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WHERE ARE THE OTHER RESONANT EXTRASOLAR PLANETS?

C. BEAUGÉ[1], C. A. GIUPPONE[1], M. TADEU DOS SANTOS[2],
S. FERRAZ-MELLO[2] and T.A. MICHTCHENKO[2]
[1] *Observatorio Astronómico, Universidad Nacional de Córdoba, Argentina*
[2] *Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, Brazil*

Abstract. The present orbital configurations of extrasolar planets, characterized by small semimajor axes and high eccentricities, are believed to be evidence of a past large-scale orbital migration caused by tidal interactions with the gaseous disk. The existence of multiple-planet systems locked in mean-motion resonance (MMR) has been presented as evidence of planetary migration, as opposed to in-situ formation scenarios. Hydrodynamical simulations indicate that resonance lock is a high-probability outcome, especially in the $1/3$ and $1/2$ MMR, and the eccentricity distribution of resonant systems appears consistent with such evolutionary mechanism.

However, over the past three years there has been no significant increase in the resonant population, even though the detected exoplanets and multiple- planet systems has nearly doubled. In this communication we review possible explanations for the apparent lack of resonant bodies. In particular, we concentrate on the conditions for detectability of two planets in a 2/1 MMR from radial velocity data, as a function of their masses, number of observations and signal-to-noise ratio.

We find that, even for a data set of the order of one hundred observations and standard deviations of the order of a few meters per second, resonant systems are difficult to detect if the masses of the planets differ by a factor larger than ~ 3. This is consistent with the present population of real exosystems in the 2/1 commensurability, most of which have resonant pairs with similar minimum masses, and could indicate that many other resonant systems exist, but are presently beyond the observational limit.

Furthermore, we analyze the error distribution in masses and orbital elements of orbital fits from synthetic data sets for resonant planets. For various mass ratios and number of data points we find that the eccentricity of the outer planet is systematically over estimated, although the inner planet's eccentricity suffers a much smaller effect. If the initial conditions correspond to small amplitude oscillations around stable apsidal corotation resonances (ACR), the amplitudes estimated from the orbital fits are biased towards larger amplitudes, in accordance to results found in real resonant extrasolar systems.

1. INTRODUCTION

Although the population of extrasolar planets continues to increase rapidly, the number of multiple-planet systems with members in mean motion resonances (MMR) appears to have reached a (hopefully temporary) plateau. The existence of a 3/1 commensurability in 55Cnc has been recently questioned by the five planet fit of

(Fischer et al. 2008) whose best solution now indicates a non-resonant motion for 55Cnc-c and 55Cnc-d. Even in the 2/1 MMR, the most dynamically important and populated commensurability, the number of planetary systems is still restricted to four well known cases: GJ876, HD82943, HD73526 and HD128311. Goździewski et al. (2007) proposed that HD160691 may also have two planets in the 2/1 MMR, although this is unconfirmed, and just two years before the best fit solution seemed to favor a 5/1 mean motion resonance between the same two planets (Goździewski et al. 2005). Fig. 1 shows the distribution of exoplanet pairs according to their mass ratio and orbital period ratio. Except for the vicinity of the 2/1, they appear more or less at random.

For several years a large proportion of resonant planets was expected as a consequence of a assumed large scale planetary migration of exoplanets due to interactions with the gaseous disk. In fact, the existence of resonant systems has many times been advanced as observational evidence that such a migration actually took place, implying that many planets were formed farther from the star than their present location. Hydrodynamical simulations seem to indicate that resonance trapping in low-order commensurabilities (particularly the 2/1 MMR) are high-probability outcomes for a wide spectrum of planetary masses, initial conditions and disk parameters. Several works, particularly focused on GJ876 (see, e.g., Kley et al. 2005), point that the present configuration of planets b and c can only be explained assuming such a scenario.

