Publ. Astron. Obs. Belgrade No. 96 (2017), 223 - 227 *Poster*

SEARCHING FOR CHAOTIC PATTERNS IN THE X-RAY LIGHT CURVES OF AGN

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Abstract. We explored short-term X-ray light curves of several Type I active galaxies from XMM - *Newton* in order to search for signatures of low-dimensional chaotic behavior. For our analysis we used 8 light curves of 4 AGN, each consisting of a total of ~ 2000 to ~ 5000 equidistantly located points. The correlation integral method was applied to search for chaos. The preliminary results show possible indications for the presence of a low-dimensional attractor ($D < 5$) in some of the light curves, however the results are not entirely conclusive.

1. INTRODUCTION

Time series of many astrophysical objects, including AGN, show variability patterns that can often be described as "random" and "unpredictable". An example of "true" random behavior is e.g. the *Brownian* motion, where the dynamical system has an immensely large number of degrees of freedom, being in order of the number of particles in the liquid. In addition, however, there are cases of dissipative systems, where the dynamics is governed by only a few (3 - 5) degrees of freedom (variables), yet the behavior appears to be "random" or "chaotic". In the famous *Lorenz attractor* case (Lorenz, 1963), a system of $N = 3$ non-linear differential equations has a solution, which once entering some volume of the phase space, stays bound and never leaves a sub-space of (non-integer) dimension $d < N$. These solutions are so called "strange attractors" and such a behavior is often described as "deterministic chaos". Trajectories of a strange attractor evolve into a finite volume in the phase space, never returning to the same points. The divergence between two close trajectories increase exponentially in time and the long-term predictions are impossible. Consequently, each variable of a strange attractor system, represented as function of time, can mimic random behavior.

To check if the short-term X-ray variability of AGN could possibly be governed by a low-dimensional dynamical system, instead of a truly random process, we applied the *correlation integral* (CI) method (see Vio et al., 1992; Lehto et al., 1993; Provenzale et al., 1994) to XMM Newton publicly available time series of several AGN. This method is based on the construction of new (empirical) phase space from the available

Table 1: The list of used objects.

Object	*Observatory*	*Reference*
Akn 564	XMM-Newton	Vignali et al., 2004; McHardy et al., 2007
MCG 6-30-15	XMM-Newton	Vaughan & Fabian, 2004
Mkn 766	XMM-Newton	Mason et al., 2003
NGC 4395	XMM-Newton	Vaughan et al., 2005

N discrete data points. The data set (e.g. the light curve) is separated into strings of length d. Each string can be considered as a d-dimensional vector (X_i), embedded in the d-dimensional empirical phase space. The number of vector pairs with a distance smaller than r, as a function of r, is computed for different d and related to the total number of pairs (n_p) for that d. Thus, the correlation integral can be written as:

$$C_d(r) = \frac{1}{n_p} \sum_{i,j=1;j>i}^{N} \Theta(r - |X_i - X_j|), \quad (1)$$

where Θ is the *Heaviside* function. So, if the dimension of the attractor is D, then:

$$C_d(r) \propto \begin{cases} r^d, & d < D \\ r^D, & d > D. \end{cases} \quad (2)$$

Therefore, increasing the embedded dimension d leads to saturation when $d > D$, which is used for the estimation of the attractor dimension. Generally:

$$D_c = \lim_{r \to 0} \frac{d \ln C(r)}{d \ln r}, \quad (3)$$

where D_c is the correlation dimension of the attractor and can be a non-integral value. Knowledge of D_c allows determination of the number of differential equations, describing the dynamical system, N, i.e. the first integral value, larger than D_c and therefore makes possible drawing conclusions about the physical process driving the variability. As an example, Fig. 1 shows the 3D Lorenz attractor. The left panel shows the Lorenz system, the right ones – the temporal behavior of X, Y and Z, as well as the trajectory of the solution in the Y-Z plane of the phase space. The application of the CI method for the Lorenz attractor (right) and random noise (left) is demonstrated in Fig. 2. For the former case saturation is clearly seen at a slope of about 2.06, which is the correlation dimension of the attractor.

