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ORBITS IN GALAXIES, A HISTORICAL PERSPECTIVE

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Abstract. We describe the development of the theory of orbits in galaxies with emphasis on the distinction between ordered and chaotic orbits. A recent development refers to the outer spiral arms of barred galaxies, which are density waves consisting mainly of sticky chaotic orbits.

1. INTRODUCTION

I am happy to announce that Dr. J. Hadjidemitriou has been elected a few days ago corresponding member of the Academy of Athens. This is a particular honor, and I am very proud for him, because Dr. Hadjidemetriou has been my student and assistant. However very soon he developed his own scientific profile, for which he is honored presently.

My review on "Orbits in Galaxies" today refers to the advances in this field, that were done by our group in Thessaloniki and later in Athens. Of course only a few significant points will be mentioned.

2. THIRD INTEGRAL

The story starts with my first visit to Stockholm in 1956. Professor Bertil Lindblad invited me there to work with him on the theory of epicyclic orbits in the plane of our galaxy. He had already developed a first order theory and he wanted an extension of this theory to higher orders. Very soon I developed the theory to all orders, and then I tried to extend the theory to the third dimension. This was not easy. Very little had been done before on this problem. So, I decided to calculate two orbits on the meridian plane of a simple galactic model. These orbits were calculated at a rather simple computer of the University of Stockholm and were published in 1958 (Contopoulos 1958) (Fig.1).

These figures were surprising. As the problem was nonlinear I expected that the orbits should fill, in an ergodic way, the whole area inside the curve of zero velocity. Instead of that each orbit filled a different "curvilinear parallelogram" like a Lissajous figure.

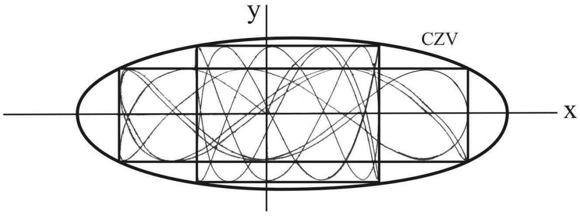

Figure 1: The first two orbits on the meridian plane of a galaxy (Contopoulos, 1957). CZV is the curve of zero velocity.

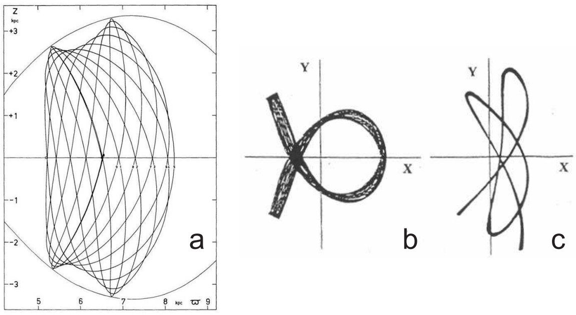

Figure 2: (a) A box orbit (Ollongren, 1965) and (b,c) tube orbits.

I presented these calculations at the IAU General Assembly in Moscow. In the audience there was a soviet astronomer, Dr. G. Kuzmin, who made the remark that these figures indicated the existence of a third integral of motion. I replied that this was improbable for two reasons: (1) Because of a theorem based on Poincaré's proof of the nonexistence of analytic integrals (besides the energy) and (2) because the problem was so simple that if there was a third integral someone would have found it already. Everyone laughed and we decided to pursue this problem further.

A few months later Dr. Kuzmin wrote me that he looked at the Poincaré theorem and he was convinced that no third integral existed in this case. But meanwhile I had shown that Poincaré's theorem was not applicable in this particular case and I had found a formal series that represented the third integral to arbitrarily high orders. Thus I wrote to Dr. Kuzmin that he was right and I was wrong in Moscow and I sent him my formula.

Later I found that I was wrong also as regards my second argument, because some people had already found some forms of the third integral (Whittaker 1916, 1937 and Cherry 1924a,b, 1928).

