Publ. Astron. Obs. Belgrade No. 96 (2017), 337 - 342 *Contributed Paper*

ON MHD WAVE COUPLING BETWEEN TERRESTRIAL IONOSPHERE AND MAGNETOSPHERE

V. M. ČADEŽ[1] and A. NINA[2]
[1]*Astronomical Observatory, Volgina 7, 11000 Belgrade, Serbia*
E–mail: vcadez@aob.rs

[2]*Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia*
E–mail: sandrast@ipb.ac.rs

Abstract. We model the terrestrial ionosphere-magnetosphere system by a high plasma-β (low magnetic field) isothermal region separated by a horizontal boundary from a low plasma-β (strong magnetic field) domain above it. Perturbations induced by sudden impacts of the solar wind can generate MHD perturbations in the magnetosphere (the low plasma-β domain) that propagate downward and through the boundary into the ionosphere (the high plasma-β domain). Applying the VLF (very low frequency) technique, we are able to identify such hydrodynamic waves in the ionosphere by computation of characteristic oscillation spectra from amplitudes of reflected VLF radio-waves recorded in real time. As not all MHD waves can cross from the magnetosphere into the ionosphere, we present in this contribution the mathematical conditions required for MHD waves to enter the ionosphere and enable a magnetosphere-ionosphere coupling mechanism.

1. INTRODUCTION

The terrestrial atmosphere is a complex gaseous system separating the planet from the outer space. Its structure is not uniform, its physical properties depend on location above the Earth surface which results in standard division of the atmosphere into characteristic regions. In this work we consider two distinct domains: the lower ionospheric D-region and the highest magnetosphere located at the frontier to the outer space. These two regions are characterized by relatively high charge particle density, relatively low temperature and negligible geomagnetic field effects in the D-region, and a very low charged particle concentration, much higher temperature and pronounced influence of magnetic field in local phenomena. Both domains are subject to various external disturbances causing numerous local perturbations ranging from temporal alternations in ionization processes at atom and molecular levels, to macroscopic collective features such as magnetohydrodynamic (MHD) waves.

Our particular aim in this paper is to estimate a possibility of MHD wave transmission through a horizontal boundary separating the ionosphere from magnetosphere in a model of idealized two region terrestrial atmosphere. Such domain coupling by

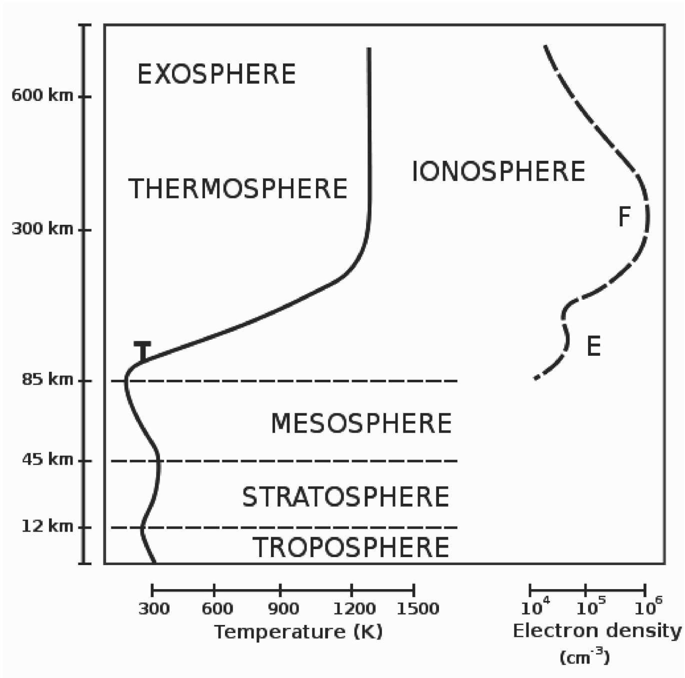

Figure 1: Typical vertical temperature profile in the atmosphere.

MHD waves is important in studying atmospheric (ionospheric as well as magnetospheric) responses to disturbances induced by various external perturbation sources. In particular, we are interested in resulting types of MHD waves and how far they can propagate through the nonuniform atmospheric plasma medium, i.e. whether the hydrodynamic waves, detected in the lower ionosphere by the VLF (very low frequency) radio wave reflection technique (for details see Nina & Čadež, 2014), can be generated far away in the magnetosphere by, say, some sudden solar wind impacts.

