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COMPARISON OF LYAPUNOV CHAOS INDICATORS

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Abstract. For a long time, the estimation of the Lyapunov Characteristic Exponents (LCEs) had been used in Celestial Mechanics to caracterize the chaoticity of orbits. With the aim of gaining speed and accuracy in detecting this chaoticity, several indicators based on the theory of Lyapunov exponents have been developped. Here we present a comparison in terms of precision, CPU speed, and practicability of several of these indicators ; the FLI (Froeschlé *et al*, 1997), MEGNO (Cincotta & Simó, 2000), and the GALI (Skokos *et al*, 2007). The $GALI_3$ (using three tangent vectors) is the version of the GALI used here. While the FLI and MEGNO have been commonly used, the GALI has not yet been applied in Astronomy. However, this indicator has its own qualities and specificities. The final aim of the comparison of these indicators is the production of stability maps in the case of irregular satellites of giant planets, the examples and applications are shown in this sense.

1. INTRODUCTION

The three indicators presented here are variants of the Lyapunov Characteristic Exponent (LCE) which is defined by :

$$\sigma = \lim_{t \to \infty} (\frac{1}{t}) \ln \frac{\|\overrightarrow{w}(t)\|}{\|\overrightarrow{w}(0)\|} \tag{1}$$

with the use of a tangent vector obtained from the variational equation : $\dot{\overrightarrow{w}} = \frac{\partial \overrightarrow{F}}{\partial \overrightarrow{X}} \overrightarrow{w}$

2. BEHAVIOUR OF THE INDICATORS IN DIFFERENT SITUATIONS

We compare the indicators with several criterions such as the contrast obtained for differentiating chaotic orbits, the integration and CPU times. The general behaviour of the three indicators for quasi-periodic and different chaotic orbits is shown in Fig. 1 for 4 different orbits. The range of the semimajor axis of the examples on Fig. 2 et Fig. 3 is chosen to be representative of different regimes undergone by a Jovian satellite in the restricted circular and planar three-body problem (Sun, Jupiter, satellite). The first example (Fig. 2) represent a part of the system dominated by chaotic