Then we started an extensive study of orbits and integrals. The main types of orbits are boxes (Fig.2a, Ollongren 1965) and tubes of various types (Figs 2b,c). Finally there are chaotic orbits. Tube orbits represent oscillations close to stable periodic orbits at particular resonances. In each resonance case a different form of the third integral should be applied (Contopoulos 1963, 1965, Contopoulos and Moutsoulas 1965. 1966).

Our calculations of orbits were done in a very unorthodox way. I had written a program that integrated orbits at the NASA Institute for Space Studies in New York. From time to time I was sending some initial conditions, in the form of punched cards, to a friend in New York and one month later I was receiving the results in the form of computer plots. Despite this delay our method worked. I remember when we calculated for the first time some tube orbits. I had a theoretical prediction that some particular initial conditions should give tube orbits instead of box orbits and I hoped to check it numerically. When the results came, we put the successive plots on the floor of the corridor of our Department. The results were exactly as we expected. We were so happy that all the staff joined hands and danced a greek dance around the plots in the corridor.

In 1962 I was at the Yale University and we organized a small Workshop on orbits and integrals. There were people like Moser, Hénon, Szebehely etc. Moser spoke about the KAM theorem. After his talk I asked him whether we could apply his theorem to the earth-moon system, i.e. whether the moon had a stable trajectory despite the perturbations of the Sun. But Moser did not want to make any prediction. Then Hénon made some calculations in his hotel and next day he presented his results. According to Moser's theorem the moon would be stable if the mass ratio moon/earth was smaller than 10^{-48} (Hénon 1966). This meant that the mass of the moon should be of the order of a few atoms. Or, with the known mass of the moon its stability would be secured only if its distance would be less than 1mm from the center of the earth! Later, of course, much improved estimates were made. And recently Efthymiopoulos and Sandor (2005) have found the stability region of the Trojan asteroids around L_4 and L_5 that extends to distances larger than one astronomical unit and contains a substantial fraction of the real asteroids.

In 1963 I was at the Yerkes Observatory, working with Dr. S. Chandrasekhar on post-Newtonian approximations. Then Dr. L. Woltjer visited us and we had a hot discussion about the third integral. Woltjer thought that my formulae applied only to smooth potentials, as in the neighbourhood of the Sun. But in more complicated potentials, like the spiral arms of a galaxy, the third integral would not be applicable. Dr. Chandrasekhar was supporting Woltjer. But then I took Woltjer to dinner and at the end of the dinner I had convinced him.

The next day Chandrasekhar told me: "George, I thought about this problem and I think that Woltjer is right and you are wrong". And I replied "Chandra, you are supposed to stand between me and Woltjer as an arbiter. But now Woltjer agrees with me. So you are eliminated".

After this discussion Woltjer and I wrote a paper together (Contopoulos and Woltjer 1964) where we applied the third integral to a rough model of spiral arms. This work, done independently of the work of Lin and Shu (1964) around the same time on galactic density waves, proved the existence of ordered orbits that can form the spiral arms in galaxies.

Our next problem was how to calculate higher order terms of the third integral. This required so many operations that it was not possible to do it by hand. On the other hand there were no computer algebra packages, like Mathematica today, to do this work. Thus we developed a Fortran program that did the necessary algebra for us (Contopoulos and Moutsoulas 1966). At that time we had a small IBM computer

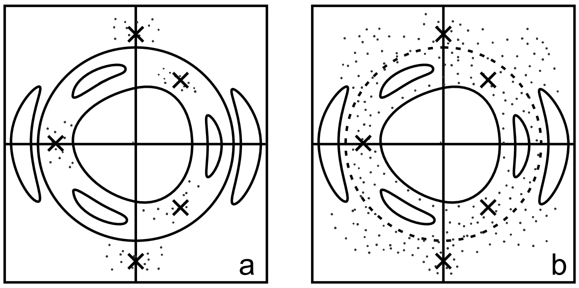

Figure 3: Interaction of resonances. (a) The resonant regions 3/1 and 2/1 are separated by KAM curves around the center. (b) When the last KAM curve is destroyed, at a larger perturbation, a large chaotic sea is formed.

