Publ. Astron. Obs. Belgrade No. 96 (2017), 251 - 256 *Contributed Paper*

HALO MASS FUNCTION: FROM CALCULATIONS TO COSMOLOGICAL SIMULATION AND BACK

N. MARTINOVIĆ and M. MIĆIĆ

Astronomical Observatory, Volgina 7, 11000 Belgrade, Serbia
E–mail: nmartinovic@aob..rs
E–mail: micic@aob..rs

Abstract. One of the important ways in which we can study general distribution of the dark matter halos in Universe is through halo mass function. In this talk we will explore and discuss a means of calculation of theoretical halo mass functions and we will derive and compare several important halo mass function fits. We will discuss connection that permeates cosmological simulations and theoretically calculated fits. Theoretical fits will ultimately be compared with halo mass function derived from our cosmological pure N-body simulation. We will acknowledge that agreement on smaller redshifts is good, but, as will be seen, there is discrepancy on higher redshifts, both between different theoretical halo mass functions and from halo mass function derived from simulation.

1. INTRODUCTION

Most of the mass of matter in the Universe is in form of dark matter. It is a carrier of structure in the Universe and is organized into clusters, groups, filaments and voids dubbed "cosmic web" (Bond et al. 1996) whose building blocks are halos of dark matter. One way to examine properties of structure in the Universe, and its building blocks (halos) is through cosmological simulations.

Through the cosmological simulations we can observe formation of dark matter halos, their evolution, clustering and ultimately formation of large scale structure (Springel et al. 2006). In the current dominant, ΛCDM paradigm, small dark matter halos form and merge into larger halos and structures (eg. White 1994). Questions arise - when do the halos start to form? What is the mass range of halos? When can we expect halo of certain mass to appear in the history of the Universe? All these questions (and some more) can be answered with the halo mass function. It can be defined as number of halos in a volume of space and per unit of mass (Lukić 2008).

Halo mass function can be calculated analytically, derived empirically or directly from the cosmological simulations Lukić et al. (2007). Here we will show standard way for analytical and empirical derivation and briefly compare it to halo mass function from cosmological simulation.

2. CALCULATING HALO MASS FUNCTION

For representing and calculating halo mass function we will use the same principles as used in Lukić et al. (2007) and Murray et al. (2013). We define halo mass function as:

$$\frac{dn}{d\,lnM} = M \; \frac{\rho_0}{M^2} \; f(\sigma) \; |\frac{d\,ln\,\sigma}{d\,lnM}| \tag{1}$$

here n is the number density of the halos, M is the mass of the halos in question, ρ is the critical density of the Universe, $f(\sigma)$ is a fitting function (which can be either analytically or empirically derived) and σ is the mass variance. Basically we calculate number density in halo mass bins. Main issue here is deriving mass variance. Unlike for the variance of the density perturbations, here we are interested in mass variance across the mass bins, given as:

$$\sigma^2 = \frac{1}{2\,\pi^2} \int_0^\infty k^2\, P(k)\, W^2(kR)\, dk \tag{2}$$

where $P(k)$ is a power spectrum and $W(kR)$ is an introduced filter which we use to constrain variance over a certain mass range. Many filters can be chosen (Percival 2001), but usually top-hat filter is used, due to the fact that is robust enough and easy enough to implement:

$$W(kR) = \frac{3\;[sin(kR) - kR\; cos(kR)]}{(kR)^3} \tag{3}$$

As we mentioned, we were noting mass range, but it is obvious that top-hat filter is used against wave-number (k) and within a certain radius (R). That is because we can directly connect mass with the radius with simple equation:

$$M = \frac{4}{3}\; \pi\, \rho_0\; R^3 \tag{4}$$

To complete calculations of mass variance, obviously power spectrum calculations are needed. We use form of power spectrum that is similar to the expected form that power spectrum had during inflation after Big Bang:

$$P(k) = A\; k^n\, T^2(k) \tag{5}$$

Transfer function ($T(k)$) is used to translate power spectrum's smallest matter density perturbations across all scales. Here, n is the spectral index and A is the normalization constant, derived from mass variance retrieved for $R = 8Mpc/h$. There are several codes for the calculation of transfer function, here we use CMBfast (Seljak and Zaldarriaga 1996).

