Dynamics of Celestial Bodies, 159 - 164 *Contributed paper*

CHAOS IN A BL-Lac DYNAMICAL MODEL

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Abstract. We construct a dynamical model in order to study the motion in a BL-Lacertae active galaxy. The model consists of a logaritmic potential with an internal perturbation. Two cases are investigated. (i) The time independent model and (ii) the evolving model. A large number of orbits are chaotic in both cases. Responsible for the chaotic motion are the internal perturbation, the flattening parameter and the dense nucleus. Theoretical arguments support the numerical outcomes. Comparison of the results with data from observations is also made.

1. INTRODUCTION

BL-Lacertae (BL-Lac) objects are active galaxies characterized by rapidly varying luminosity no emission lines in the spectrum and point Vlike appearance. The first observed BL-Lac galaxy was mistaken for a peculiar kind of variable star in our own galaxy and assigned that kind of name. The mystery of the BL-Lac was solved about fifty years later in the late 1970s when a group of astronomers demonstrated unequivocally that the prototype of the BL-Lacs is located in a luminous galaxy (see Miller & Hawley 1977, Miller et al. 1978). Particularly, Miller and his collaborators showed that the light from the nucleus of the prototype of the BL-Lacs had a spectrum similar to that of the small elliptical galaxy M32. This result, together with the fact that we can see much further into the heart of a BL-Lac object, strongly suggests that BL-Lac objects are elliptical galaxies with bright active nuclei.

Given the above, it seems challenging to construct a dynamical model in order to study the motion in a BL-Lac object. In particular, we shall try to segregate the critical parameters of the system, that is the parameters that play an important dynamical role in the system and define this role. Furthermore, it would be of particular interest to compare the results obtained using our dynamical model with results available from observations.

In order to describe the motion in our BL-Lac model we use the potential

$$V_{BL}(x,y) = \frac{1}{2}\upsilon_0^2 \ln[x^2 + ay^2 - \lambda x^3 + c_n^2] \quad . \tag{1}$$

The dynamical model (1) represents an elliptical galaxy with a flattening parameter $1 \leq \alpha < 2$ and a nucleus of radius c_n. The term $-\lambda x^3, \lambda << 1$ represents an internal perturbation (see Binney & Tremaine 1987). The parameter υ_0 is used for the consistency of the galactic units. We use a system of galactic units, where the unit of length is $1kpc$, the unit of time is $0.97746 \times 10^8 yr$ and the unit of mass is $2.325 \times 10^7 M_{\odot}$. The velocity and the angular velocity units are $10km/s$ and $10km/s/kpc$ respectively, while G is equal to unity. The energy unit (per unit mass) is $10(km/s)^2$. In the above units we use the values, $\upsilon_0 = 15, c_n = 0.25$, while α, λ, Ω are treated as parameters.

In a clockwise rotating frame, with an angular velocity Ω, potential (1) writes

$$V_{eff}(x, y) = V_{BL}(x, y) - \frac{1}{2}\Omega^2(x^2 + y^2) \quad , \tag{2}$$

while the corresponding Hamiltonian is

$$H_J = \frac{1}{2}(p_x^2 + p_y^2) + V_{BL}(x, y) - \frac{1}{2}\Omega^2(x^2 + y^2) = \frac{1}{2}(p_x^2 + p_y^2) + V_{eff}(x, y) = h_J \quad , \tag{3}$$

where p_x, p_y are the momenta, per unit mass, conjugate to x and y, h_J is is the numerical of the Hamiltonian which is conserved and is it the well known Jacobi integral. The equations of motion in this rotating coordinate system are

$$\ddot{x} = -2\Omega p_y - \frac{\partial V_{eff}}{\partial x} \ , \ \ddot{y} = 2\Omega p_x - \frac{\partial V_{eff}}{\partial y} \quad , \tag{4}$$

where the dot indicates derivative with respect to the time.