in our University (IBM1620). We worked with punched cards. But the computer had not enough memory to do all the work in one stretch. So we split the program in two parts. The first part provided an interim output in the form a deck of cards 1 meter long. This deck was introduced as initial datum in the second part and finally we had two meters of cards that contained the coefficients of the higher order terms of the third integral. This deck was then used to calculate periodic and nonperiodic orbits with spectacular results.

But how far was the third integral applicable? We knew that orbits in systems with large perturbations are chaotic, and the third integral cannot apply to them. The question was how chaos is introduced. The answer is by the overlapping of resonances. The resonant regions increase in size as the perturbation increases and finally they tend to overlap each other. I calculated theoretically this phenomenon and I presented my results at a Symposium in France during 1966 (my paper was published in the Proceedings of this Symposium, Contopoulos 1967). Later I realized that a group of physicists had found the same mechanism and published their results in the journal Nuclear Fusion (Rosenbluth et al. 1966). Later much work has been done on resonance overlap by Chirikov (1979) and many people call this mechanism "Chirikov's criterion". But Chirikov refers to my work and the work of Rosenbluth et al. (Zaslavsky and Chirikov 1972). Furthermore in a letter of Chirikov to me dated 23-10-1968 he wrote "My criterion of stochasticity by resonance overlapping is essentially the same as yours in your paper in Bulletin Astronomique (1967)".

The details of the resonance overlap can be seen in Fig.3. In a given system we see various resonances like 3/1 and 2/1 (Fig.3a). These regions contain stable periodic orbits with islands around them and unstable periodic orbits surrounded by chaotic orbits. When the perturbation is relatively small the two resonant regions are separated by invariant curves around the center and no interaction of resonances appears. But when the perturbation increases (Fig.3b) the separating invariant curves are destroyed and the chaotic regions of the two domains interact, forming a large chaotic sea.

3. DENSITY WAVE THEORY

The basic theory about the spiral arms in galaxies is the density wave theory. Namely the spiral arms are not material arms, composed always of the same material, but density waves. That means that the stars go through the spiral arms, but stay longer close to them, so that the density is maximum there.

The density wave theory was formulated by B. Lindblad (1940, 1941 etc), but it was revived by Lin and Shu (1964, 1966), Shu (1970) and after that it was firmly established. Many people like Kalnajs (1971), Toomre (1969, 1977), Mark (1974), Roberts (1969), Lynden-Bell and Kalnajs (1972), Yuan (1969) etc made significant contributions to this theory.

During 1963 I met Lin at MIT and he told me that he wanted to develop a theory of density waves. I told him that B. Lindblad had already formulated such a theory. So we went to the MIT library and we borrowed several volumes of the Stockholms Observatoriums Annaler. But when I met again Lin, a few days later, he told me that he found Lindblad's work rather difficult to follow, so that he decided to develop the theory from scratch.

The basic theory of density waves is a linear theory. It considers the potential V as composed of an axisymmetric background V_0 plus a spiral perturbation V_1, and higher order terms

$$V = V_0 + V_1 + V_2 + \dots \tag{1}$$

On the plane of a galaxy we have a zero order potential $V = V_0(r)$ and we consider a spiral perturbation

$$V_1 = A(r)\exp[i(\varphi(r) + \omega t - m\vartheta)] \tag{2}$$

where $A(r)$ is the amplitude, $\phi(r)$ is the phase of the spiral, m is the number of spiral arms (in general $m = 2$) and $\omega = m\Omega_s$, with Ω_s the pattern velocity of the system. If the derivative $\phi'(r) = k(r)$ (wave number) is positive the spiral is leading, while if $\phi'(r) = k(r) < 0$ the spiral is trailing.

