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# PERIODIC ORBITS IN THE MAIN LUNAR PROBLEM

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**Abstract.** We investigate the arrangements of periodic orbits in a restricted, circular, 3-dimensional 3-body problem in which the primaries are the Sun and the Earth, and the massless body represents the Moon. By fixing the duration of the periodic orbits to a multiple of the synodic lunar month, we set up a numerical procedure that systematically finds all the periodic orbits of given duration, not only close to the current lunar orbit, i.e. at low eccentricity and low inclination, but also far from it.

## 1. INTRODUCTION

The study of the role of high-integer near commensurabilities among lunar months (Perozzi et al., 1991; Steves et al., 1993) has led to the discovery that the lunar orbit is very close to a set of 8 long-period periodic orbits of the restricted circular 3-dimensional Sun-Earth-Moon problem in which also the secular motion of the argument of perigee is involved (Roy et al., 1991; Valsecchi et al., 1993a).

In each of these periodic orbits 223 synodic months are equal to 239 anomalistic and 242 nodical ones, a relationship that approximately holds in the case of the observed Saros cycle, and the various orbits differ from each other for the initial phases. These integer ratios imply that, in one cycle of the periodic orbit, the argument of perigee makes 3 revolutions, i.e. the difference between the 242 nodical and the 239 anomalistic months; in fact, these two months differ from each other just for the prograde rotation of the argument of perigee.

## 2. POINCARÉ'S CONJECTURE ON PERIODIC ORBITS

The periodic orbits associated with the Saros cycle appear to be of long duration when compared to those usually found in literature; however, they are by no means the longest ones that can be found close to that of the Moon, as was suggested by Valsecchi et al. (1993b) and is shown hereafter.

According to a conjecture of Poincaré[1] there should be infinitely many periodic orbits, of longer and longer period, approximating better and better any bounded motion in a problem like the one at hand.

If we fix the duration of the synodic month, for instance to that of the currently observed one (29.530589 d), it is possible to show, with the help of Delaunay's expressions for the motion of the lunar perigee and node, that these longer and longer periodic orbits are arranged in the mean eccentricity vs. mean inclination plane in a rather characteristic pattern (Valsecchi et al. 1993b). This pattern, in turn, is simply a deformation of the arrangement, in frequency space, of the set of points corresponding to the frequencies of the periodic orbits themselves.

This finding allows us to set up a numerical scheme to find the periodic orbits systematically; this tool can then be used to make an exploration of of orbits in the Main Lunar Problem.

## 3. OUR PROCEDURE

A Saros comprises 223 synodic months $T_S$, thus lasting

$$223 \cdot 29.530589 = 6585.321347 \text{ d}.$$

Eclipses repeat because 242 nodical months $T_N$ last

$$242 \cdot 27.212220 = 6585.357240 \text{ d},$$

and their characteristics repeat because 239 anomalistic months $T_A$ last

$$239 \cdot 27.554551 = 6585.537689 \text{ d};$$

thus,

$$223 T_S \approx 239 T_A \approx 242 T_N.$$

It can be shown that, if at a certain time the Moon is either new or full, and is very close to both the line of apsides and the line of nodes of its geocentric orbit, it will be in the same situation a Saros later.

We can generalize the angular relationships involved in the Saros; given three integers $N_S$, $N_A$, $N_N$, we can look for periodic orbits such that

$$(N_N - N_S)\dot{\lambda} - N_N \dot{\lambda}' + N_S \dot{\Omega} = 0$$

and

$$(N_N - N_A)\dot{\lambda} - N_A \dot{\lambda}' + N_S \dot{\varpi} = 0,$$

[1] Il y a même plus: voici un fait que je n'ai pu démontrer rigoureusement, mais qui me paraît très vraisemblable.

Étant données des équations de la forme définie dans le n° 13 et une solution particulière quelconque de ces équations, on peut toujours trouver une solution périodique (dont la période peut, il est vrai, être très longue), telle que la différence entre les deux solutions soit aussi petite qu'on le veut, pendant un temps aussi long qu'on le veut.

D'ailleurs, ce qui nous rend ces solutions périodiques si précieuses, c'est qu'elles sont, pour ainsi dire, la seule brèche par où nous puissions essayer de pénétrer dans une place jusqu'ici réputée inabordable (Poincaré 1892, p. 82).

