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DISCRETE BREATHERS IN A CHAIN OF COUPLED SYMPLECTIC MAPS

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Abstract. An one-dimensional chain of weakly coupled symplectic maps is studied. We examine the existence and the size of the stability regions around specific periodic orbits in the phase space of the system with respect to the length of the chain and the coupling strength between the oscillators. In order to accomplish this, we consider a set of orbits defined by a grid of initial conditions in a proper section of the phase space of the system and classify them according to their regularity using the fast Lyapunov indicator (FLI) as a chaos indicator. The correlation between the existence of stability islands and the linear stability of the corresponding periodic orbits is demonstrated. The results of our study are used in order to locate Discrete Breathers and examine their stability properties.

1. INTRODUCTION

By the term Discrete Breather (DB) we denote a spatially-localized and time-periodic motion in a spatially periodic and extended (virtually infinite) system. By the term localized we refer to a state of the system in which the energy is mainly concentrated around one site of the chain or the lattice, which is called the “central” oscillator. The other sites also oscillate but the amplitude of the oscillation decays, usually exponentially, as we move away from the central oscillator. In the present work, we consider the system to be an one dimensional chain of k two-dimensional symplectic mappings. We define a section of the phase space of the system by fixing initial conditions for all the non-central oscillators and we take a grid of initial conditions for the central oscillator in this section. Then we can construct a map by classifying the above defined orbits as *regular* or *chaotic* using a suitable chaos indicator (the *Fast Lyapunov Indicator* or *FLI* in our case) (Froeschlé, et al. 1997, Froeschlé & Lega 2000). Although the system is high dimensional, in a DB solution the major portion of the energy is concentrated in the mapping which represents the central oscillator. So, the result of this method is expected to be similar to a 2D phase portrait. Consequently, there will be stability regions around the stable isolated periodic orbits which correspond to DBs. This fact will be used in order to locate the

DBs. In addition, the existence and size of these stability islands provides us with important information about the stability properties of the specific solutions.

2. THE MODEL OF COUPLED SURIS MAPPINGS

Consider a representative of the Suris maps family (Suris 1989) (Fig. 1a):

$$\begin{aligned} x_i' &= x_i + 4\pi^2 y_i' \qquad \bmod 2\pi \\ y_i' &= y_i - \frac{1}{\pi^2}\arctan\left(\frac{\sin(x)}{3+\cos(x)}\right). \end{aligned} \tag{1}$$

This mapping is symplectic and integrable. Using a sinusoidal weak nearest-neighbour coupling, we construct the chain:

$$\begin{aligned} x_i' &= x_i + 4\pi^2 y_i' \qquad \bmod 2\pi \\ y_i' &= y_i + V'(x_i) + \varepsilon\sin(x_{i+1} - x_i) + \varepsilon\sin(x_{i-1} - x_i) \end{aligned} \tag{2}$$

where i denotes the oscillator in a chain of length k and ε stands for the strength of the coupling. Note that we always consider the oscillators at the edges of the chain to be fixed at the stable fixed point $(x, y) = (0, 0)$.

3. STABILITY ANALYSIS

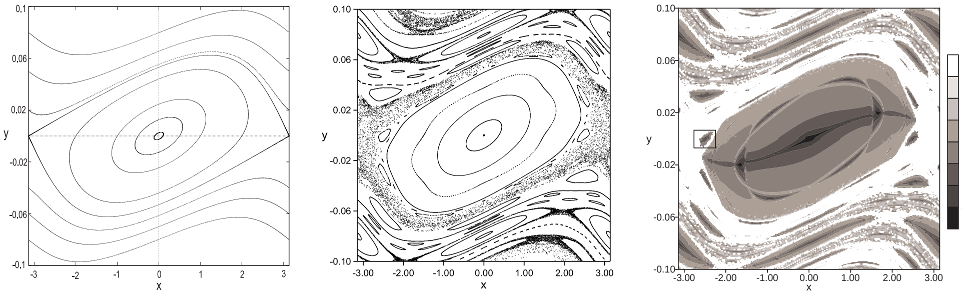
First we consider a $k = 3$ oscillators length chain, and compute the FLI stability map for the central oscillator. In Fig. 1 we compare the phase space of the single oscillator with the Poincaré surface of section of the three oscillator system and the corresponding stability map. The stability analysis is performed for the period-8 orbit and the corresponding island around it which is shown in Fig. 1c.

We performed the stability analysis for chains of length $k = 5, 7, \ldots, 31$ oscillators, where we consider initial conditions for the non-central oscillators corresponding to the period-8 DB (Koukouloyannis & Ictiaroglou 2002), while the central one takes initial values from the defined grid and established that the results of the linear stability analysis are in total agreement with the results derived by the method of the stability maps as it can be shown in Fig. 2.

In addition, we construct the diagram of the number of oscillators versus the critical value of $\varepsilon = \varepsilon_{cr}$ for which the instability occurs (Fig. 3). It seems that for chain length $k > 10$ the size of the chaon does not affect the stability results.

