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FREQUENCY ANALYSIS OF THE DYNAMICS OF HIGH AREA-TO-MASS RATIOS GEOSYNCHRONOUS SPACE DEBRIS

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Abstract. European Space Agency 1 m telescope in Tenerife (Canary islands) has discovered a new unexpected population of 10 cm size space debris near the geostationary orbit (GEO), sometimes associated to highly eccentric orbits (Schildknecht et al., 2004, 2005). Recent numerical and analytical investigations (Anselmo and Pardini, 2005; Liou and Weaver, 2005) prove that this newly discovered population consists of near geosynchronous objects with high area-to-mass ratios. These objects have a dynamical behavior dominated by the solar radiation pressure: starting with very small eccentricities and inclinations, they can reach very high values after few years. In this paper we develop further the analysis done by Valk et al. (2008) by using a frequency analysis to study both the stability and the resonances patterns of such particular objects.

1. INTRODUCTION

As the GEO region is indisputably all-important for both commercial and scientific missions, ESA has recently initiated an optical search for fragments in the geostationary ring in order to improve the knowledge about the debris population and to understand their future evolution (Schildknecht et al., 2005). These observations have been performed, on behalf of ESA, by the Astronomical Institute of the University of Bern (AIUB) by using the 1 meter telescope located in Tenerife (Canary Islands). The recent observational discoveries in high altitude Earths orbit (for the largest part in geosynchronous orbits) stimulated the revisit of direct radiation pressure models. In particular, numerical investigations were recently performed in order to assess the time evolution of objects subject to such extreme situations (Anselmo & Pardini, 2005, Liou & Weaver, 2004). In this framework, short-term as well as long-term evolutions of geosynchronous space debris were studied in detail. These objects sometimes present highly eccentric orbits with eccentricities as high as 0.55 (Schildknecht et al., 2004, 2005). In addition, these authors and others, such as Chao (2006) and later Valk et al. (2007) presented some detailed analytical results concerning the short- and long-term evolution of high area-to-mass ratios geosynchronous space debris subjected to direct solar radiation pressure. More specifically, these latter authors mainly focused their attention on the long-term variation of both the eccentricity and the

inclination vector. Recently, Valk et al. (2008) presented investigations of the long-term stability (intrinsic stability of such uncommon orbits) of high area-to-mass ratios space debris subjected to the direct solar radiation pressure, by means of the MEGNO criterion, moreover a relevant class of additional resonances in the phase space (with respect to the main resonance due to C_{22} spherical harmonic) have been brought to the fore.

In this paper, we improve such results, by performing a preliminary frequency analysis of these peculiar orbits. The paper is organised as follows, first we briefly recall the results of Valk et al. (2008). Secondly, we sketch the main ideas of the FMA (Frequency Map Analysis) theory and we lastly conclude by showing our results based on the frequency analysis.

2. SOME REVIEW OF RECENT RESULTS

In a recent paper (Valk et al., 2008) we studied the global dynamics of high area-to-mass ratios geosynchronous space debris, applying a recent technique developed by Cincotta et al. (2000): Mean Exponential Growth factor of Nearby Orbits (MEGNO), which provides an efficient tool to investigate both regular and chaotic components of the phase space. For the purpose of this study, we considered the modeling of a space debris subjected to the influence of the Earth gravity field (includes the central body attraction, the second degree and order harmonics J_2, C_{22} and S_{22}), as well as the combined attractions of the Sun and the Moon. The perturbing effects of the direct solar radiation pressure were also taken into account for a high area-to-mass ratio fixed to $A/m = 10$ m^2/kg. The GEO objects are submitted to a resonance due to the C_{22} harmonic, thus main dynamics of a large set of GEO object can be described by the double pendulum. In Fig. 1 (top) we present a former result (Valk et al., 2008). We considered a part (semi-major axis, resonant angle) of the phase space (double pendulum) close to a stable point of the resonance due to C_{22} spherical harmonic on the geosynchronous orbits. For each orbit whose initial conditions lie on the horizontal black solid line (that is the subset of orbits having the same initial resonant angle value), we computed the MEGNO indicator and in Fig. 1 (middle) we reported the value reached at the end of the simulation as a function of the semi-major axis.

We performed also (Valk et al., 2008) a frequency analysis investigation (see Laskar 1990, 1995 and Noyelles et al. 2008) aimed to study the behaviour of the proper frequency of the resonant angle $\sigma = \lambda - \theta$ (λ being the mean longitude and θ the Greenwich time), whose results are reported in Fig. 1 (bottom). We remark the well-know 1:1 resonance due to C_{22}. Indeed, both the MEGNO and the fundamental period show distinctively a minimum close to the stable equilibrium point. Near the separatrices, the MEGNO clearly exhibits high values and the fundamental period erraticly reaches significant values which confirm the presence of chaotic orbits. Considering the behavior of the fundamental period with respect to the semi-major axis, we were able to show the presence of the so-called additional resonances, associated, regarding the angle σ, with periods commensurate with 1 year. More precisely, the major additional resonance located at approximately 40 km on both sides of the pendulum-like pattern are related to a 2 years fundamental period of σ. Concerning the farther patterns located at ± 80 km, the fundamental period of σ turns out to be very close