Two explanations have been presented recently to account for the lack of a larger resonant population. One possibility is that not all planets approaching the 2/1 resonance may have been captured. If the mass ratio between the inner and outer planet was sufficiently small (of the order of the Saturn over Jupiter ratio), then a very fast orbital decay of the outer smaller body (e.g. Type III migration) may have avoided resonance capture in the 2/1 and led to a later trapping in the 3/2 MMR (Masset & Snellgrove 2001). A similar effect has been proposed by Morbidelli & Crida (2007) as a first step to explain the current orbital architecture of the outer planets of our own Solar System. However, it is not clear under what circumstances such a fast migration would occur (see D'Angelo & Lubow 2008). Alternatively, turbulence effects in the gaseous disk may have caused significant orbital perturbations in the decaying bodies to inhibit resonance trapping (Adams et al. 2008).

A second possibility may lie in the survival rate of resonant planets during their evolution within the commensurability. As shown originally by Lee and Peale (2002), once inside the resonance domain, tidal interactions with a gaseous disk will increase the eccentricities of the planetary bodies until a disruptive collision or ejection of one body occurs. This is due to the topology of the stable ACR families in the eccentricity domain (see, e.g., Beaugé et al. 2006, Hadjidemetriou 2008) within the 2/1 MMR. The only way two resonant planets may survive a large scale orbital migration is if the driving mechanism introduced a significant damping of the orbital eccentricities, leading to equilibrium values of these elements comparable with the observed values (Kley et al. 2005, Beaugé et al. 2006). Although this effect is expected from nominal disc parameters, especially if a inner disc is assumed (Crida et al. 2008), there is no evidence that this must be true in all cases, and perhaps most of the systems originally trapped in the 2/1 could have been ejected. Moorhead and Adams (2005)

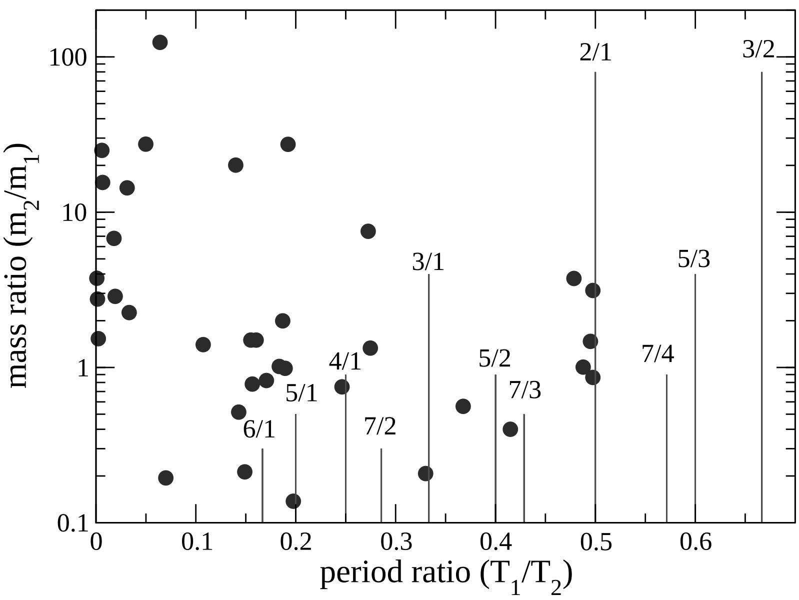

Figure 1: Distribution of pairs of consecutive planets in multiple-planetary systems, according to mass ratio over ratio of orbital periods. Mean motion resonances are indicated by vertical lines, whose length is inversely proportional to the order of the MMR. Data correspond to most current orbital fits.

proposed such a mechanism to explain the present semimajor axis and eccentricity distribution of exoplanets.

A third possibility, little considered up to now, is that the apparent lack of resonant planets may be due to detectability limitations. Recall that resonant motion causes an almost periodic repetition of the radial velocity curve of the star which, under certain circumstances, may not allow a good separation of components. Perhaps, many additional systems may actually lie within the 2/1 resonance but are currently not discernible due to limited observations or small signal-to-noise ratios. Recently, Anglada-Escudé et al. (2008) discussed cases where a two planet resonant system with almost circular orbits may appear masked as a single planet in a highly elliptical orbit. In this communication we address this question in more detail constructing a detectability criteria.