2. BASIC STATE AND EQUATIONS

According to the typical temperature profile in Fig.1 we model the terrestrial atmosphere as a simple system of two quasi-isothermal domains with the following properties:

- The ionospheric D-region with model values: T_0=250K, $V_s \equiv V_{si}$=700m/s $\gg V_{Ai}$.
- In the magnetospheric region with model values: T_0=1200K, $V_s \equiv V_{sm}$=12 km/s$\ll V_{Am}$=6500 km/s,

where:

$V_A^2 \equiv B^2/(\mu_0\rho_0)$, $V_s^2 \equiv \gamma R T_0$ and $V_c^2 \equiv V_A^2 V_s^2/(V_A^2 + V_s^2)$ are squares of the standard Alfvén, sound and cusp speed, respectively.

The considered atmosphere is assumed vertically stratified (along the z-axis), being in a hydrostatic equilibrium, and having constant Alfvén speed in both domains separated by a horizontal boundary. The resulting plasma density and magnetic field vertical distributions are given by:

$$\rho_0(z),\, B_0^2(z) \sim \exp^{-z/H}, \quad H \equiv \frac{V_s^2}{\gamma g} + \frac{V_A^2}{2g} = \text{const.}$$

Starting from standard MHD equations for linear, adiabatic perturbations in a dissipationless fully ionized stratified plasma we obtain the following general dispersion relation (Pinter & Čadež, 1999):

$$k_z^2 = A_2 A_3 - \left(A_1 - \frac{\gamma g}{2V_s^2 + \gamma V_A^2}\right)^2 \tag{1}$$

with:

$$A_1 = \frac{g\omega^2}{(V_A^2 + V_s^2)(\omega^2 - V_c^2 k_x^2)},$$

$$A_2 = \frac{\omega^2 - V_s^2 k_x^2}{(V_A^2 + V_s^2)(\omega^2 - V_c^2 k_x^2)} - \frac{k_y^2}{\omega^2 - V_A^2 k_x^2},$$

$$A_3 = (\omega^2 - V_A^2 k_x^2)\left[1 + \frac{g^2}{(V_A^2 + V_s^2)(\omega^2 - V_c^2 k_x^2)}\right] - \frac{2\gamma g^2}{2V_s^2 + \gamma V_A^2}$$

with perturbed quantities $\delta\psi$ being harmonic in space and time in the following way:

$$\delta\psi = \hat{\delta\psi}(z) \mathrm{e}^{i(k_x x + k_y y - \omega t)}.$$

Clearly, waves propagate if the condition $k_z^2 > 0$ holds for given values of $k_{x,y}$ and ω.

Dispersion relation Eq.1 with parameters typical of the model ionospheric D-layer reduces to:

$$k_z^2 = \frac{\omega^2 - V_{si}^2 k_0^2}{V_{si}^2}\left[1 + \frac{g^2(\gamma - 1)}{V_{si}^2 \omega^2}\right] - \frac{g^2}{V_s^4}\left(1 - \frac{1}{\gamma}\right)^2$$

where:

$$k_0^2 \equiv k_x^2 + k_y^2, \quad \lambda_0 = \frac{2\pi}{k_0}, \quad \lambda_z = \frac{2\pi}{k_z}, \quad \tau = \frac{2\pi}{\omega},$$

as treated by A. Nina & V. M. Čadež (2014). Fig.2 shows the resulting diagrams for propagating p- and g-modes with several weave periods τ obtained from the Fourier spectral analysis of recorded VLF radio waves after being reflected from the ionospheric D-layer. The plotted ranges of the horizontal and vertical wavelengths λ_0 and λ_z, respectively, reflect the allowed perturbation dimensions that do not violate physical conditions and geometrical dimensions of the medium assumed by the model atmosphere. Now, the question we are interested in is whether such waves that propagate in the ionospheric domain can also propagate in the domain with conditions typical of the magnetosphere.