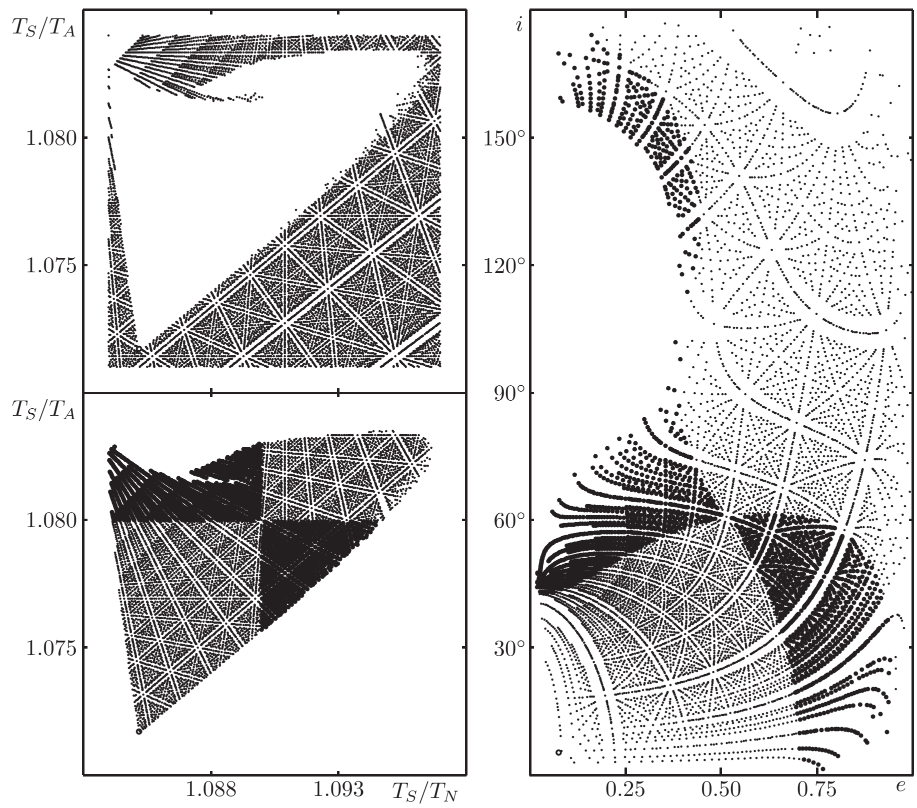


Figure 1: Left panels: all the points with rational coordinates in the space $T_S/T_A$ versus $T_S/T_N$, for $T_S \leq 750$; the lower panel contains the rational points for which a periodic orbit has been found, the upper panel contains the remaining rational points. Right panel: the location, in the space $i$-$e$, of the initial conditions of periodic orbits of duration up to $750 \cdot T_S$, with initial phases $\omega = 0°$, $M = 0°$, and $\lambda - \lambda' = 0°$. The open circle in each left panel shows the frequency ratios of the real lunar orbit, the one in the right panel shows the lunar orbit when $\omega = 0°$, $M = 0°$, and $\lambda - \lambda' = 0°$.

where $\dot{\lambda}$, $\dot{\lambda}'$, $\dot{\varpi}$, $\dot{\Omega}$, are respectively the time derivatives of the geocentric mean longitudes of the Moon and of the Sun, and of the longitudes of the lunar perigee and node.

A way to understand how these orbits are arranged in the space of lunar orbital elements is to fix the duration of the synodic month, thus fixing the average mean motion of the Moon, and use Delaunay's expressions for $\dot{\varpi}$ and $\dot{\Omega}$, as done by Valsecchi et al. (1993b). Approximate initial conditions (osculating $a$, $e$, $i$) of periodic orbits can be found by interpolating in three dimensions (two frequency ratios and the value of the element to interpolate); details of the interpolation procedure are given in a paper in preparation. Once approximate initial conditions are established, the periodic orbits can then be found in the way described in Valsecchi et al (1993a).

## 4. PRELIMINARY RESULTS

We are currently performing a systematic exploration, for all the possible values of the initial phases, of the Saros-like periodic orbits of this problem. Hereafter we show our preliminary results for the periodic orbits with initial phases $\omega = 0^\circ$, $M = 0^\circ$, and $\lambda - \lambda' = 0^\circ$.

The right panel of Fig. 1 shows the location, in the space osculating inclination $i$ vs. osculating eccentricity $e$, of the initial conditions of periodic orbits of duration up to $750 \cdot T_S$. The left panels contain all the points with rational coordinates in the space $T_S/T_A$ versus $T_S/T_N$, for $T_S \leq 750$; in particular, the lower panel contains the rational points for which a periodic orbit has been found, while the upper panel contains the remaining rational points.

Thus, each point in the lower left panel corresponds to a point in the right panel; to guide the eye in identifying the correspondence, for $T_S/T_A < 1.080$ and $T_S/T_N < 1.090$ we use small dots in both the lower left panel and the right panel, and do the same for $T_S/T_A > 1.080$ and $T_S/T_N > 1.090$; otherwise, i.e. for $T_S/T_A > 1.080$ and $T_S/T_N < 1.090$ or for $T_S/T_A < 1.080$ and $T_S/T_N > 1.090$, we use larger dots.

With the help of this dot-size coding, it is relatively easy to identify the correspondence between the alignements of points along straight lines in the lower left panel, that correspond to resonances, and the corresponding points in the right panel. Some of these resonances start at relatively low values of eccentricity and inclination, and go all the way up to eccentricity close to 1 and inclination close to 180°.

### References

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