We also discuss the errors in the planetary masses and orbital parameters of those resonant systems which can actually be detected. Although this is different problem, it shares many common points and can be studied using the same approach. Of the four systems currently inhabiting the 2/1 MMR, only GJ876 has a dynamically stable best fit, while the rest are characterized by orbits leading to a disruption of the system in time scales much smaller than the age of the star. Although stable orbital solutions are possible for these troublesome cases, having residuals similar to the best fit, they usually correspond to large amplitude ACR, not easily compatible with a smooth orbital migration (Sándor et al. 2007, Crida et al. 2008).

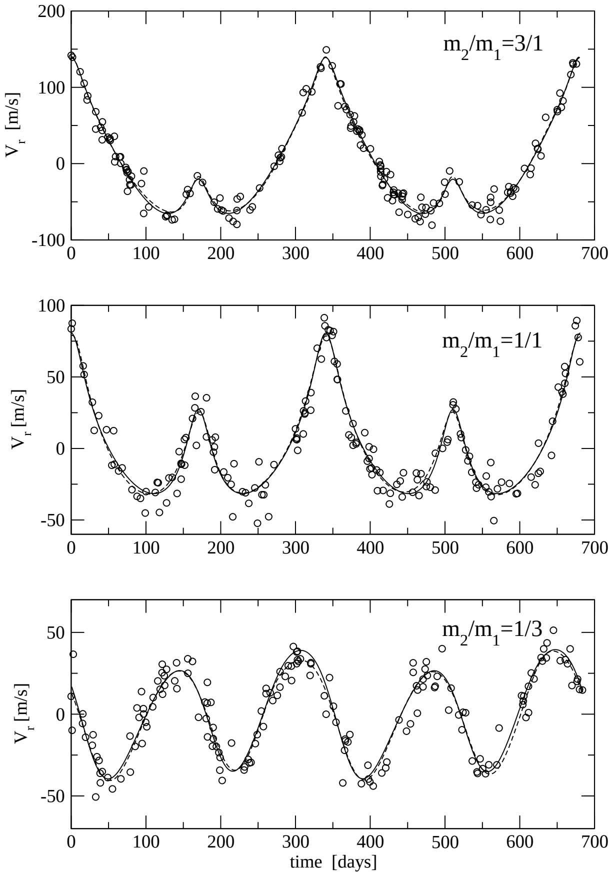

Figure 2: Black continuous line shows synthetic RV curves for the three ACR. Mass ratio decreases from top to bottom. Open circles are $N = 165$ fictitious points with random time distribution and standard deviation $\sigma = 10$ m/s.

2. TWO-PLANET SYSTEMS IN A 2/1 MMR

Consider two planets of masses m_1 and m_2 orbiting a star m_0 with semimajor axes a_i, eccentricities e_i, mean longitudes λ_i and longitudes of pericenter ϖ_i. The index $i = 1$ will be used for the inner planet, while $i = 2$ will denote the outer body (i.e. $a_1 < a_2$). We assume that both bodies share the same orbital plane oriented edge-on with respect to the observer, and that the orbital configuration leads to a small amplitude oscillation around a stable ACR in the 2/1 MMR. Hydrodynamical solutions (e.g. Kley et al. 2005) seem to indicate that ACR constitute a natural outcome of planetary migration due to disk-planet interactions.

To quantify how the uncertainties in the orbital fit will affect the observed motion of the system, we analyzed three fictitious systems consisting of two planets orbiting a Solar-like star ($m_0 = 1M_\odot$) with mass ratios m_2/m_1 equal to 3/1, 1/1 and 1/3. In all cases m_1 was taken equal to one Jupiter mass and $a_1 = 0.63$ AU. Initial conditions were chosen to correspond to small-amplitude oscillations around a stable ACR. In the first two cases the ACR are of type $(0, 0)$ (see Beaugé et al. 2006), while for $m_2/m_1 = 1/3$ the solution corresponds to an asymmetric ACR.