The surface density σ is also the sum of an axisymmetric background σ_0 and a spiral perturbation σ_1 etc. In the same way the phase space density (called also distribution function) is:

$$f = f_0 + f_1 + f_2 + \dots \tag{3}$$

Finally

$$\sigma = \int f d\bar{v} \tag{4}$$

The function f satisfies Liouville's equation which expresses the fact that f is an integral of motion. The density is connected to the potential through Poisson's equation.

The linear density wave theory had a remarkable success, both theoretically and in applications to real galaxies. However it could not be applied near the main resonances

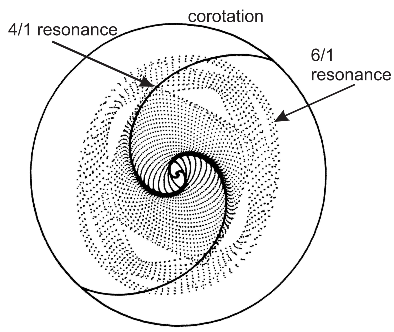

Figure 4: Periodic orbits in a spiral galaxy.

of a galaxy. Resonances occur whenever the angular velocity in the rotating frame of reference $(\Omega - \Omega_s)$ divided by the epicyclic frequency κ is a rational number

$$\frac{\Omega - \Omega_s}{\kappa} = \frac{m}{n} \tag{5}$$

The most important resonances are the Lindblad resonances (when $\frac{\Omega-\Omega_s}{\kappa} = \pm\frac{1}{2}$) and corotation $(\Omega = \Omega_s)$. At these resonances the perturbation f_1, as given by the linear theory, tends to infinity. Therefore the linear theory is not applicable.

The distribution function f is a particular case of the third integral. Its form changes at every resonance. In particular at the main resonances of a galaxy (Lindblad resonances and corotation) its form is very different from the unperturbed function f_0.

We developed the nonlinear theory near the Lindblad resonances (Contopoulos 1970, 1975, 1979; Contopoulos and Mertzanides 1977) and near corotation (Contopoulos 1973) in the same way as in the third integral.

An important application of the nonlinear theory refers to the termination of the spiral arms of a normal galaxy. In Fig.4 we give the main periodic orbits from the inner Lindblad resonance outwards. Close to the inner Lindblad resonance the orbits are like ellipses, that make two radial oscillations during one rotation. The orientations of these orbits are changing, as we proceed outwards, in order to support the spiral arms. Further out we have the 4/1 resonance and the orbits are like squares with rounded corners. But beyond the 4/1 resonance the original family of stable orbits is terminated and a new family of stable orbits takes over.

These orbits start as nearly squares, and further out they become like hexagons, at the 6/1 resonance, and so on. But the orientations of these orbits are very different from the previous ones, and they do not support any more the spiral arms. Because of that the spiral arms terminate near the 4/1 resonance (Contopoulos 1985, Contopoulos and Grosbol 1986, 1988, Patsis et al. 1994).

These considerations apply to relatively open spirals (of type Sb, Sc) in which the amplitudes of the spiral arms are of the order of 10% of the axisymmetric background. Only weak spirals (of type Sa), which are also tight, can extend all the way to corotation.

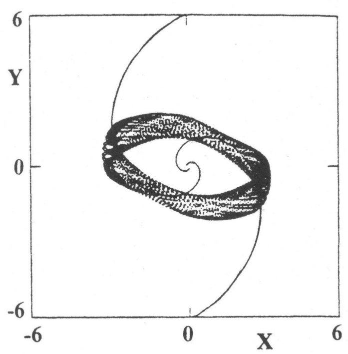

Figure 5: A nonperiodic orbit near the Inner Lindblad Resonance.

