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ON THE INNER BORDER OF PHOCAEA GROUP OF ASTEROIDS

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Abstract. In this work we check wether or not the inner boundary of the region in term of semi–major axis, namely the 7J:-2A resonance with Jupiter, could be crossed under the influence of gravitational and non-gravitational forces. The obtained results show that a significant fraction of our test particles successfully transit across the resonance, without being removed from the region. This means that, despite being relatively effective in pumping up asteroid eccentricities in this region, the 7J:-2A resonance is not an absolute dynamical boundary.

1. INTRODUCTION - PHOCAEA REGION

The Phocaea group is located in the inner asteroid belt, and consists of asteroids having orbital inclination higher than about 20 degrees, with semi-major axis from 2.25 AU to 2.5 AU and eccentricity ranging between 0.15 and 0.3. The asteroids in this region are characterized by a very interesting dynamics. The region of the Phocaea group is delimited by the 7J:-2A mean motion resonance (MMR) at low a, by the 3J:-1A MMR at high a and by the $\nu_6 = g - g_6$ (where g is the secular frequency of precession of the pericentre, and the suffix 6 refers to Saturn) secular resonance at low i (Knežević & Milani 2003). In addition, as explained by Knežević & Milani (2003), the Phocaea group is characterized by a region of shallow close encounters with Mars at $e > 0.3$, which displays significant chaotic behavior.

Moreover, the Phocaea group itself is characterized by its interaction with the secular resonance $\nu_6 - \nu_{16} = (g - g_6) - (s - s_6)$, where s is the secular precession frequency of the asteroid node. For this resonance is shown to be important for the dynamics of the Phocaea asteroids, and that a significant fraction of them is locked inside it (Knežević & Milani 2003).

1. 1. THE AIM OF OUR WORK

In Phocaea region there are many mean motion and secular resonances. The one very interesting is mentioned 7J:-2A, which is located at about 2.256 AU, and seems to set the lower boundary of the region in terms of semi–major axis.

The impact of Yarkovsky thermal force (Neiman at al., 1965, Vokrouhlický, 2001) has been usually examined separately from the influence of resonances on the motion of an asteroid. For example, Carruba (2010) found the 7J:-2A resonance to be very efficient in pumping up orbital eccentricities, leading to close approach with Mars.

He investigated dynamical behavior of objects in neighborhood of this resonance, but without taking into account the impact of the Yarkovsky effect.

Our main purpose here is to analyze the dynamical behavior of objects close to the 7J:-2A resonance, but taking into account the Yarkovsky effect. More precisely, our aim is to answer the question: could 7J:-2A resonance be crossed under the combined impacts of gravitational and non-gravitational forces?

2. THE METHOD

We want to study a possibility of the resonance crossing to occur. For this reason we performed numerical integrations on set of 20,000 fictitious asteroids initially located very close to the outer boundary of the 7J:-2A (Figure 1).

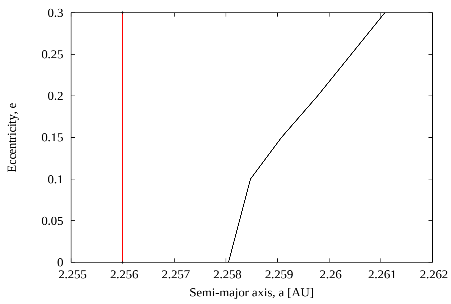

Figure 1: The outer (”entering”) border (black lines) and center (red line) of the 7J:-2A resonance.

The orbital motion of fictitious bodies was tracked for 20 Myr using the public domain ORBIT9 software (Milani & Nobili 1988), embedded in the multipurpose OrbFit package4. The dynamical model includes seven planets, from Venus to Neptune, as perturbing bodies. To account for the indirect effect of Mercury, its mass is added to the mass of the Sun and the barycentric correction is applied to the initial conditions. The Yarkovsky thermal force was also included in the model.

In order to generate this fictitious objects, we first took 1000 different asteroids, which lie on four straight lines with 300, 200, 200, 300 asteroids on this lines respectively. These lines are chosen to follow the ”entering” boundary of the resonance (Figure 1). The osculating eccentricity was equally distributed in each of four intervals $[0.0, 0.10]$, $[0.10, 0.15]$, $[0.15, 0.20]$, $[0.20, 0.30]$. Then the osculating semi–major axis was chosen in intervals $[2.25805, 2.258475]$, $[2.258475, 2.259075]$, $[2.259075, 2.259775]$, $[2.259775, 2.261075]$ in a such a way to satisfy equation $e = ka + n$ in each segment of a and e. k is coefficient of our line of the best fit and n is free parameter of equation. The osculating inclination was generated with random values within the interval $[18°, 29°]$. The longitude of node, the longitude of perihelion, and the mean anomaly are all taken randomly from the interval $[0°, 360°]$.