It should be clear by now that there is no direct link with time or more precisely scale factor (or redshift). Fitting function itself is derived in a way that it is insensitive to the redshift, where we introduce link to it by asuming that mass variance depends both on considered mass of the halos and current redshift, that is mass variance is connected with the redshift through the linear growth:

$$\sigma(M, z) = \sigma(M)\; d(z) \tag{6}$$

where linear growth factor is given as:

$$d(z) = \frac{D^{+}(z)}{D^{+}(z=0)} \tag{7}$$

and $D^{+}(z)$ can be calculated as:

$$D^{+}(z) = \frac{5\Omega_m}{2}\frac{H(z)}{H_0}\int_z^{\infty}\frac{(1+z')dz'}{[(H(z')/H_0]^3} \tag{8}$$

$H(z)$ is a value of Hubble constant at the redshift z, given as:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)} \tag{9}$$

2. 1. FITTING FUNCTIONS

For the fitting functions, we have selected several charateristic solutions, starting from the Press-Schechter (1974) (PS), a spheroidal halo aproximation, over Sheth et al. (2001) (SMT), which introduces eliptical correction for the shape of dark matter halos, to the empirically derived functions of Warren et al. (2006) and Angulo et al. (2013) for which they have used cosmological simulations - Warren used a series of zoom-in simulations with increasing mass resolution, while Angulo used one very large box with high number of particles which allowed him to span many orders of magnitude in halo size with a single simulation.

Fitting functions forms are given in Table 1. It should be noticed that with adequate factors SMT could be reduced to PS.

Table 1: Fitting functions used for calculating halo mass and halo growth function.

Press-Schechter (1974)	$f_{PS}(\sigma) = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma}\, exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$
Sheth et al. (2001)	$f_{SMT}(\sigma) = A\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_c^2}\right)^p\right]\frac{\delta_c}{\sigma}\, exp\left[-\frac{a\delta_c^2}{2\sigma^2}\right]$
Warren et al. (2006)	$f_W(\sigma) = 0.7234\,(\sigma^{-1.625}+0.2538)\; exp\left[\frac{-1.1982}{\sigma^2}\right]$
Angulo et al. (2013)	$f_A(\sigma) = A\left[\left(\frac{b}{\sigma}\right)^a + 1\right] exp\left[-\frac{c}{\sigma^2}\right]$

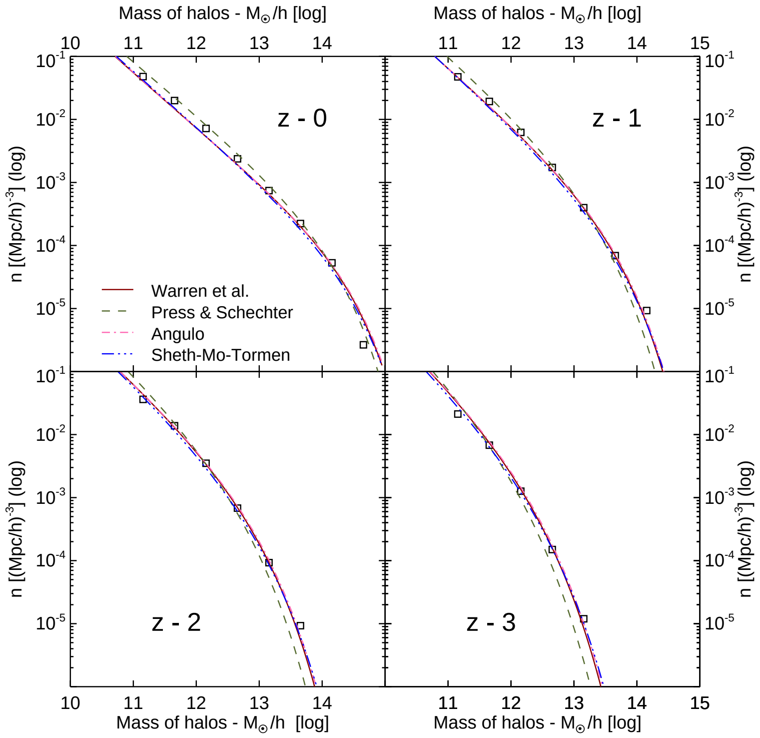

Figure 1: Plot of mass function: number density of halos as a function of halo mass (binned), plotted for 4 different redshifts. Results from simulation are compared to 4 different analytical fits (from Martinović (2015)).

2. 2. SIMULATION

For comparison with the calculated halo mass functions we use halo mass function derived from the $130 Mpc/h$ periodic box simulation with 512^3 particles executed with the GADGET2 code (Springel 2005), where we used ROCKSTAR (Behroozi et al. 2013) as the halo finder. Cosmological parameters used for the simulation are: $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $\Omega_b = 0.04$, $h = 0.7$ with $\sigma_8 = 0.8$ and $n_s = 1$.

Retrieved halos were binned for each simulation snapshot after which their number density for each bin was calculated. Halo mass function from the simulation was represented against the ones calculated, and those results are presented in Figure 1. (from Martinović 2015). Halo growth function (Heitmann et al. 2006) was calculated as the $n(M_1, M_2, z) = \int_{M_1}^{M_2} F(M, z)\, d\log M$ and those results are presented in Figure 2. (from Martinović 2015.) again with the rest of the calculated fitting functions.