For each nominal solution, we generateed a synthetic RV curve mimicking the star movement around the barycenter of the system (shown as continuous curves in Fig. 2). Each curve is the sum of two periodic signals, each with amplitude K_j related to the radial velocity caused by the j-th planet. The synthetic curves were then sampled choosing N observation times t_i distributed randomly according to a homogeneous distribution. For every t_i the corresponding RV value was defined as a random displacement of the nominal $V_r(t_i)$; this displacement followed a Gaussian distribution with constant variance σ^2. The resulting synthetic data set was then used as input in our orbital fitting procedure (Beaugé et al 2008). Typical examples of synthetic RV data sets with $\sigma = 10$ m/s are shown as open circles in Fig. 2.

2. 1. DETECTABILITY OF TWO RESONANT PLANETS

To calculate the detectability limit for two planets in the 2/1 MMR, we use the procedure developed in Giuppone et al. (2008). Although the reader is referred to that paper for further details, here we summarize some of the main characteristics. We assumed that the body generating the largest amplitude in the RV signal (i.e. K_a) is already known, and wished to calculate under which conditions the signal of the second body (of amplitude K_b) also satisfies the given detectability limit.

A look at the different plots of Fig. 2 shows that there is no unique separation of the signals. When the mass ratio m_2/m_1 is large, the RV amplitude of the outer planet is the larger component (implying $K_a = K_2$), and the detectability of the resonant pair reduces to the discernability of the inner mass. In the opposite case, when $K_2/K_1 < 1$, it is the inner planet that dominates the RV curve ($K_a = K_1$) and the existence of the outer body must be deduced from the difference between successive maxima.

For a given false alarm probability FAP and a detectability probability P_{detect}, it can be shown (see also Cumming 2004) that the discernability of the secondary signal depends only on the number N of observations, the variance σ^2 of the dispersion in the RV data and on the amplitude ratio K_2/K_1. This is valid as long as the orbital periods of all the planets are smaller than the total observational time span.

Fixing m_1 to one Jovian mass and its semimajor axis to $a_1 = 0.63$ AU, and considering near-circular orbits for both planets, we can transform the limiting values of K_2/K_1 to values of mass ratio m_2/m_1. Fig. 3 shows the resulting detectability limits, as function of N, for four different values of the standard deviation. Each value of σ gives origin to two curves, one localed above and one below $m_2/m_1 \simeq \sqrt[3]{2}$. For a resonant companion to be detected, the mass ratio must lie between both curves corresponding to the same value of σ (i.e. curves of equal shades of gray). For example, for RV data with typically errors of the order of 10 m/s it appears very difficult to detect planetary systems in the 2/1 MMR if the number of observations is smaller than $N \sim 40$. Even for larger data sets, the discernable mass ratios always lie very close to unity.

In general terms, this criterion places strong restrictions on the mass ratios of detectable resonant systems. Except for very accurate radial velocity data, bodies