Second, for each of 1000 objects generated in the previous step we used 20 different Yarkovsky clones, by setting Yarkovsky drift to vary from -5.0×10^{-5}AU/Myr to -1.0×10^{-3} AU/Myr with the step of -5.0×10^{-5}AU/Myr. The negative Yarkovsky drift was adopted because we followed movement of asteroids from larger to smaller osculating semi–major axes. In this way we produced an input catalog with 20,000 fictitious objects in total.

ORBIT9 integrator has an option to perform on-line digital filtering in order to remove short periodic oscillations. In this way, mean orbital elements are obtained, and then used in all our analyses performed here.

Our next step is to define resonance crossing with following three conditions. The first condition is if an array of 1000 mean semi–major axes and mean eccentricities meets the one of the next seven terms (Table 1 and Figure 2):

Table 1: Terms for resonance crossing in terms of a_m and e_m. These limits follow very well the "leaving" border of the resonance.

1	$0.35 \leq e_m,$	$a_m < 2.2520$
2	$0.30 \leq e_m < 0.35,$	$a_m < 2.2525$
3	$0.25 \leq e_m < 0.30,$	$a_m < 2.2530$
4	$0.20 \leq e_m < 0.25,$	$a_m < 2.2535$
5	$0.15 \leq e_m < 0.20,$	$a_m < 2.2540$
6	$0.10 \leq e_m < 0.15,$	$a_m < 2.2545$
7	$0.00 \leq e_m < 0.10,$	$a_m < 2.2550$

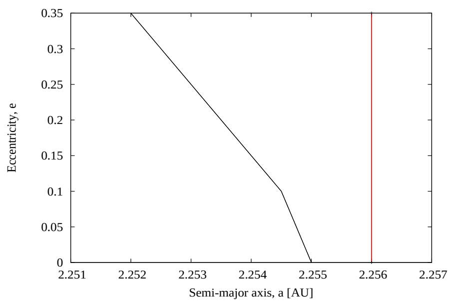

Figure 2: The inner ("leaving") border (black lines) and center (red line) of the 7J:-2A resonance.

From numerical integration we obtained mean orbital elements with the step of 200 yr. In order to define the second condition, we have applied the method of least squares to fit a linear model $a_m = kt + n$ at time t. From the fit we have calculated parameters k, n and their errors σ_k, σ_n.

The second condition is then fulfilled if coefficient of our line of the best fit k is negative, i.e. in agreement with assumed semi-major axis drift due to the Yarkovsky, for a period of at least 0.75 Myr and at most of 1 Myr, including 0.2 Myr from the first condition. Also, the ratio between coefficient of the linear trend and its error has to be greater than 30, because we decided to work with reliable values of k.

Finally, to characterize the third condition, we compared nominal values of expected and obtained semi–major axis drift. It means that obtained Yarkovsky drift speed has to be in interval $(k - \sigma_k, k + \sigma_k)$. If test particle meets all the three conditions, we considered that examined object crossed the 7J:-2A resonance.

3. RESULTS

Here, we applied the above described methodology to the set of 19,800 test particles that survived 20 Myr of numerical integrations. Let us first show an example of a test particle, which very quickly crossed the resonance (see Figure 3).

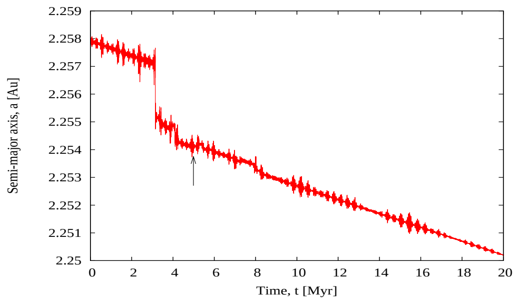

Figure 3: An example of test particle which successfully crossed the resonance without having close approach. The moment of crossing is shown by an arrow at $t \approx 5.0$ Myr. Assumed Yarkovsky drift speed of this particle is -2.5×10^{-4} AU/Myr, while from the linear fit we got $k \approx -2.46 \times 10^{-10}$ AU/yr, and $\sigma_k \approx 3.92 \times 10^{-12}$ AU/yr.