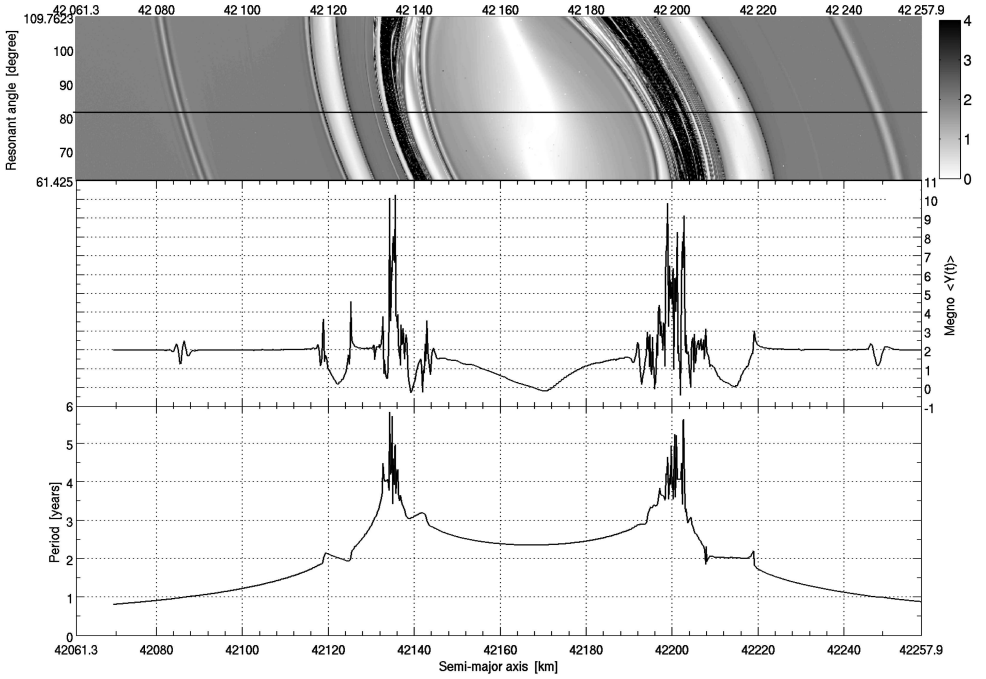

Figure 1: Blow-up of the phase space around 1:1 resonance due to C_{22} [Graphics and caption from Valk et al. (2008)]. For a resonant angle section (horizontal black solid line), evolution of the MEGNO (middle panel) and the fundamental period of σ with respect to the initial semi-major axis a_0 (bottom panel).

to 1 year. As a consequence, we can conclude that these additional resonances are actually related to a commensurability between σ and the 1 year period angle $\lambda_\odot$, that is the ecliptic longitude of the Sun.

3. EXTENDED NUMERICAL ANALYSIS: FREQUENCY MAP ANALYSIS

We now introduce a second original approach to study the stability of the high area-to-mass ratio space debris. We use a powerful method FMA (Laskar, 1990, 1993, 1995) previously introduce and used to study, for example, the chaotic motion of the solar system (Laskar, 1990). To the best of our knowledge the present study provides the first application of FMA to the geodesy.

Let us briefly sketch the main idea of the FMA. Let $f(t)$ be a quasiperiodic regular function of the time t, with values in a complex domain, whose Fourier coefficient a_k have decreasing amplitudes with k. The FMA is a efficient numerical method of frequency analysis allowing to obtain an approximation $f'(t) = \sum_{k=1}^{N} a'_k e^{i\nu'_k t}$ of the given signal with a given number (N) of harmonics, from the numerical knowledge of $f(t) = \sum_{k=1}^{\infty} a_k e^{i\nu_k t}$ over a finite time span $[-T, T]$ (Laskar, 1990, 1995). For our

results, we use the FMA software provided by J. Laskar (1990, 1993) that we kindly acknowledge. Still for the resonant angle section (Fig. 1) but within a simpler model (where J_2, C_{22} and radiation pressure with circular sun, over a time span of 500 years have been considered) we calculate the first fundamental period of the resonant angle (Fig. 2 bottom). To improve the direct analysis of the fundamental frequency ($\nu_1 a$)), we calculate (Laskar, 1993) the numerical second derivative ($\delta\delta\nu_1 a) = \nu_1 a) - 2\nu_1 a - h) + \nu_1 a - 2h)$) of the frequency of the resonant angle (Fig. 2 middle pannel). We also calculate the diffusion of the main frequency with respect to the time (Laskar, 1993). This is obtained by computing a first value $\nu_1^{(1)}$ of the main frequency of the resonant angle over a time span T_1 (for example to $-T$ untill 0) and a second value $\nu_1^{(2)}$ of this main frequency of the same signal but over a time span T_2 (for example to 0 untill T). Then we estimate the diffusion rate (Fig. 2 top) by computing a numerical indicator of diffusion of frequency with respect to the time by $\log|1-\nu_1^{(2)}/\nu_1^{(1)}|$: the smaller the value is the smaller the diffusion is. We can observe the same structure on both plots for derivative and for diffusion indicator. But the latter being less noisy. One can easily recognize the main additional resonances when the period is equal to 2 years

