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THE COPENHAGEN CASE WHEN THE PRIMARIES ARE OBLATE SPHEROIDS WITH DIPOLE-TYPE MAGNETIC FIELDS

T. KALVOURIDIS

National Technical University of Athens

E–mail: tkalvouridis@gmail.com

Abstract. The Copenhagen case of the restricted three-body problem is the simplest system with more than two bodies and still remains a problem of interest after the discovery of many extra solar planetary systems, the majority of which consist of two major bodies. In this work we consider an extension of the case, where the two primaries are no longer spherical but oblate spheroids and furthermore that they dispose magnetic fields of dipole-type. Here we study some aspects of the dynamics of a charge with negligible mass which is moving under the influence of the Lorentz's forces of the primaries, but is also affected by the changes due to the oblatenesses of these bodies.

1. INTRODUCTION

A look back over a century of investigation of a charged particle dynamics in an electromagnetic field and the various phenomena associated with it that occur in the neighborhood of Earth reveals an era of rigorous growth not only in depth but also in sheer volume. From the simple Störmer's problem at the beginning of the 20th century until the most complex models of the last decade, many configurations, theories, experiments and mathematical processes have been proposed and carried out in order to give universal answers. Moving in this framework, some years ago a model commonly known as the problem of two rotating magnetic dipoles was proposed. It was based on the combination of both old Störmer's problem and the restricted three-body problem. The fundamental configuration consists of two massive bodies with dipole-type magnetic fields that rotate around their common center of mass in circular orbits with constant angular velocity under their mutual Newtonian attraction. A charged particle with negligible mass moves in the created by the primaries electromagnetic field. After the discovery of many extra-solar planetary systems during the last 15 years, this model comes again on stage and acquires theoretical interest since many of these systems have two members and some of them may dispose magnetic fields. In this presentation we further elaborate on the problem by assuming that the two primaries are oblate spheroids with equal masses (Copenhagen case) and we present some aspects of it that are related to the parametric evolution of the regions where $3-D$ particle motions exist.

2. EQUATIONS OF MOTION

We consider a synodic coordinate system $Oxyz$ where the plane Oxy is the plane of the primaries' motion, and oblate and homogeneous primaries that create dipole-type magnetic fields. The particle moves acted upon by the Lorentz's forces created by the two primaries P_1 and P_2. Its normalized equations of motion have the form (Kalvouridis, 1991),

$$\begin{aligned} \ddot{x} - f^*\dot{y} + g^*\dot{z} &= U_x^* \\ \ddot{y} - h^*\dot{z} + f^*\dot{x} &= U_y^* \\ \ddot{z} - g^*\dot{x} + h^*\dot{y} &= U_z^* \end{aligned} \tag{1}$$

where

$$U^* = \frac{1}{2}\omega^{*2}(x^2+y^2) + \omega^*(xA_y - yA_x) + \frac{1}{2}\omega^*(xC_y - yC_x) - \frac{3}{2}\omega^* z^2(xD_y - yD_x) \tag{2}$$

and the Jacobian integral of motion

$$\frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = U^* + C \tag{3}$$

The analytical expressions of the quantities that appear in (1) and (2) are found in the above reference. Hereafter we shall assume that the magnetic moments of both primaries are perpendicular to the plane Oxy of the synodic system (Fig. 1).

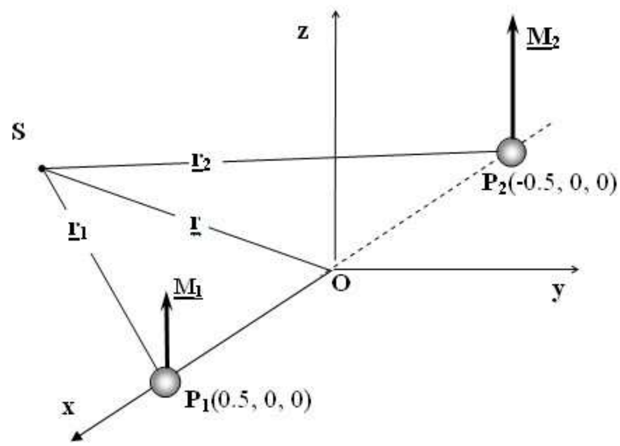


Figure 1: The configuration of the considered model:P_1 and P_2 are the primaries-dipoles and S is the charged particle

3. EQUILIBRIUM LOCATIONS OF THE PARTICLE AND ZERO-VELOCITY SURFACES

We can summarize the principal conclusions regarding the existing equilibrium locations of the particle (L_i, $i = 1, 2, \ldots$) for the considered configuration:

- All the equilibrium positions lie on the Oxy plane and are unstable.
- There are either three, five or seven equilibrium points depending on λ.
- Three of them L_1, L_2, and L_3, are collinear, that is, they are located on the axis Ox and appear for any value of $\lambda > 0$.
- The rest four points are triangular and appear in pairs in symmetric positions with respect to the x-axis. For some values of λ one or both pairs are missing.
- Oblateness increases the Jacobian constants of the equilibrium points and pushes them towards the origin (the center of mass of the system).