This article is organized as follows: In Section 2, we study the structure of the phase plane and present the different families of orbits in the Hamiltonian (3). Some theoretical arguments, supporting the numerical results, are given in the same Section. In Section 3, we present results when our model is time dependent. We close with Section 4, where a linking to observation data together with a discussion and the conclusion of this research are given.

2. ORBITS IN THE TIME-INDEPENDENT POTENTIAL

This Section is devoted to the study of the structure of the $x - p_x, y = 0, p_y > 0$ phase plane. The values of parameters are $h_J = 500, \alpha = 1.1, c_n = 0.25$. When $\Omega = 0, \lambda = 0$ the motion is regular, while chaotic regions, if any, are negligible. There are two main families of orbits producing invariant curves around the 1:1 direct and retrograde periodic points. For the above values of the other parameters and $\Omega = 0.5$, we observe a considerable chaotic layer which appears between the regular areas, while the at the same time small islands are produced by secondary resonances. Thus, we can say that the role of rotation is to produce chaotic motion. Fig. 1a shows the phase plane, when $\lambda = 0.01, \Omega = 0.5$, while other parameters are as above. Here the chaotic region is much larger, while some secondary resonance are also present. Therefore, it is evident that the internal perturbation enlarges the chaotic regions. Fig. 1b is same as Fig. 1a but when $\alpha = 1.7$. In this case one observes a large chaotic sea, while the regular areas around the 1:1 periodic points are much smaller.

In the following we shall give a semi-theoretical explanation about the structure of the phase plane descibed above and given in Fig. 1a-b. The star's angular momentum in the rotating case is given by

$$L = xp_y - yp_x - \Omega(x^2 + y^2) \quad . \tag{5}$$

Although the angular momentun is not conserved, we have proven (see Caranicolas and Papadopoulos 2005 and references therein) that its averaged value decreases as Ω increases. It is also well known that stars of small angular momentun can display chaotic motion in galaxies with dense nuclei. Therefore, the one expects to observe larger chaotic layers in the phase plane as as Ω increases.

The role of the vertical force is very important in order to observe chaotic motion in galaxies with massive nuclei. The stronger is the vertical force the larger chaotic regions are observed (see Caranicolas and Innanen 1991, Caranicolas and Papadopoulos 2003). The role of vertical force for the potential (1) is played by the F_y force component, which is

$$F_y = -\frac{\alpha v_0^2 y}{x^2 + \alpha y^2 - \lambda x^3 + c_n^2} + \Omega^2 y \quad . \tag{6}$$

It is evident that the attractive F_y force near the nuclear region for given but small values of x and y, when all other parameters are fixed, increases as λ increases. The F_y force also increase rapidly as the flattening parameter α increases. This fact justifies the increase of the chaotic regions observed in Figs. 1c and 1d. Another interesting result coming from equation (6) is that the F_y force increases as the radius of nucleus decreases. Thus, one would expect larger chaotic regions from galaxies with denser nuclei. Numerical calculations, not shown here, justify this result.

3. ORBITS IN THE TIME-DEPENDENT POTENTIAL

In this Section we shall study the behavior of orbits in the time dependent potential. In particular, we shall consider the case when our BL-Lac model evolves in such a way so that the parameters α and λ are linear functions of the time given by the equations

$$\alpha(t) = a_{in} + k_1 t, \lambda(t) = \lambda_{in} + k_2 t \quad , \tag{7}$$

where $\alpha_{in}, \lambda_{in}$ are the initial values of α and λ, while k_1, k_2 are parameters. Let us consider an orbit starting at $x_0 = 6, y_0 = p_{x0} = 0$ with initial value of energy $h_{J0} = 500$. The value of p_{y0} in all cases is found from the energy integral. It is evident that this orbit stars near the retrograde periodic point. All other parameters are as in Fig. 1c, while the flattening parameter α evolves following the first of equations (7) with $\alpha_{in} = 1.1$ and $k_1 = 0.01$. During the evolution the energy is not conserved because the potential is time dependent. It is assumed that the evolution stops when α reaches a final value $\alpha_{fin} = 1.7$. This means that for the first 60 time units the galaxy evolves and after that the evolution stops and we have a final value of energy. The results for a period of 100 time units are shown in Fig. 2. We observe a well defined quasi-periodic tube orbit. The final value of energy is $h_{JF} = 527.85$. Therefore, we