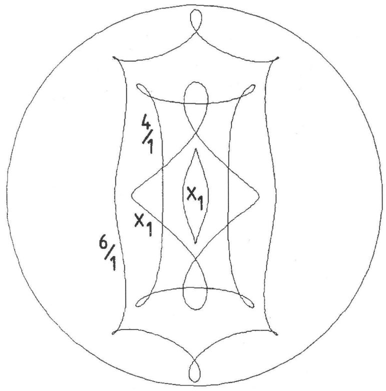

Figure 6: Periodic orbits in a barred galaxy.

The nonperiodic orbits follow the periodic orbits (Fig.5) forming rings around the center.

On the other hand in barred galaxies the perturbation (amplitude) is of the order of 50 - 100%. In these cases the orbits are chaotic and neither the linear, not the nonlinear theory of density waves can be applied.

In barred galaxies we have again two main populations of orbits, one with orbits elongated along the bar. This family terminates at the 4/1 resonance with orbits like squares but with large extra loops along the bar (Fig.6). Beyond the 4/1 resonance the orbits are like parallelograms with loops at the four corners and further out hexagonal orbits etc. But in contrast with the normal spirals, all these orbits support the bar.

However near and beyond corotation the orbits are chaotic. Such orbits either form rings around the center (Fig.7a) or they follow the spiral arms up to about 180 degrees, but also populate the outer envelope of the bar (Fig.7b) (Kaufmann and Contopoulos 1996).

In any case the bars of barred galaxies terminate near corotation for two reasons: (1) Because the ordered orbits beyond corotation do not support the bar, as they do

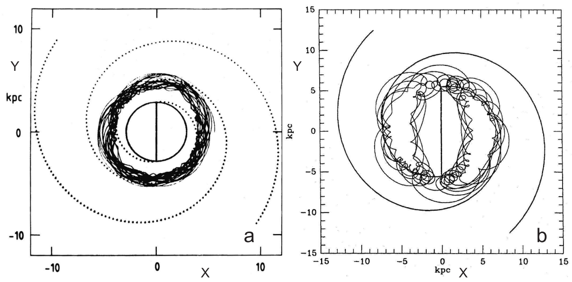

Figure 7: Chaotic orbits near corotation (a) an orbit forming a ring around the center (b) an orbit following the spiral arms and the envelope of a bar.

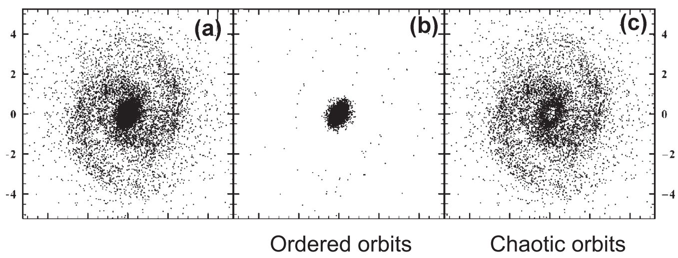

Figure 8: Distribution of the stars in a N-body model of a barred galaxy. (a) All the orbits (b) Ordered orbits (c) Chaotic orbits.

inside corotation, and (2) because most orbits near corotation are chaotic (Contopoulos 1980). That is now accepted in general (Sanders and Tubbs 1980, Elmegreen and Elmegreen 1985, Sellwood and Wilkinson 1993).

A recent development in the theory of density waves refers to the spiral arms in a barred galaxy outside corotation which are composed of chaotic orbits.

In Fig.8 we give the distribution of the stars in a N-body model of a barred galaxy (Fig.8a). We separate the stars in ordered orbits, which are mainly in the main body of the bar, (Fig.8b) and those in chaotic orbits, which are mainly in the spiral arms and in the outer envelope of the bar (Fig.8c) (Voglis et al. 2006a).