In this paper we analyzed several segments of XMM-Newton equally spaced X-ray light curves of several AGN in order to search for the presence of low-dimensional chaotic signatures in the light curves.

2. DATA AND RESULTS

The objects we tested for chaos (listed in Tab. 1) were chosen on a basis of the availability in the literature of long enough, good quality X-ray datasets.

The results, in terms of log(CI) vs. slope are shown in Figs. 3 and 4. Although some saturation is evident for some of the light curves, we were unable to firmly

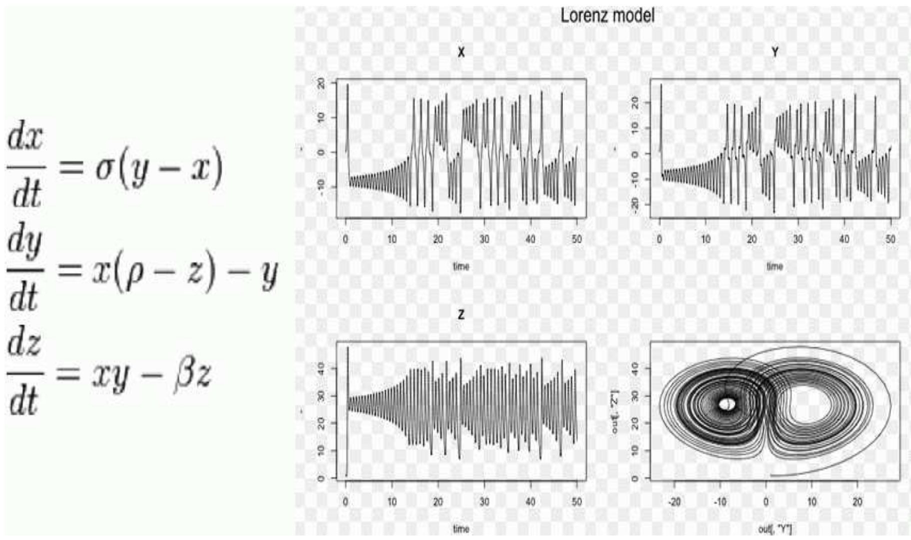

Figure 1: Lorenz attractor system and the temporal behavior of the variables (from Internet).

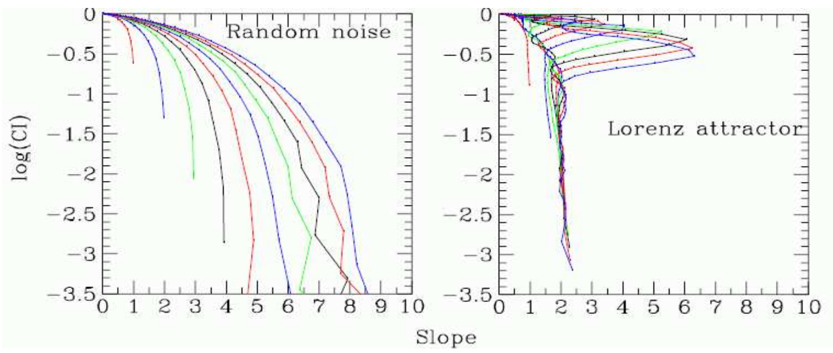

Figure 2: Application of the CI method to random noise and Lorenz attractor. Different embedded dimensions are shown with different colors. One sees that for the Lorenz attractor case there is saturation of the embedded dimensions for a slope of about $D \sim 2.06$, which is the dimension of the Lorenz attractor.

confirm the presence of low-dimensional attractor. The presence of noise in the light curves additionally complicates the situation.