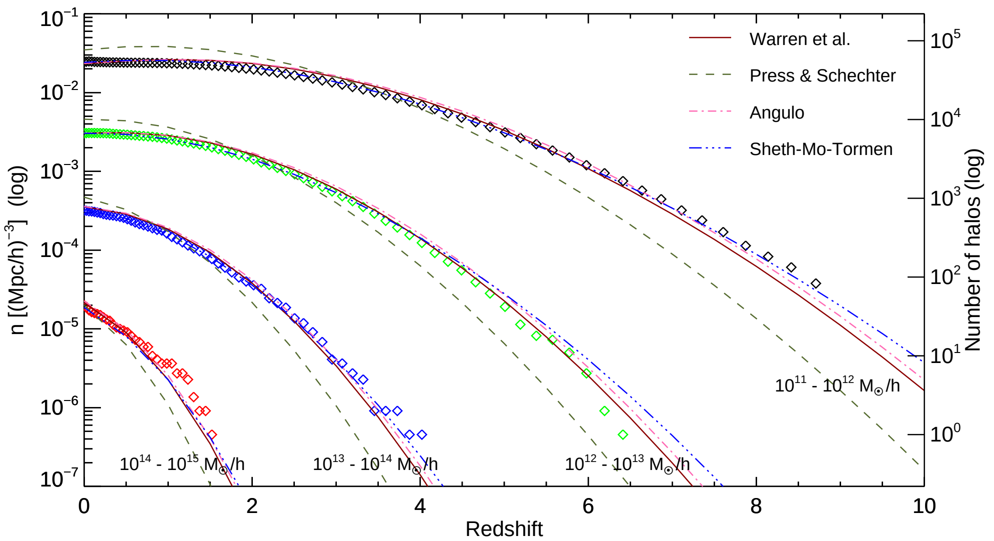

Figure 2: Plot of halo growth functions: number density of halos as a function of redshift presented for 4 mass bins. Results from simulation are compared to 4 different analytical fits (from Martinović (2015)).

3. DISCUSSION

We have used a method from Lukić et al. (2007), Murray et al. (2013), etc. for calculating dark halo mass function with various fitting functions. That method is used in calculating halo growth function as well and both functions are compared to the results derived from the cosmological simulation.

For halo mass function it can be seen in Figure 1. that there is a good agreement between all the calculated fitting functions and the one from cosmological simulation except for Press-Schechter one. As is obvious, it understimates the number of massive halos and overestimates number of less massive halos on all redshifts. Halo growth function of Figure 2. amplifies this result even further, where the same discrepancy between Press-Schechter and the rest of the fitting function is seen more clearly.

For more extensive analysis we point to the paper of Martinović (2015).

Acknowledgments

During the work on this paper the authors were financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia through project no. 176021 'Visible and invisible matter in nearby galaxies: theory and observations'.

Numerical results were obtained on PARADOX cluster at the Scientific Computing Laboratory of the Institute of Physics Belgrade, supported in part by the national research project ON171017, funded by the Serbian Ministry of Education, Science and Technological Development.

References

Angulo, Raul E.; Hahn, Oliver; Abel, Tom: 2013, *Mon. Not. R. Astron. Soc.*, **434**, 3337.
Behroozi, Peter S.; Wechsler, Risa H.; Wu, Hao-Yi: 2013, *Astrophys. J.*, **762**, 109.
Bond, J. Richard; Kofman, Lev; Pogosyan, Dmitry: 1996, **Nature**, **380**, 603.
Heitmann K., Lukic Z., Habib S., Ricker, P. M.: 2006, *Astrophys. J. Letter*, **642**, 85.
Lukić, Zarija: 2008, PhD thesis, Univ. Illinois at Urbana-Champaign.
Lukić, Zarija; Heitmann, Katrin; Habib, Salman; Bashinsky, Sergei; Ricker, Paul M.: 2007, *Astrophys. J.*, **671**, 1160.
Martinović, N.: 2015, *Serb. Astron. J.*, in preparation.
Murray, S. G., Power, C., Robotham, A. S. G.: 2013, *Astronomy and Computing*, **3**, 23.
Percival, Will J.: 2001, *Mon. Not. R. Astron. Soc.*, **327**, 1313.
Press, William H.; Schechter, Paul: 1974, *Astrophys. J.*, **187**, 425.
Seljak, Uros; Zaldarriaga, Matias: 1996, *Astrophys. J.*, **469**, 437.
Sheth, Ravi K.; Mo, H. J.; Tormen, Giuseppe: 2001, *Mon. Not. R. Astron. Soc.*, **323**, 1.
Springel, Volker: 2005, *Mon. Not. R. Astron. Soc.*, **364**, 1105.
Springel, Volker; Frenk, Carlos S.; White, Simon D. M.: 2006, *Nature*, **440**, 1137,
Warren, Michael S.; Abazajian, Kevork; Holz, Daniel E.; Teodoro, Luis: 2006, *Astrophys. J.*, **646**, 881.
White, Simon D. M.: 1994, arXiv:astro-ph/9410043.