Near the ends of the bar there are the unstable Lagrangian points L_1 and L_2 (Fig.9). Around these points there are the unstable periodic orbits PL_1, PL_2 and out of them emanate the unstable and stable manifolds. On a surface of section $\dot{r} = 0$, representing the apocentra of orbits, we have two unstable asymptotic curves from PL_1, U along a spiral and UU along the envelope of the bar, and two stable curves S and SS. The orbits close to L_1 approach the point PL_1 along lines near S and SS and then they deviate from PL_1, close to U and UU. When the asymptotic curve U approaches the unstable point PL_2 it makes several oscillations close to the asymptotic curves U' and UU'. The spiral arms consist of stars coming close to U and U' at their apocentra (Fig.10) (Voglis et al. 2006b).

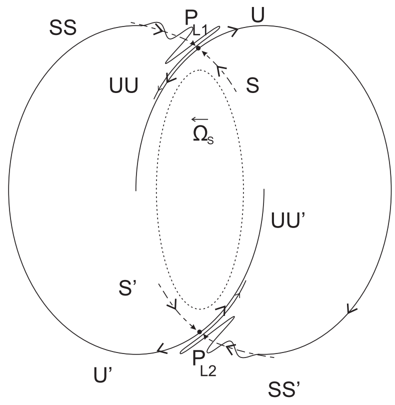

Figure 9: The asymptotic curves of the short period orbits PL_1, PL_2 around the Lagrangian points L_1, L_2.

Besides the short period orbits PL_1, PL_2 around L_1, L_2 there are several other unstable periodic orbits like 3/1 and 4/1 inside corotation and -4/1, -2/1 (outer Lindblad resonance), outside corotation. Their unstable asymptotic curves cannot cross themselves, or each other. Thus they follow paths similar to these of U and U' from PL_1 and PL_2 (Fig.11). In this way we form thick spiral arms (Fig.11) (Tsoutsis et al. 2007).

The chaotic stars do not conserve the energy E and the angular momentum J, but they conserve the Jacobi constant

$$H = E - \Omega_s J \tag{6}$$

For every value of H we have a set of unstable periodic orbits. If we superimpose the unstable manifolds for many values of the Jacobi constant we find thick spiral arms.

The spiral arms are composed of chaotic orbits, but they are waves, because they are not composed always of the same stars. These spiral arms are long-lived, lasting for several billions of years (Fig.12).

A way to check the density wave theory is by finding the velocity fields of stars in spiral galaxies (Fig.13a,b). In normal spirals the velocities are close to circular, but there are abrupt deviations close to the spiral arms (Fig.13a). On the other hand in barred galaxies the velocities outside the bar are along the spiral arms (Fig.13b) (Patsis 2006). Such velocity fields can be observed in various galaxies.

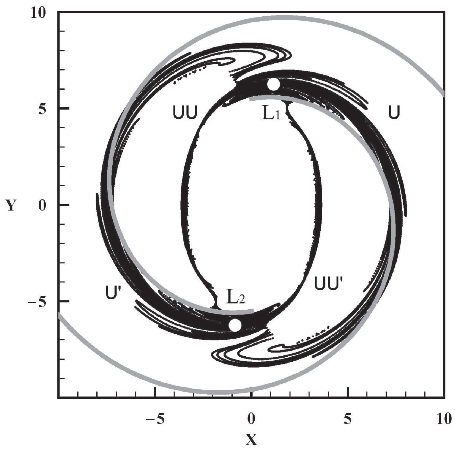

Figure 10: Spiral arms composed of stars along the asymptotic curves U, U’ from the short period orbits PL_1, PL_2.

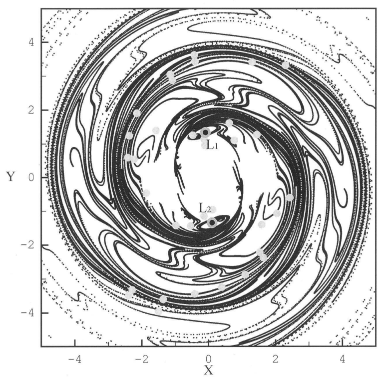

Figure 11: Thick spiral arms due to the superposition of the unstable manifolds of the unstable periodic orbits PL_1, PL_2, 3/1, 4/1, -4/1, -2/1 and - 1/1 for a given value of the Jacobi constant.