Furthermore, when searching for low-dimensional chaos one is to take into account the so called *Ruelle criterion*, stating that the maximal attractor dimension that can be determined from a series of N points is $D_{max} \leq 2 \log N$, which in our case means about 5-6.

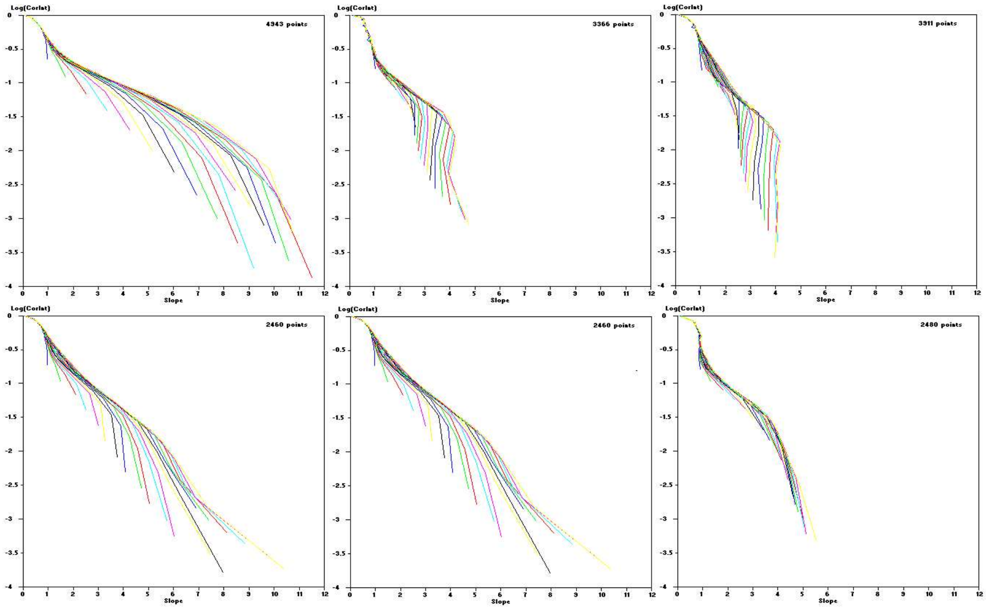

Figure 3: Log (CI) vs. its slope diagrams for 3 segments of Akn 564 X-ray LC (upper 3 diagrams) and MCG 6-30-15 (lower 3 segments).

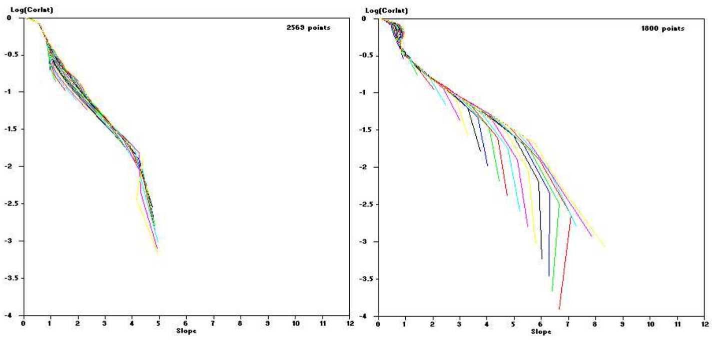

Figure 4: The same for Mkn 766 (left) and NGC 4395 (right).

3. SUMMARY

Despite we were not able to confirm the presence of low-dimensional attractor, our first results are still encouraging. If chaotic behavior is confirmed in the future studies it may have significant consequences for the accretion theory. Indeed, one may expect to find chaotic behavior in accretion systems as the process of accretion is governed only by a few parameters (equations), e.g. 5 or 6. As this is more or less our Ruelle limit, this may simply indicate that more data points are needed for a firm detection of the attractor. This work is in progress and the results are preliminary.

Acknowledgments

This work has been partially supported through grant BIn NTS 01/09 (2013) from the Bulgarian NSF.

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