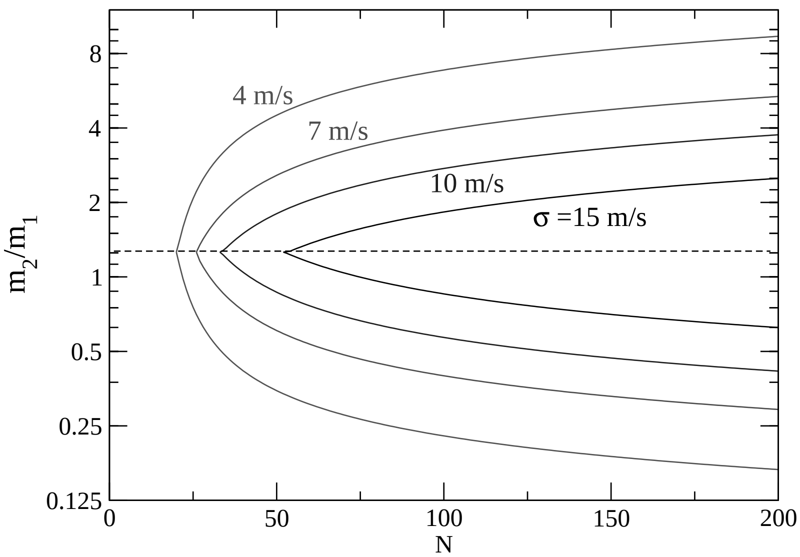

Figure 3: Detectability limits of mass ratio m_2/m_1 (with $P_{detect} = 0.99$), as a function of number N of data points and for four values of σ. Both eccentricities are chosen as $e_i = 0.1$. FAP is taken equal to $\mathcal{F} = 10^{-4}$. For each value of σ, resonant planets can be detected only if located inside the corresponding curves.

with mass ratios m_2/m_1 larger than $\simeq 4$ or lower than $\simeq 0.3$ always seem to be out of bounds. It is important to stress that this does not mean that it is impossible to detect systems outside these borders; but this detectability will not be guaranteed for any data set and will depend on the specific time distribution and/or errors of individual data points.

It is also worthwhile to stress that the value of σ to consider in these calculation is not just the one deduced from the observational techniques, but the total standard deviation including stellar jitter and the possible incompleteness of the 2-planet model. Typical values of stellar jitter (Wright 2005) are of the order of $3 - 7$ m/s, and instrumental uncertainties are also of similar magnitude. The square sum of both implies that typical values of σ for these systems should be of the order of $7 - 10$ m/s. A good qualitative estimate of σ may be given by the value of the *wrms* resulting from the orbital fit (Shen & Turner 2008) which also leads to $\sigma \sim 7 - 10$ for most multiple-planet systems.

The apparent extreme difficulty in detecting resonant planets with significantly different masses is in accordance with currently known systems. Recall that Fig. 1 shows the distribution of mass ratios against orbital period ratio for all consecutive planetary pairs in multi-planetary systems. All planetary systems near the 2/1 MMR have mass ratios m_2/m_1 grouped near unity, in accordance to the analytical findings of the present model. The interesting conclusion is that it is possible that resonant systems may exist for other mass ratios, but are presently undetectable.

Table 1: Percentage of different dynamical outcomes of the best fits for different mass ratios m_2/m_1 and number of data points. For symmetric ACR the resonant angle is taken as θ_1, while θ_2 is adopted for asymmetric solutions.

m_2/m_1	N	non-resonant	θ_i-librator	ACR
3/1	200	0%	2%	98%
	100	0%	12%	88%
1/1	200	1%	7%	92%
	100	9%	21%	70%
1/3	200	42%	8%	50%
	100	68%	6%	26%

3. ERROR ESTIMATION IN THE FITTED PARAMETERS

Even if a two-planet resonant system is actually detected, there is no guarantee that the masses and orbital parameters will be estimated with any accuracy. Of those synthetic data sets that satisfied our detectability criterion, we noted a significant dispersion in the mean motions around the nominal value $n_1/n_2 = 2$, with an appreciable percentage of the systems which would be qualified as near-resonant but not locked in MMR. This effect is more dominant for small values of the mass ratio m_2/m_1 (see Table 1). Thus, it is possible to mistakenly assume that two planets are not in resonance lock solely due to imprecisions in the orbital fits.