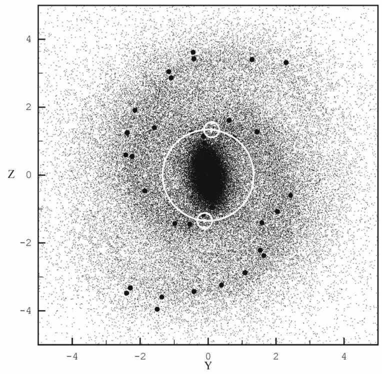

Figure 12: N-Body spiral arms that last for at least one Hubble time. The circle represents corotation and the small circles represent the short period orbits PL_1, PL_2. The white dots are L_1, L_2 and the black dots are the maxima of density at successive rings around the center.

4. ESCAPES AND EVOLUTION

The stars outside corotation are not restricted by any curve of zero velocity, thus they may escape to infinity.

A particular example of an escaping orbit is given in Fig. 14a. This orbit has a fixed Jacobi constant, but its energy varies (Fig.14b). When the orbit goes far away from corotation it describes an almost elliptical arc with constant energy, but when it comes close to the main body of the galaxy its energy changes. After many increases and decreases its energy becomes positive (Fig.14b). Then the orbit escapes along a nearly hyperbolic path (Fig.14a).

In the case of a barred galaxy the chaotic orbits may or may not escape to infinity. In Fig.15a,b we show two models of galaxies in the phase space (θ, p_θ), where θ is the azimuth and p_θ the angular momentum, which is a monotonic function of the distance r from the center. In Fig.15a we see many ordered orbits forming islands, or KAM curves that extend all the way from θ=0 to θ=2π, thus surrounding the whole galaxy. We see also many chaotic orbits (scattered dots). In particular the unstable asymptotic curves from PL_1, PL_2 are at the bottom of the figure. These asymptotic curves do not reach the first closed KAM curve, but after a long time they approach it. However they cannot cross this KAM curve and go outwards to infinity.

On the other hand in Fig.15b there are no closed KAM curves from θ=0 to θ=2π. There are only small islands of stability, but these do no restrict the motions outwards. The asymptotic curves are again restricted in the lower part of the figure, but later on they diffuse outwards and many stars close to them escape to infinity. However this is done after several billions of years.

In both cases we see that chaotic orbits of stars near the spiral arms, are sticky, i.e. they remain close to the spiral arms for very long times, before escaping outwards, and sometimes reaching infinity (Fig.15b).

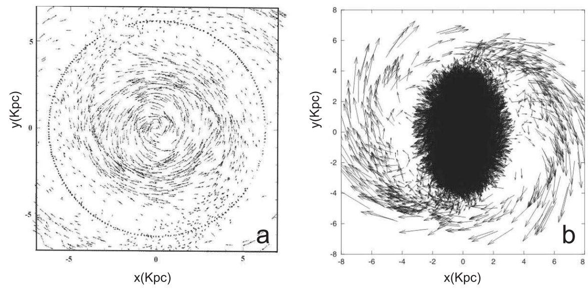

Figure 13: Velocity fields of spiral galaxies. (a) Inside corotation (circle) in a normal spiral and (b) outside corotation in a barred spiral.