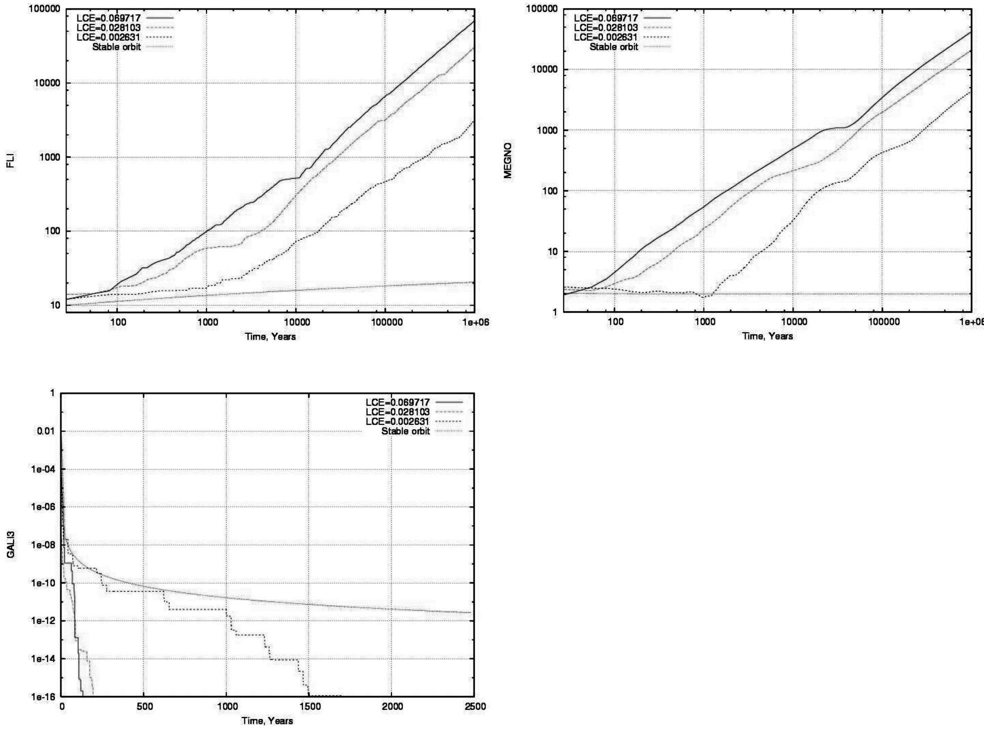

Figure 1: Evolution of the FLI (**Top left**), MEGNO (**Top right**) and $GALI_3$ (**Bottom left**) for 4 different orbits. The LCE of the chaotic orbits is indicated in the figures.

and resonant orbits, while the second (Fig. 3) contains mainly quasi-periodic orbits, together with a few resonant and unstable orbits. In the figures, the final values over different integration times of the FLI and the MEGNO is shown, in addition with the inverse of the time τ_{GALI_3} needed by the $GALI_3$ to attain a particular treshold.

3. DISCUSSIONS

We are making different objective tests to the methods, concerning mainly contrast between orbits and CPU time which are important for creating maps. Firstly, and as it is already thought, we found that the strong points of the FLI is its simplicity of use and its rapid computation. The FLI and MEGNO seems equivalent (Figs. 1 and 3), although it appears that the MEGNO can show structures and resonances faster in integration time than the FLI, but with more CPU time. Despite its slower speed, the MEGNO is perfectly adapted for the studies of resonant orbits thanks to its quasi-fixed value for stable orbits (Fig. 1). Indeed, for a given time, the value of the FLI and $GALI_3$ for stable orbits depend (although faintly) on the location of the orbits (Fig. 3).

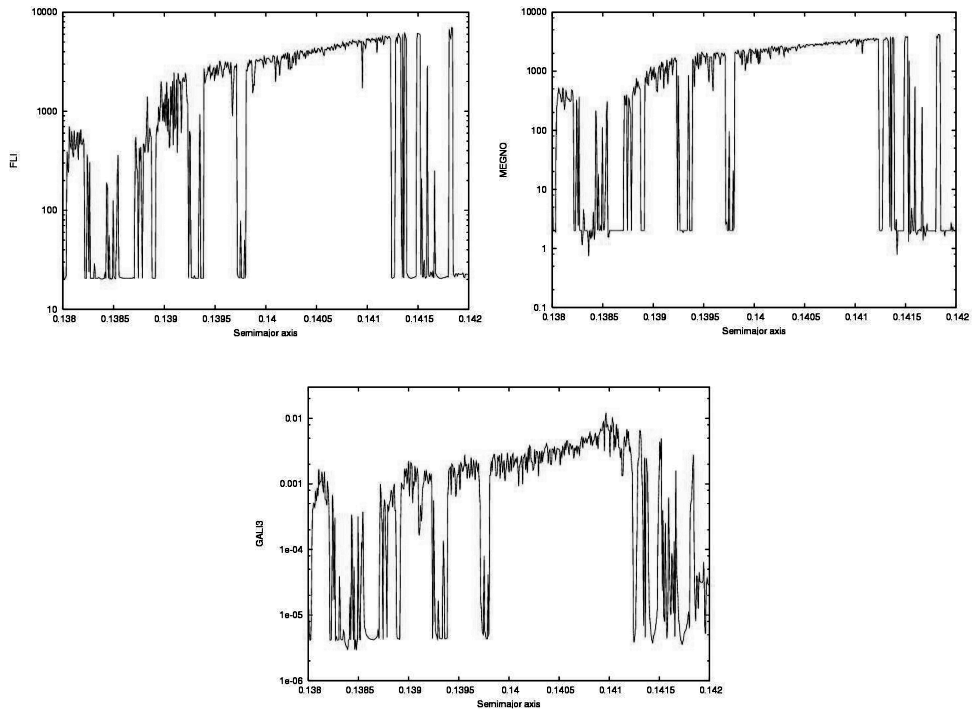

Figure 2: Final values of the indicators for an integration time of 100,000 years (or treshold value of 10^{-16} for the $GALI_3$), varying the semi-major axis in [0.138-0.142 UA] with e(0)=0.68 and f(0)=w(0)=0. FLI (**Top left**), MEGNO (**Top right**) and $\frac{1}{\tau_{GALI_3}}$ (**Bottom center**). The CPU time is respectively 3.5h, 10.5h and 7.25h.