Moreover, even if correctly identified as being locked in resonance, the planetary orbital elements may still contain significant errors and affect the deduced dynamics for the fitted systems. To quantify this effect, we constructed 1000 synthetic RV data sets for each of the fictitious planetary systems shown in Fig. 2, all leading to small amplitude oscillations around stable ACR. Each data set contained $N = 200$ data points distributed randomly over four orbital periods of the outer planet (i.e. four years). The values of each individual radial velocity was taken randomly following a Gaussian distribution around the exact value and a variance given by a constant standard deviation for $\sigma = 7$ m/s. These values guarantee a detectability of the two planets and places the deduced mean-motion ratio within the commensurability region.

A best fit solution was calculated for each data set, whose orbital configuration was then numerically integrated over a time span of 10^4 orbital periods. We then obtained for each data set its dynamical stability as well as the amplitudes of oscillation of the resonant angles $\theta_i = 2\lambda_2 - \lambda_1 - \varpi_i$ and the difference in longitudes of pericenter $\Delta\varpi = \varpi_2 - \varpi_1$. The critical angle θ_1 corresponds to an interior resonance (see Michtchenko et al. 2008a), as found in symmetric ACR, while θ_2 is the librating angle for exterior resonances (see Michtchenko et al. 2008b) and is thus applicable to asymmetric ACR. The proportion of solutions leading to different dynamical behaviors is also shown in Table 1.

Fig. 4 shows the distribution of fitted eccentricities and amplitudes of oscillation of the angular variables for $N = 200$ for the mass ratio $m_2/m_1 = 1/1$. The nominal values corresponding to the original system are marked with vertical dashed lines.

The two top frames correspond to the eccentricities; e_1 on the left and e_2 on the

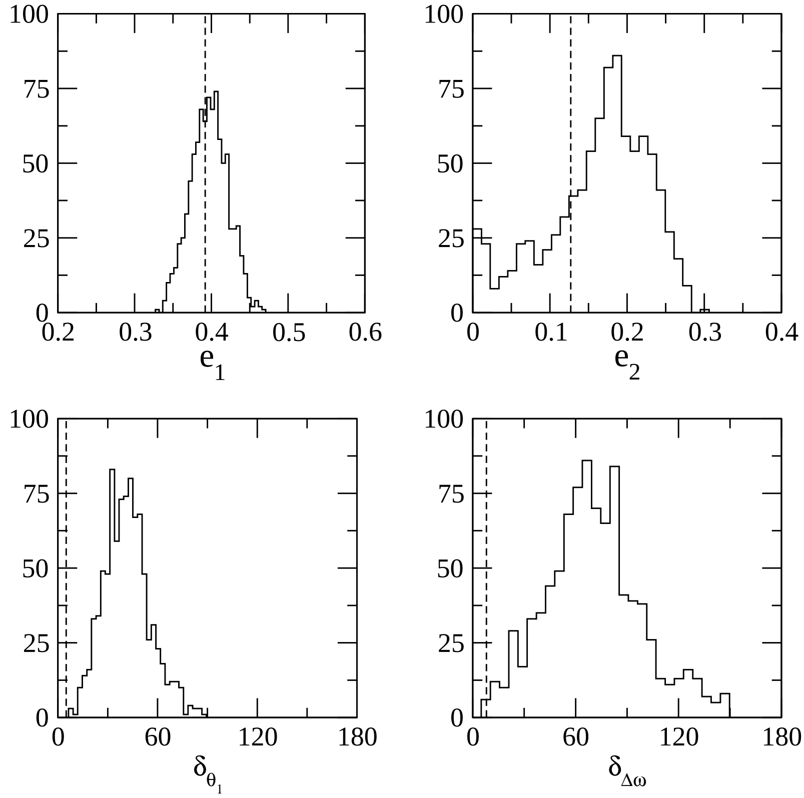

Figure 4: Distribution of calculated eccentricities and amplitudes of oscillation of the resonant angle θ_1 and $\Delta\varpi$, for 1000 synthetic data sets constructed from a nominal solution corresponding to a small amplitude oscillation around stable ACR for $m_2/m_1 = 1/1$. Nominal values for each variable is indicated by dashed vertical lines. Color codes are used to identify different sizes N of the data sets.

right. The distribution of eccentricities of the inner planet is fairly symmetric with respect to the true value (~ 0.39), and is reminiscent of the results shown for single planet systems (Shen & Turner 2008). Although the dispersion increases slightly for smaller N, there appears to be little systematic error and the peak of the histogram is close to the dashed vertical line. The same, however, is less evident in the case of the eccentricity of the outer planet, where the distribution is less symmetric and there is a notorious bias towards larger values of e_2.