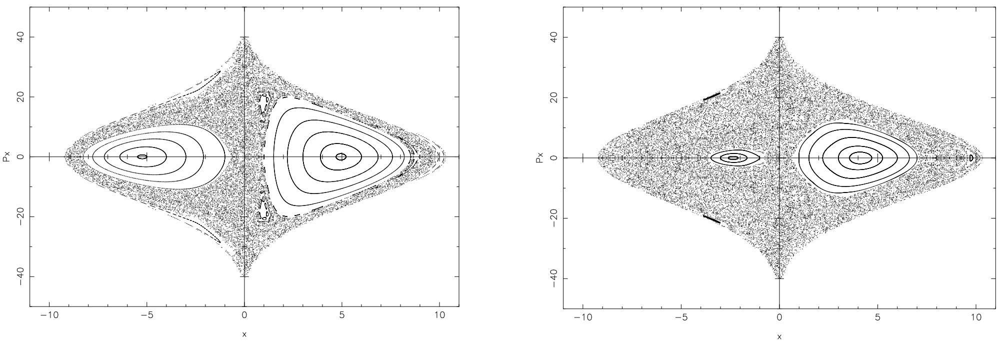


Figure 1: a-b: a-left, b-right. The $x - p_x$ Poincare phase plane for the Hamiltonian (3). The values of parameters are $h_J = 500, \Omega = 0.5, c_n = 0.25$. Details are given in text.

can say the orbit starts as regular, remains regular during the evolution and continues as regular after the end of the galactic evolution. Furthermore, we conclude that the structure of the phase plane, near the retrograde periodic point, continues to be the same as that of Fig. 1a.

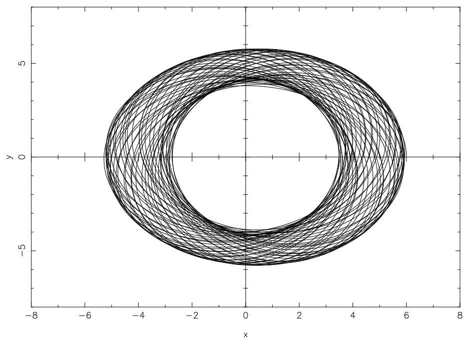
Fig. 3 is same as Fig. 2 but the orbit has different initial conditions. This orbit starts at $x_0 = -6, y_0 = p_{x0} = 0$ with initial value of energy $h_{J0} = 500$. The final value of energy is $h_{JF} = 539$. Here, the numerical calculations suggest that the orbits starts as regular, becomes gradually chaotic during the galactic evolution and remains chaotic after the galactic evolution has stopped. In this case, we can say that the structure of the phase plane, near the direct periodic point, changes drastically from regular to chaotic.

4. COMPARISON WITH OBSERVATIONAL DATA

It is today clear that the name BL-Lacertae is actually a remnant from an original misidendification. The first BL-Lac galaxy was mistaken for a variable star in our own galaxy. Now we know that BL-Lacs are distant galaxies with active nuclei. The excellent seeing conditions obtained by the large telescopes give interesting information of the properties and environment of those galaxies during the last years (see Burbidge 1996, Falomo et al. 1996, Falomo et al. 2002, Grundahl et al. 1995, Heidt et al. 1996, Nilsson et al. 1996, Vladilo et al 1997, Wright et al. 1998, Barth et al. 2002, Falcone et al. 2004).

On this basis we felt it would be of interest to construct a dynamical model for BL-Lac galaxy, study its dynamical behavior and compare theoretical with observational data. The model and some of its dynamical properties were presented in the preceding Section. In this Section, we shall try to compare some theoretical outcomes with data from observations.