The horizontal boundary, separating the ionosphere from magnetosphere in our simplified model, imposes boundary conditions on harmonic waves in the two domains saying that the wave frequency ω and the horizontal wavevector components $k_{x,y}$ do

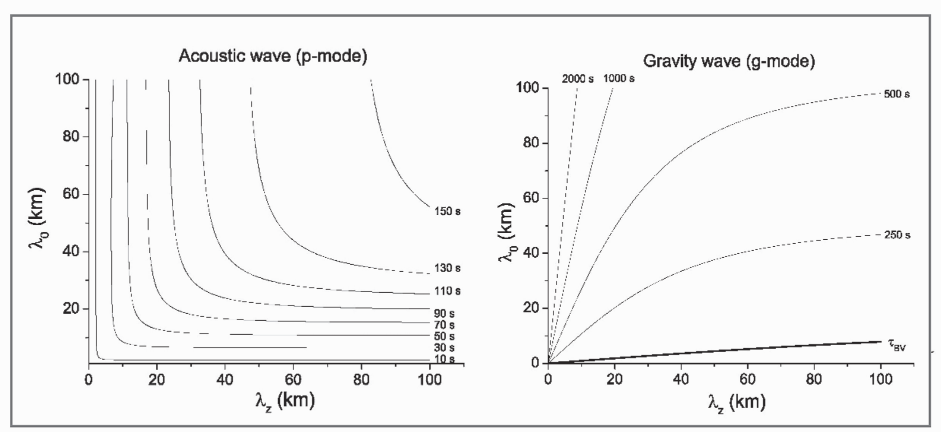

Figure 2: Computed dispersion curves for the acoustic p-mode (left panel) and gravity g-mode (right panel), respectively, under the assumed ionospheric conditions.

not change across the interface, while this is not true for the vertical component k_z. If k_z^2 remains positive, the two domains are coupled by such MHD waves meaning that disturbances generated in the magnetosphere can be present and detected also in the ionosphere. Consequently, we apply wave parameters for the D-region (plotted in Fig.2) in the dispersion relation (Eq. 1) with physical quantities relevant to magnetospheric conditions in the assumed model. A straightforward mathematical analysis shows that the resulting sign of k_z^2 is generally negative with an exception of a narrow domain close to the so called Alfvén resonance:

$$\omega^2 - k_x^2 V_{Am}^2 \approx 0, \quad \text{or} \quad \omega^2 - k_0^2 \cos^2\phi \, V_{Am}^2 \approx 0. \tag{2}$$

where it can be positive, which is the condition for the considered MHD waves to propagate in both regions. In this case, the dispersion relation Eq.(1) can be expanded for the horizontal propagation angle ϕ close to the resonant condition ϕ_{re} defined by:

$$\omega^2 - k_0^2 \cos^2\phi_{re} \, V_{Am}^2 = 0. \tag{3}$$

The resulting asymptotic expression is given by:

$$k_z^2 \approx \frac{2g^2 k_0^2}{V_{Am}^2(\omega^2 - V_{Am}^2 k_0^2 \cos^2\phi)} \qquad \text{if} \qquad \cos\phi \approx \cos\phi_{\mathrm{re}} \equiv \frac{V_{si}}{V_{Am}} \ll 1,$$

or:

$$\lambda_z^2 = \frac{V_{Am}^2(\lambda_0^2 - V_{Am}^2 \tau^2 \cos^2\phi)}{\tau^2} \qquad \text{if} \qquad \cos\phi \approx \cos\phi_{\mathrm{re}} \equiv \frac{V_{si}}{V_{Am}} \ll 1 \tag{4}$$

with

$$\delta_\phi \equiv \frac{\phi - \phi_{\mathrm{re}}}{\phi_{\mathrm{re}}}$$

being a normalized propagation angle.