The two lower plots give the distribution of amplitudes of oscillation of the resonant angle θ_1 (left-hand graph) and $\Delta\varpi$ (right-hand graph). There is an important increase in the amplitudes leading to a systematic bias towards solutions with larger amplitudes of oscillation, and this bias increases for smaller data sets. Practically none of the synthetic data sets showed best fits with amplitudes of oscillation similar to the original system. Even for larger data sets, some solutions show variations of $\Delta\varpi$ characteristic of circulations of that angle.

3. 1. APPLICATION TO REAL EXOSYSTEMS

Table 1 summarized the possible outcomes of the orbits fits for the three mass ratios. Although all data sets were constructed with the same standard deviation $\sigma = 7$ m/s, there is a marked difference with the mass ratio. For $m_2/m_1 = 3/1$ most fits correctly identified the ACR and led to stable motion of both planets in timescales of the order of 10^4 orbital periods. The inverse case, $m_2/m_1 = 1/3$ is significantly less precise, and is probably due to the existence of several domains of stable motion, each associated to a different family of ACR (Michtchenko et al. 2008b). Each domain is separated by a chaotic layer, and are less robust than those found for mass ratios larger than unity.

Depending on the number N of data points, approximately half of the best fits failed to recognize the resonant configuration, and the ACR was identified only in $26 - 50\%$ of the cases. Thus, it appears that resonant planets with mass ratios lower than unity are not only more difficult to detect, but even when detected the orbital fits usually lead to unstable motion. Although these results are valid for the 2/1 resonance and the 55Cnc-c and 55Cnc-d planets are located in the vicinity of the 3/1 commensurability (mass ratio $m_2/m_1 \simeq 1/5$), it seems plausible to expect similar results, particularly since asymmetric ACR are involved in both cases. This could be a possible explanation of why successive orbital fits of the 55Cnc planets alternatively point towards resonant or non-resonant motion, while the same uncertainty is not present for exosystems with other mass ratios. Another example is given by the HD37124 system with two planets in the vicinity of the 2/1 MMR. Although the best fit indicates $m_2/m_1 \simeq 1.2$ and an asymmetric ACR (Baluev 2008), other solutions exist in which the bodies are outside this commensurability.

Fig. 5 compares the distribution of eccentricities found for all three mass ratios. Each ensemble is identified by the value of m_2/m_1. Only those orbital fits leading to stable ACR were plotted. The continuous curves correspond to the zero-amplitude ACR for each mass ratio. For $m_2/m_1 = 1/3$ we note that most of the solutions lie relatively close to the family of stable ACR, although it is important to recall that these correspond to less than half of the total data sets. A similar trend is observed for $m_2/m_1 = 3/1$, except that all orbital fits were now plotted. In the case of equal masses, however, there is no apparent correlation between the eccentricity dispersion and the ACR families. It then appears that, for large mass ratios, there is a much larger probability that even uncertain orbital fits will remain close to stable ACR configurations. For mass ratios close to unity, this is in no way guaranteed, and even small standard deviations may lead to larger amplitude of libration or unstable motion.

4. CONCLUSIONS

In this communication we have presented recent results on the detectability limit of two planets in the vicinity of a 2/1 mean-motion resonance, as a function of the number N of data points and the standard deviation σ of the RV values. For values of σ and N comparable to real exosystems we found an important bias towards cases with $m_2/m_1 \sim 1$. Planets with mass ratios much larger or smaller than unity are much more difficult to separate from Doppler data.