A first step for this comparison is to obtain the rotational velocity $\Theta = \Theta(r)$ corresponding to the axially symmetric $(\alpha = 1, \lambda = 0)$ dynamical model (1). This is given by the equation



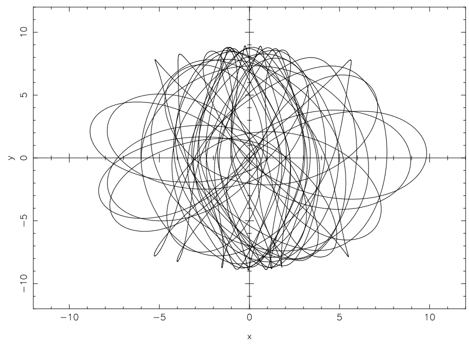


Figure 2: - Figure 3: Fig. 2-left. A regular orbit in the time dependent potential. Initial conditions starting at $x_0 = 6, y_0 = p_{x0} = 0, h_{J0} = 500$. All parameters are as in Fig. 1a, while the flattening parameter α evolves following the first of equations (7) with $\alpha_{in} = 1.1$ and $k_1 = 0.01$. Fig. 3-right. Same as Fig. 2 but with initial conditions $x_0 = -6, y_0 = p_{x0} = 0$. The orbits starts as regular, becomes gradually chaotic.

$$\Theta(r) = \upsilon_0 \sqrt{r \frac{dV_{BLA}(r)}{dr}} = \upsilon_0 \sqrt{\frac{r^2}{r^2 + c_n^2}} \quad , \tag{8}$$

where $V_{BLA}(r)$ the axially symmetric potential. An estimation of the mass M_0 inside the radius c_n can be made using the formula

$$M_0 = r_0 \frac{\Theta_0^2}{G} \quad . \tag{9}$$

When $r_0 = c_n$, equation (8) gives $\Theta_0 = \upsilon_0/\sqrt{2}$. Setting these values of r_0, Θ_0 in (9), we obtain

$$M_0 = \frac{\upsilon_0^2 c_n}{2G} \quad . \tag{10}$$

For values of c_n between 0.1 and 0.25 equation (10) gives values of M_0 in the range $2.6 \times 10^8 M_\odot - 6.5 \times 10^8 M_\odot$. Those values are in very good agreement with the values of the active nuclei inside BL-Lac objects $10^{7.9} M_\odot - 10^{9.2} M_\odot$ obtained by Barth et al. (2003) using observational data.

It is also of interest to compare the maximum theoretical velocity with that obtained using observational data. Setting $y = 0$ in the integral (3) we find

$$\frac{1}{2}(p_x^2 + p_y^2) + V_{eff}(x) = h_J \quad . \tag{11}$$

Equation (11) gives

$$\frac{1}{2} p_y^2 \geq V_{eff}(x) + h_J - p_x^2 \quad , \tag{12}$$

defining the area on the $x - p_x$ phase plane inside which motion is permitted. The equation

$$p_x^2 = 2[h_J - V_{eff}(x)] = 2h_J - \upsilon_0^2 \ln(x^2 - \lambda x^3 + c_n^2) \quad , \tag{13}$$

defines the limiting curve. The maximum p_x velocity occurs at $x = 0$ and since we are on the limiting curve, where $y = p_y = 0$, this is the maximum total velocity. Thus

$$\upsilon_{max} = \sqrt{2(h_J - \upsilon_0^2 \ln c_n)} \quad . \tag{14}$$

Note that for a fixed value of the energy h_J the maximum velocity increases as the radius of nucleus decreases that is higher velocities are expected in galaxies with dense nuclei. For the values $h_J = 500, c_n = 0.25, \upsilon_0 = 15$ we find $\upsilon_{max} = 403km/s$. This value is very close to the maximum velocity observed by Barth et al (2003), which was found equal to $370km/s$

Therefore, one can say, that our dynamical model describes in a satisfactory way the properties of motion in a BL-Lac active galaxy and its theoretical outcomes are in good agreement with the data available from observations. Since active galaxies is a fast developing branch of Observational Astronomy, we hope to be able to construct better dynamical models for those systems in the near future.

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