Concerning the CPU time vs integration time, the MEGNO and $GALI_3$ are slower by at least a factor 3 than the FLI, but in this case the GALI method is fundamentally different in its use : using in fact the time as indicator, it allows to deal with chaotic orbits very fast, making it more efficient for regions where chaos is largely extended. For example it takes less than 2000 years of integration to attain the numerical zero for the less chaotic orbit in Fig. 1. This fact can be for example interesting for the studies of orbits which escape rapidly from a giant planet's Hill radius. Conversely, the CPU time increase drastically for the case of stable orbits, although this can be overcome by adding more tangent vectors to the computation of the GALI, thus using a more convergent indicator. We must add that this indicator cannot be computed up to a certain threshold (i.e. integration time) corresponding to the numerical limit.

To continue the comparison, we are adding more tests as objective as possible, in particular comparing the detection of resonances by the different methods in function of time.

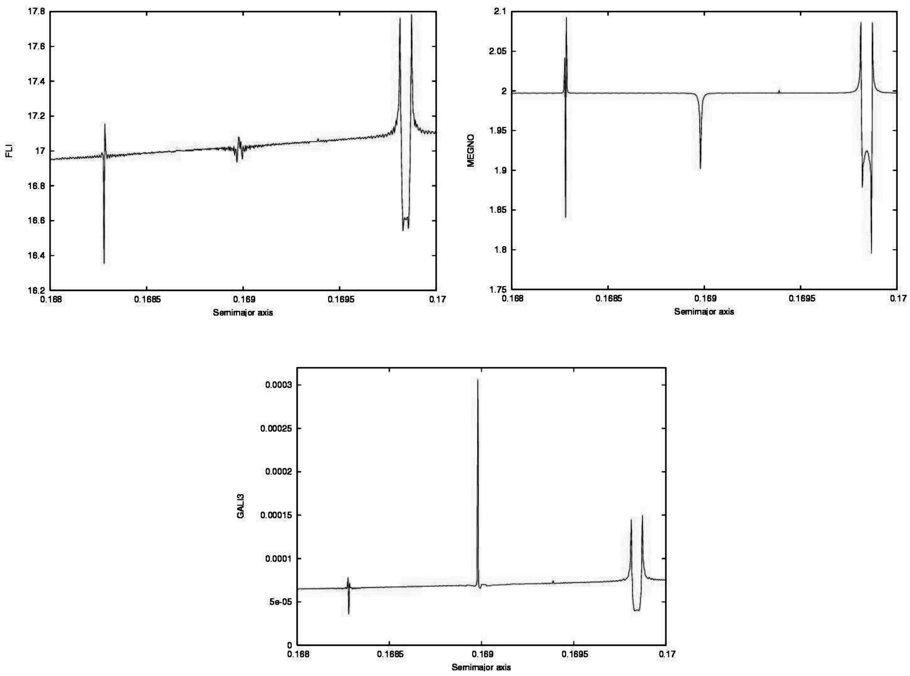

Figure 3: Final values of the indicators for an integration time of 10,000 years (or treshold value of 10^{-11} for the $GALI_3$), varying the semi-major axis in [0.168-0.17 UA] with e(0)=f(0)=w(0)=0. FLI (**Top left**), MEGNO (**Top right**) and $\frac{1}{\tau_{GALI_3}}$ (**Bottom center**). The CPU time is respectively 0.25h, 1h and 1.5h.

References

Cincotta, P. M., Simó, C.: 2000, *Astron. Astrophys. Suppl.*, **147**, 205.
Froeschlé, C., Gonczi, R., Lega, E.: 1997, *Planetary & Space Science*, **45**, 881.
Skokos, C., Bountis, T. C., Antonopoulos, C.: 2007, *Physica D*, **231**, 1, 30.