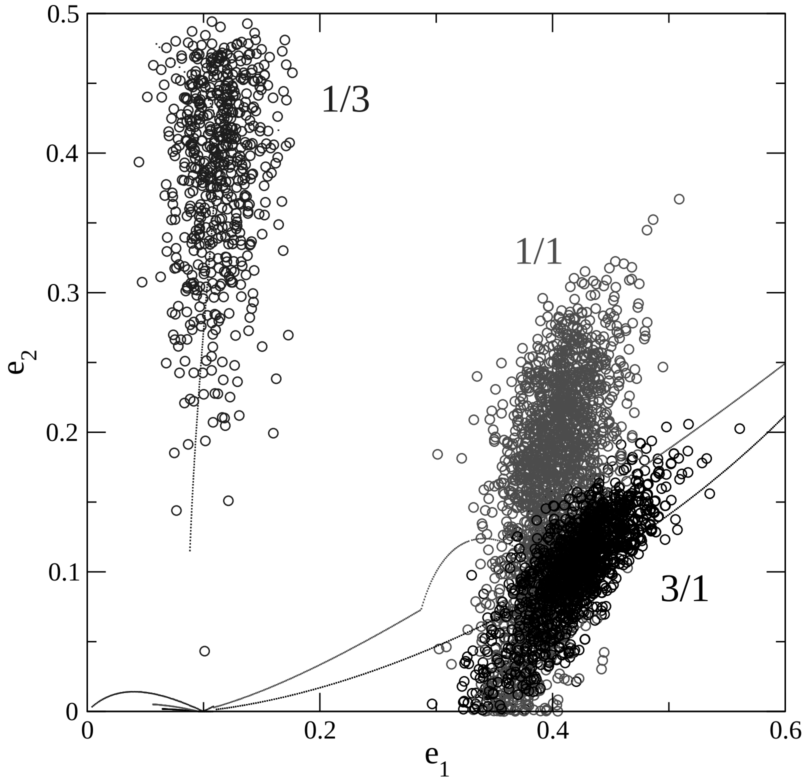

Figure 5: Distribution of planetary eccentricities from best fits of synthetic data sets, compared with the families of ACR. Shades of gray identify the ensembles corresponding to each mass ratio m_2/m_1.

Even if both planets are actually detected, the deduced orbital configuration may not identify the resonance relation. This effect seems to be particularly strong for low values of m_2/m_1, where many of the detected planetary pairs lead to an erroneous non-resonant configuration. Even within the subset of resonant orbits, there is also a large dispersion in the estimated values of the eccentricity of the outer planet, and the median of the distribution is always higher than the nominal value of e_2. This is in accordance with several real exosystems in the 2/1 MMR, particularly HD82943. Moreover, nominal solutions corresponding to small amplitude librations around ACR are usually accompanied by an important bias towards incorrect large amplitudes that may even compromise their dynamical stability on long timescales. Even if this is not the case, it presents a possible answer to the question of why, with the exception of the resonant bodies in GJ876, all other resonant planets have orbital fits consistent with large amplitude librations. In other words, this dynamical characteristic may not be real, but a simple consequence of dealing with small data sets with large uncertainties in the RV values.

For multiplanetary systems it is important to stress that the absolute numerical precision of the orbital elements is not, in itself, the only datum to consider, as much as the diversity in dynamical behaviors within the uncertainty region. In other words, if the two planets are non resonant with low eccentricities, even significant changes in the orbital elements will not necessarily lead to different regimes of motion. Conversely, if the planets are locked in a MMR and have moderate to high eccentricities, even a small change in the orbital parameters may mean the difference between stable and unstable motion.

Finally, at present it is not possible to evaluate the real proportion of resonant systems within the population of exoplanets. Although it is possible that resonant planets are not as frequent as once believed, the limited number of known cases may also be due to detectability restrictions and such configurations may actually prove to be much more common than currently believed.

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