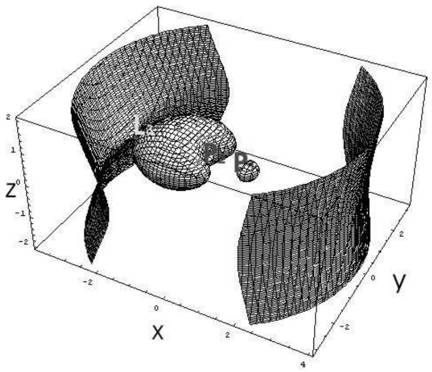
As it is known, the zero-velocity surfaces are imaginary surfaces that limit the regions of the $3-D$ space where the motion of the particle is permitted from those where this motion is forbidden. Regarding the evolution of these surfaces, we have investigated the following cases: (1) one primary (either P_1 or P_2) is oblate, (2) both primaries are oblate with the same or different oblateness. The main results obtained from the study of these surfaces can be summarized as follows:

- The evolution of the zero-velocity surfaces depends in any case on the order of the Jacobian constants C_{L_i} of the equilibrium points.
- When $C > max\{C_{L_i}, i = 1, ..n\}$, where n is the number of equilibrium locations, the zero-velocity surfaces consist of two internal closed surfaces formed in the neighborhood of the two dipoles. Inside them the particle motion is trapped. A large external shell surrounds these surfaces and the two dipoles. The particle is also free to move outside the external shell.
- Bifurcations in the topology of the zero-velocity surfaces happen at the values of C that correspond to the equilibrium positions (see Figs. 2a, 2d).
- The maximum number of trapping regions is two and the minimum is zero.
- Oblateness produces "compression" on the z-direction on the external shell and the zero-velocity surfaces in general.

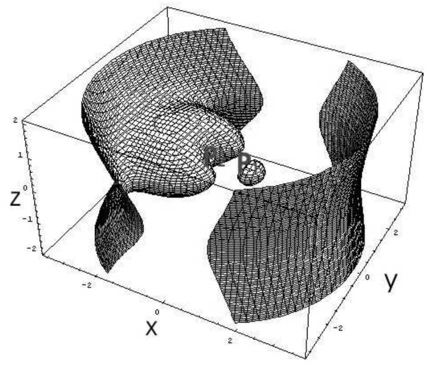
In what follows we present some indicative results obtained from the study of a case where $\lambda = 10$, $I_1 = 0.001$, $I_2 = 0$. In this case seven equilibrium points have been found, the Jacobian constants of which satisfy the inequality:

$$C_{L_3} > C_{L_4,L_5} > C_{L_1} > C_{L_6,L_7} > C_{L_2} \tag{4}$$

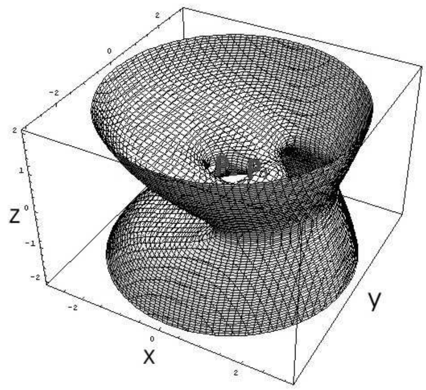
In Fig. 2a, when $C = C_{L_3}$, the closed surface of permitted motion that evolves on the left side of primary P_2 touches the external shell at the location of L_3. When $C_{L_4,L_5} < C < C_{L_3}$, a cavity is created on the left side of the external shell (Fig. 2b). If $C = C_{L_4,L_5}$ the closed surface near primary P_1 comes in contact with the internal "bulge" at the locations of L_4 and L_5. For values $C_{L_4,L_5} > C > C_{L_1}$ two transversal holes are created near primaries P_2 and P_1, as well as a channel between them that connects two opposite parts of the surface. The particle can pass through it (Fig. 2c). For $C_{L_6,L_7} > C > C_{L_2}$, the surface takes the form of two reversed cups joined together. As C further decreases, a neck is formed and when $C = C_{L_2}$, a small closed region of non permitted motion on the right side of the neck appears (Fig. 2d). The particle is neither permitted to move inside the "cups" nor inside the neck or the closed region on the right.



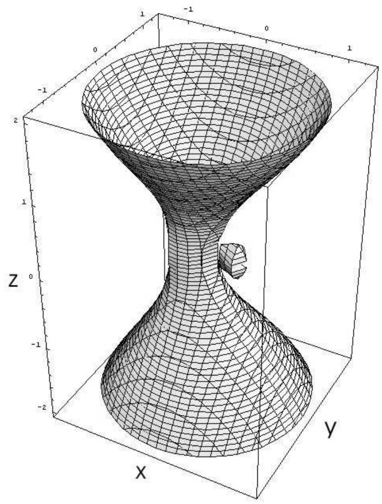
(a) $C = C_{L_3}$



(b) $C_{L_3} > C > C_{L_4,L_5}$



(c) $C_{L_4,L_5} > C > C_{L_1}$



(d) $C = C_{L_2}$

Figure 2: Various phases in the evolution of the zero-velocity surfaces for $\lambda = 10$, $I_1 = 0.001$, $I_2 = 0$.

References

Kalvouridis, T. J.: 1991, The three-dimensional equilibria of magnetic-binary systems with oblate primaries, *Celestial Mechanics and Dynamical Astronomy*, **50**, 301-311.