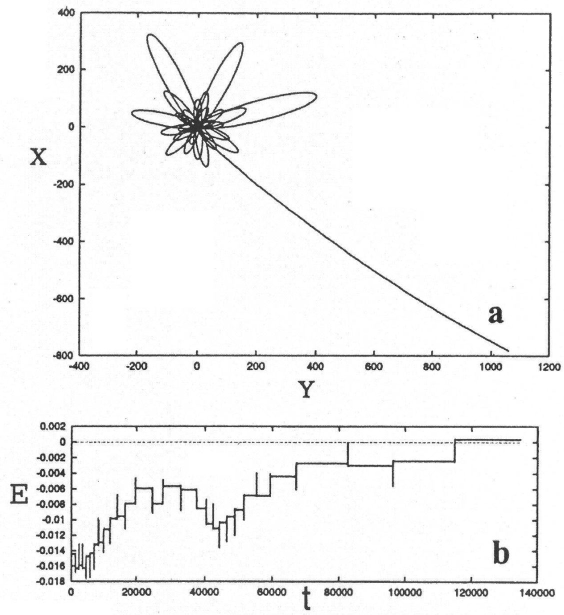

Figure 14: Escape of a star (a) The orbit, (b) The variation of its energy in time.

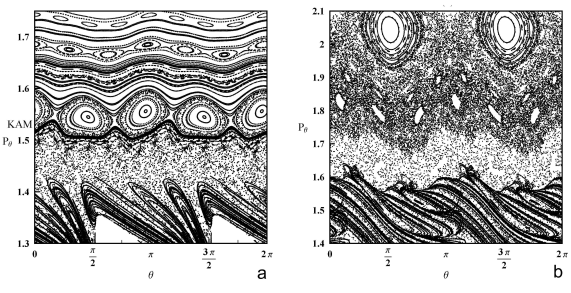

Figure 15: The phase space (ϑ, p_ϑ) in models of two barred galaxies (a) NGC 3982 (b) NGC 1398.

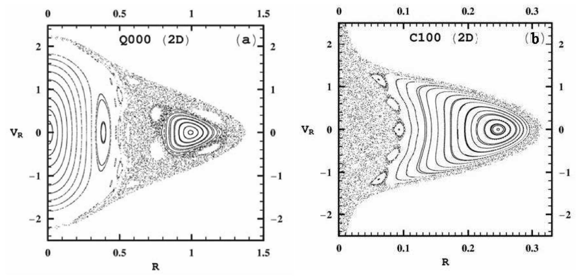

Figure 16: Surfaces of section $(x,\ x)$ in the meridian plane of an elliptical galaxy, (a) The initial distribution of ordered orbits (lines) and chaotic orbits (dots) (b) the distribution after the introduction of a large black hole at the center.

The forms of the orbits in a N-body model of a galaxy change with time. Regular orbits may become chaotic and chaotic orbits may become regular.

The best way to follow the structure of the orbits in an evolving galaxy is by freezing the potential at particular times and calculating several orbits in this frozen potential. Then we can compare the distributions of the orbits at successive times.

A particular example of the evolution of an elliptical galaxy (nonrotating) is shown in Fig.16a,b. (Kalapotharakos et al. 2004) The orbits are shown on a surface of section. Originally (Figs.16a) there are two types of regular orbits (1) box orbits near the center, and (2) tube orbits far away from the center. The closed curves there represent ring type orbits around the center. Between the two types of regular orbits there are chaotic orbits represented by many dots.

If now we introduce a massive black hole in the center of the galaxy the box orbits become chaotic (Fig.16b). However several chaotic orbits of Fig.16a become tube orbits. Thus the region of the tube orbits becomes much larger.

The evolution from Fig.16a to Fig.16b is not done abruptly. If we calculate the potential and the types of orbits at intermediate times between the times of Fig.16a,b, we see that the evolution is gradual. What is more important is that the form of the galaxy seems to be stabilized at particular times for certain time intervals, but later on the galaxy continues to evolve further. Before reaching the final stage, which is close to that represented by Fig.16b, the galaxy goes several times through quasi-stationary states. This phenomenon is a kind of self-organized criticality.

This kind of galactic evolution is an important topic for study, that has been barely explored up to now. It is remarkable that after several decades of studies of galactic dynamics, there are still new and important problems to be explored further in the near future.

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