keplerian redshift, z_{GR} is gravitational redshift calculated taking into account the first post-Newtonian correction, $f = 0$ corresponds to Keplerian (Newtonian) fit, while $f = 1$ corresponds to the first parametric post-Newtonian fit. The GRAVITY collaboration established that $f = 0.90 \pm 0.09|_{\rm stat} \pm 0.15|_{\rm sys}$ and the authors also concluded that S2 data are inconsistent with a pure Newtonian dynamics with around 10σ confidence level.

Later, the estimate for redshift parameter has been corrected and GRAVITY collaboration: Abuter et al. (2019) found $f = 1.04 \pm 0.05$ with around 20σ confidence level and in addition the authors claimed that they evaluated the distance toward the Galactic Center with unprecendented accuracy, namely they found $R_0 = 8178 \pm 13|_{\rm stat} \pm 22|_{\rm sys}$ pc.

The Keck team (Do et al., 2019) obtained similar results namely, the authors found that $f = 0.87 \pm 0.17$ and it is consistent with GR at the 1 σ level. The Newtonian model $f = 0$ has to rejected with 5σ confidence level.

8. 2. RELATIVISTIC PRECESSION FOR S2 STAR ORBIT

To evaluate relativistic precession for S2 star the GRAVITY collaboration: Abuter et al. (2020) used the Will (2008) proposal for GR testing with observations of S2 like stars. The authors used the standard β and γ parameters for PPN approximation (Will, 2018). In general relativity, $\beta_{GR} = \gamma_{GR} = 1$. It is known that relativistic precession could expression through β and γ parameters (Will, 2008)

$$\Delta\phi_{\text{ per orbit}} = (2 + 2\gamma - \beta)\frac{\pi R_S}{a(1-e^2)}, \tag{3}$$

where R_S is the Schwarzschild radius, a is the semi-major axis and e is the eccentricity of the orbit. For S2 star $a = 125$ mas ($R_0 = 8246.7$ pc) and $e = 0.88$. If we note $f_{SP} = (2 + 2\gamma - \beta)/3$, then in GR f_{SP} should be equal 1. From their observations the GRAVITY collaboration: Abuter et al. (2020) found $\beta = 1.05 \pm 0.11$ and $\gamma = 1.18 \pm 0.34$ and $f_{SP} = 1.10 \pm 0.19$, therefore observations are in concordance with GR predictions and an extended mass distribution has a low impact on relativistic precession of S2 orbit.

9. CONSTRAINTS ON ALTERNATIVE THEORIES OF GRAVITY

It is clear, that there is an extended mass distribution around the supermassive black hole due to a presence of a stellar cluster and possibly a presence of dark matter. As it was shown by Rubilar and Eckart (2001), Nucita et al. (2007), Zakharov et al. (2007) an existence of an extended mass distribution leads to pericenter shift in the direction which is opposite to relativistic one.

In the last years, theorists proposed a number of alternative theories of gravity and sometimes they theories have non-Newtonian limit in a weak gravitational field approximation. Borka et al. (2013), Zakharov et al. (2016), Zakharov et al. (2018) found constraints on parameters of Yukawa gravity and graviton mass, in particular, the graviton mass constraint $m_g = 2.9 \times 10^{-21}$ eV found by Zakharov et al. (2016) is included in Particle Data Group (Tanabashi, 2018, 2019). Zakharov (2018) showed an opportunity to evaluate a tidal charge since in this case the pericenter shift could be calculated analytically. Hees et al. (2017), the GRAVITY Collaboration: A. Amorim et al.,: 2019, the GRAVITY collaboration: Abuter et al. (2020) demonstrated opportunities to improve current constraints on alternative theories of gravity.

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