A different example of a successful crossing is shown in Figure 4. In that case considered object is captured by the resonance for a long time, from 2 Myr to about 17 Myr, before it finally crossed the resonance.

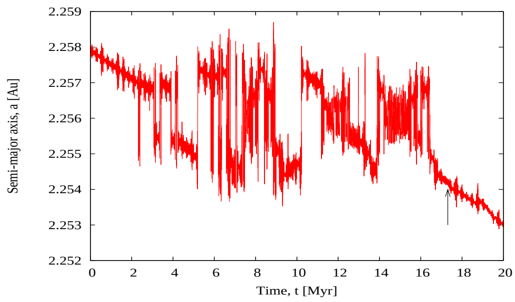

Figure 4: An example of test particle which successfully crossed the resonance without having close approach, with Yarkovsky drift speed -4.0×10^{-4} AU/Myr, $k \approx -4.03 \times 10^{-10}$ AU/yr, $\sigma_k \approx 3.16 \times 10^{-12}$ AU/yr. The moment of crossing is shown by an arrow at $t \approx 17.0$ Myr.

The distributions of moments of the resonance crossing, shown in Figure 5 and 6, are very similar. Most of the objects crossed the resonance in the first 8 Myr. The largest number of objects in both figures crossed the resonance between 4 and 6 Myr because they had certainly sufficient time for crossing.

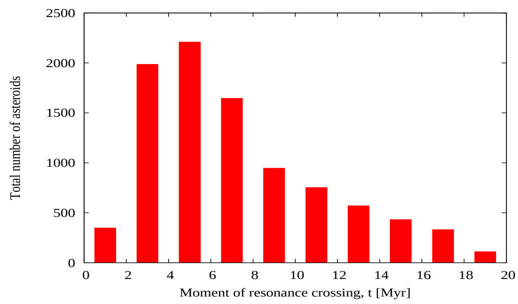

Figure 5: Distribution of the moments of crossing for 9330 particles that crossed the resonance.

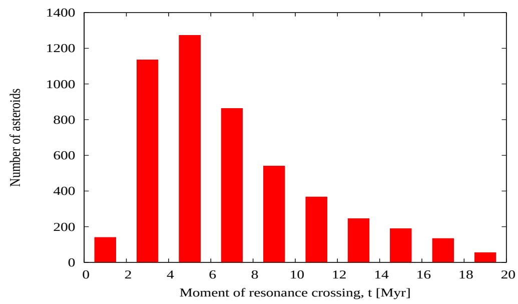

Figure 6: Distribution of the moments of crossing for 4937 particles that crossed the resonance without having close approach to any planet.

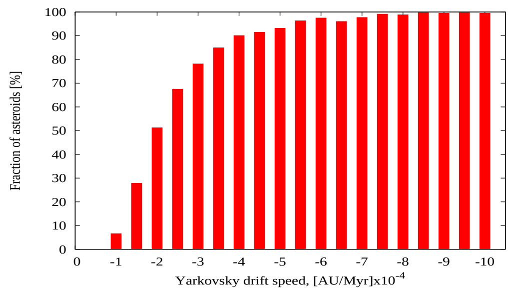

Figure 7: Fraction of objects that crossed the resonance as a function of Yarkovsky drift speed. Only those particles without having close approach to any planets are shown.

The goal of the next histogram is to analyze the number of objects that crossed the resonance as a function of Yarkovsky drift speed. We calculated the ratio between the number of objects that crossed the resonance and the total number of objects, for each of 20 different values of Yarkovsky drift speed. Here we considered only objects without having close approach to the planets. There are no objects with Yarkovsky drift speed of -5.0×10^{-5} that crossed the resonance without having close approach to the planets. Almost 100% of test particles with values of Yarkovsky drift speed higher than -5.0×10^{-4} AU/Myr successfully crossed the resonance (Figure 7).

4. CONCLUSIONS

At the beginning of the numerical integrations we had 20,000 test particles and almost all (19,800) survived integrations. A significant fraction of our test particles successfully transit across the resonance (9330), without being removed from the region, and 4937 particles crossed the resonance without having close approach to any planet. Although some authors found the 7J:-2A being powerful dynamical boundary in this region, our results show that this resonance is not an absolute dynamical boundary at all.

Acknowledgments

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