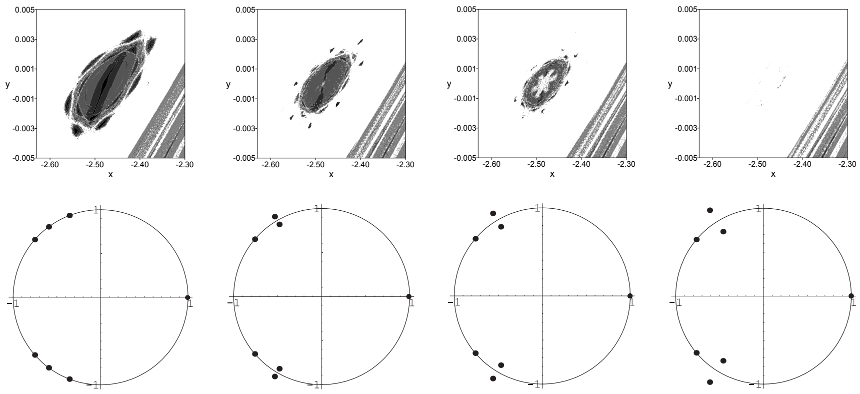
4. THE USE OF STABILITY MAPS IN LOCATING DBs

We examine if we can locate a period-10 DB using the above mentioned results. We compute a stability map of initial conditions of the central oscillator having the others at rest in a $k = 9$ chain and for $\varepsilon = 0.001$. The resulting map is shown in Fig. 4a where we can distinguish the period-10 chain and select one of its islands (Fig. 4b) as an initial estimation in order to calculate exact initial conditions for the corresponding breather which is shown in Fig. 4c.



(a) (b) (c)

Figure 1: (a) The phase space of the integrable Suris map (b) The poincare surface of section for the coupled system ($k = 3$, $\varepsilon = 0.0028$) and (c) the corresponding stability map of (Fig. 1b)where white and gray color points correspond to chaotic and regular orbits, respectively.



$\varepsilon = 0.0023$ $\varepsilon = 0.00238$ $\varepsilon = 0.0024$ $\varepsilon = 0.00245$

Figure 2: The evolution of a period-8 stability island and the eigenvalues of the linearized system corresponding to the periodic orbit in a $k = 5$ chain for increasing values of ε.

5. CONCLUSIONS

We studied linear stability of DBs in a chain of weakly coupled symplectic mappings using stability maps. We compared the results with the stability results extracted by using FLI stability maps and establish a perfect agreement between the two methods. By increasing the length of the chain under consideration we realized that the stability results remain intact for lengths larger than ten oscillators. Finally we showed how the stability maps can be used in order to locate DBs and determine their stability properties. For a more detailed description one can refer to Koukouloyannis et al. 2008.

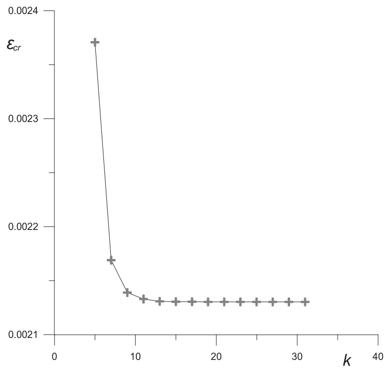
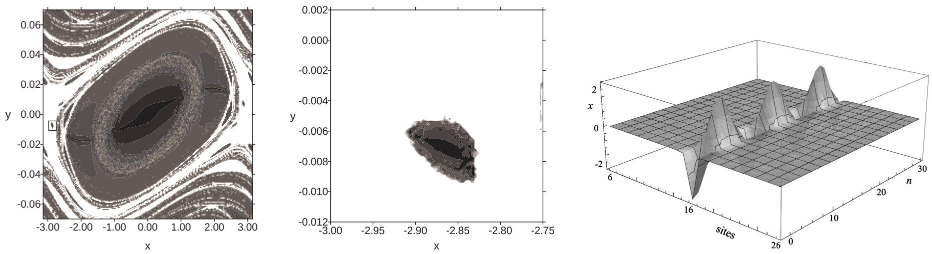


Figure 3: The value ε_{cr} vs the size k of the chain.



(a) (b) (c)

Figure 4: (a) The stability map for $\varepsilon = 0.001$ and $k = 9$ (b) a zoom around an island of the period 10 chain and (c) the time evolution of the corresponding DB.

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References

Froeschlé, C., Lega, E. and Gonczíu, R.: 1997, *Cel. Mech. and Dyn. Astr.*, **67**, 41.
Froeschlé, C. and Lega, E.: 2000, *Cel. Mech. and Dyn. Astr.*, **78**, 167.
Koukouloyannis, V. and Ichtiaroglou, S.: 2002, *Chaos, Solitons and Fractals*, **13**, 1317.
Koukouloyanis, V., Tziotzios, T. and Voyatzis, G.: 2008, *Nonlinear Phenomena in Complex Systems*, **11**, 233.
Suris, Yu. B.: 1989, *Func. Anal. Appl.*, **23**, 74.