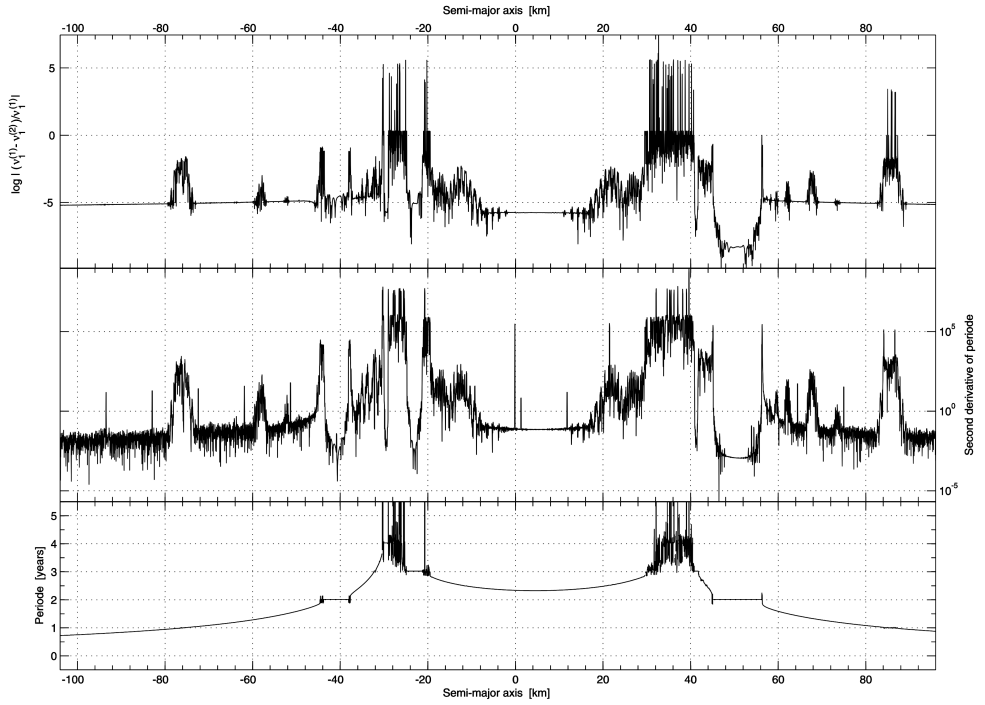

Figure 2: For the resonant angle section in Fig. 1, with a simpler model (described in the text). Evolution of the fundamental period (bottom panel), second derivative (middle panel) and diffusion indicator (top panel); with respect to the initial semi-major axis a_0.

(between 42 and 58 km above the "eye" of resonance due to C_{22} and between -36 and -44 km below the "eye" of the resonance). One can also observe an additional resonance when the period is equal to 1 year (between 84 and 86 km above the "eye" of the resonance due to C_{22} and near to -76 km below the "eye" of the resonance). Our analysis suggest that other additional resonances can be observed. A first one when the period is equal to 3 years (at $\pm 42km$ above the "eye" and $\pm -32km$ below the main resonance); a second resonance when the period is equal to 4 years (at $-30km$ below the main resonance) and a third one when the period is equal to 3/2 years and equal to 4/3 years. We can also observe a secondary resonance (inside the "eye" of the resonance due to C_{22}) when the period of the resonant angle is equal to 3 years, close to the separatrix (at $-20km$ to $-24km$ above the main resonance) and a weak evidence of secondary resonance (in the "eye" of the resonance due to C_{22}) when the period of the resonant angle is equal to 2.5 years (at $-16km$ below and $24km$ above the main resonance).

4. CONCLUSIONS

In this paper, using numerical estimator, we brought to the fore a relevant class of additional resonances in the GEO for space debris exhibits high area-to-mass ratio. Some of these resonances corresponding to commensurabilities between the primary resonant angle and the ecliptic longitude of the Sun were also underlined in Valk et al. (2008). A set of these resonances have been discovered using the `MEGNO` indicator. Because of its better resolution, a larger set of resonances has been discovered by using the FMA (corresponding to periods equal to 4/3 and 3/2 years).

An analytical analysis of these additional resonances will be investigated in a forthcoming publication. Moreover a whole atlas, as in Valk et al. (2008), has been constructed using frequency analysis confirming the existence of a large number of additional resonances that wasn't discovered using the `MEGNO` indicator.

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