Fig.3 shows numerical values of the resonant propagation angle ϕ_{re} obtained from Eq.(3) under the assumed magnetospheric conditions for the range of wave periods

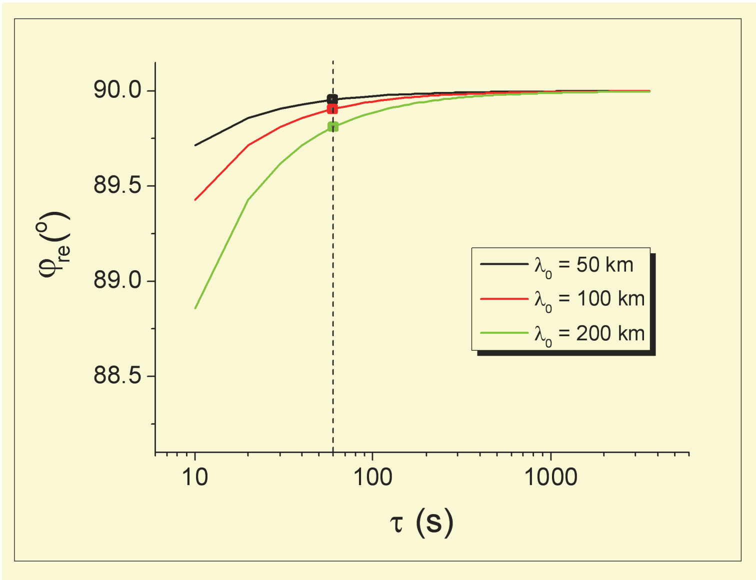

Figure 3: Resonant propagation angle ϕ_{re} dependence on wave periods τ for three typical horizontal wave lengths λ_0 at considered magnetospheric conditions.

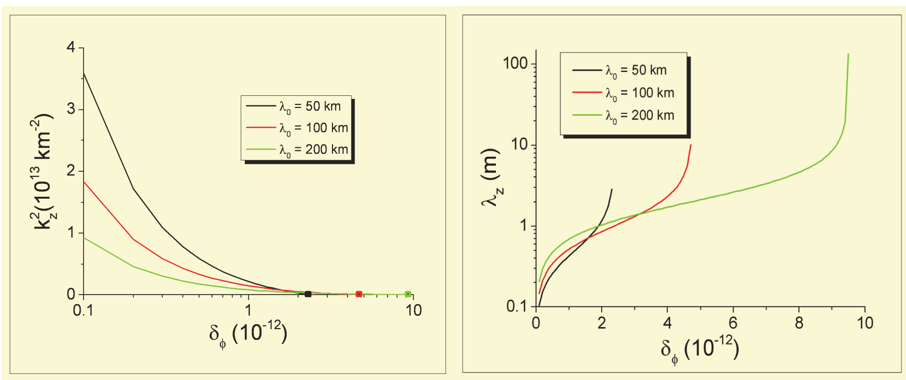

Figure 4: Vertical number k_z (left panel) and the related wavelength λ_z (right panel) dependences on the normalized propagation angle δ_ϕ for three typical horizontal wave lengths λ_0 at considered magnetospheric conditions.

τ registered in the lower ionosphere by the VLF radio-wave monitoring technique explained in A. Nina & V. M. Čadež (2014). As can be seen, the resonant condition Eq.(3) in our model can be satisfied for waves propagating almost parallel to the magnetic field.

The resulting vertical wavenumber k_z and the corresponding wavelength λ_z for the model magnetosphere are plotted in Fig.4. As can be seen, the vertical perturbation dimension defined by λ_z is real and grows rapidly with δ_ϕ for indicated horizontal perturbation dimensions λ_0.

3. CONCLUSIONS

The obtained results plotted in Figs. 3 and 4 show a possible existence of MHD wave coupling between the model ionosphere and magnetosphere through a restrictive Alfvén resonance. In other words, the coupling, i.e. wave transmission through the interface separating model ionosphere and magnetosphere, occurs for waves propagating almost perpendicularly ($\phi \approx \pi/2$) to the horizontal magnetic field of the model. In this case, hydrodynamic waves that can propagate in the ionosphere and be detected by the VLF radio wave reflection technique, may be generated in the magnetosphere the outermost part of the atmosphere, as a result of dynamical perturbations caused by solar wind variations and other phenomena depending on solar activity.

Acknowledgment

The present work was supported by the Ministry of Education and Science of the Republic of Serbia as a part of the projects no. III 44002, 176002 and 176004.

References

Nina, A., Čadež, M. V.: 2014, *Geoph. Res. Letters*, **40**, 4793.
Pinter, B., Čadež, V. M.: 1999, *Astron. Astrophys.*, **346**, 190.