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NUMERICAL CODE FOR FITTING RADIAL EMISSION PROFILE OF A SHELL SUPERNOVA REMNANT

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Abstract. Expressions for surface brightness distribution and for flux density have been theoretically derived in the case of two simple models of a shell supernova remnant. The models are: a homogenous optically thin emitting shell with constant emissivity and a synchrotron shell source with radial magnetic field. Interactive Data Language (IDL) codes for fitting theoretically derived emission profiles assuming these two models to mean profiles of shell supernova remnants obtained from radio observations have been written.

1. MODELS OF EMISSION OF SHELL SUPERNOVA REMNANTS

1. 1. HOMOGENOUS OPTICALLY THIN EMITTING SHELL WITH CONSTANT EMISSIVITY

In this paper, we investigated two models of emission of shell supernova remnants (SNRs). If we consider homogenous emitting shell with emissivity $\varepsilon_\nu = \text{const}$, for specific intensity, if the medium is optically thin, we have

$$I_\nu = \int \varepsilon_\nu \mathrm{d}s = \begin{cases} \varepsilon_\nu (r'_{2+} - r'_{1+}) + \varepsilon_\nu (r'_{1-} - r'_{2-}), & 0 < \sin\theta < \frac{R-\Delta}{d} \\ \varepsilon_\nu (r'_{2+} - r'_{2-}), & \frac{R-\Delta}{d} \leq \sin\theta \leq \frac{R}{d}, \end{cases} \tag{1}$$

where $\mathrm{d}s = \mathrm{d}r'$. Cosine theorems (see Fig. 1)

$$(R-\Delta)^2 = d^2 + r_1'^2 - 2dr_1' \cos\theta, \tag{2}$$

$$R^2 = d^2 + r_2'^2 - 2dr_2' \cos\theta, \tag{3}$$

give us

$$r'_{1\pm} = d\cos\theta \pm \sqrt{(R-\Delta)^2 - d^2 \sin^2\theta}, \tag{4}$$

$$r'_{2\pm} = d\cos\theta \pm \sqrt{R^2 - d^2 \sin^2\theta}. \tag{5}$$

Finally, we have

$$I_\nu = \begin{cases} C_\nu \left(\sqrt{\sin^2\theta_2 - \sin^2\theta} - \sqrt{\sin^2\theta_1 - \sin^2\theta} \right), & 0 < \theta < \theta_1 \\ C_\nu \sqrt{\sin^2\theta_2 - \sin^2\theta}, & \theta_1 \leq \theta \leq \theta_2, \end{cases} \tag{6}$$

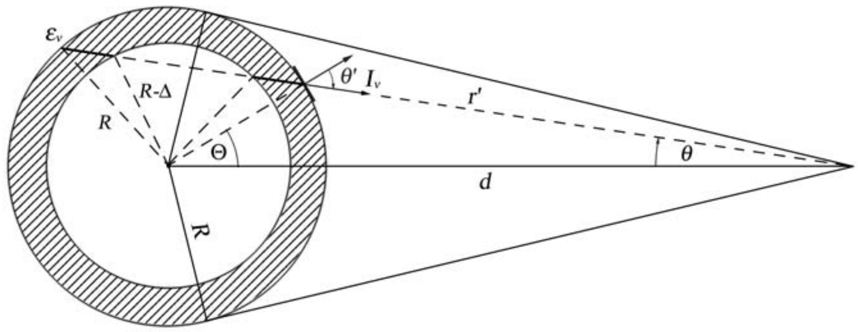

Figure 1: Radiation from an optically thin homogenous shell with thickness Δ and radius R, at the distance d from the observer. r'_1 and r'_2 are the distances from intersection of the line of sight for a given θ with the inner and outer radius of the shell, respectively.

where $\theta_1 = \arcsin \frac{R-\Delta}{d}$, $\theta_2 = \arcsin \frac{R}{d}$ and $C_\nu = 2\varepsilon_\nu d$. From the last equation one can see that $I_\nu^0 = 2\varepsilon_\nu R\delta$ and $I_\nu^{\max} = 2\varepsilon_\nu R\sqrt{\delta(2-\delta)}$, where $\delta = \Delta/R$. Brightness distribution i.e. specific intensity in units $2\varepsilon_\nu d$ given by equation (6), for $\delta = \Delta/R = 0.1$ and $R/d = 0.01$, can be seen in Fig. 2.

For the total flux density we have

$$\begin{aligned} S_\nu &= \int_0^{2\pi}\int_0^{\theta_s} I_\nu \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \\ &= 4\pi\varepsilon_\nu \int_0^{\theta_1} \left(\sqrt{R^2 - d^2\sin^2\theta} - \sqrt{(R-\Delta)^2 - d^2\sin^2\theta}\right)\cos\theta\sin\theta \mathrm{d}\theta \\ &+ 4\pi\varepsilon_\nu \int_{\theta_1}^{\theta_2} \sqrt{R^2 - d^2\sin^2\theta}\cos\theta\sin\theta \mathrm{d}\theta. \end{aligned} \tag{7}$$

After integration we obtain the expected result

$$S_\nu = \frac{4\pi}{3}\varepsilon_\nu d\Big[\Big(\frac{R}{d}\Big)^3 - \Big(\frac{R-\Delta}{d}\Big)^3\Big] = \frac{\varepsilon_\nu V}{d^2} = \frac{\mathcal{E}_\nu V}{4\pi d^2} = \frac{L_\nu}{4\pi d^2}, \tag{8}$$

where the shell volume is $V = \frac{4\pi}{3} f R^3$, $f = 1 - (1-\delta)^3$ is the volume filling factor, $\mathcal{E}_\nu = 4\pi\varepsilon_\nu$ is total volume emissivity (ε_ν is emissivity per unit solid angle) and L_ν is luminosity.

1. 2. SYNCHROTRON SHELL SOURCE WITH RADIAL MAGNETIC FIELD

If we have a synchrotron shell source with radial magnetic field, emission coefficient is $\varepsilon_\nu \propto (B\sin\theta')^{\alpha+1}\nu^{-\alpha}$ i.e.

$$\varepsilon_\nu = \tilde{\varepsilon}_\nu(\sin\theta')^{\alpha+1}. \tag{9}$$

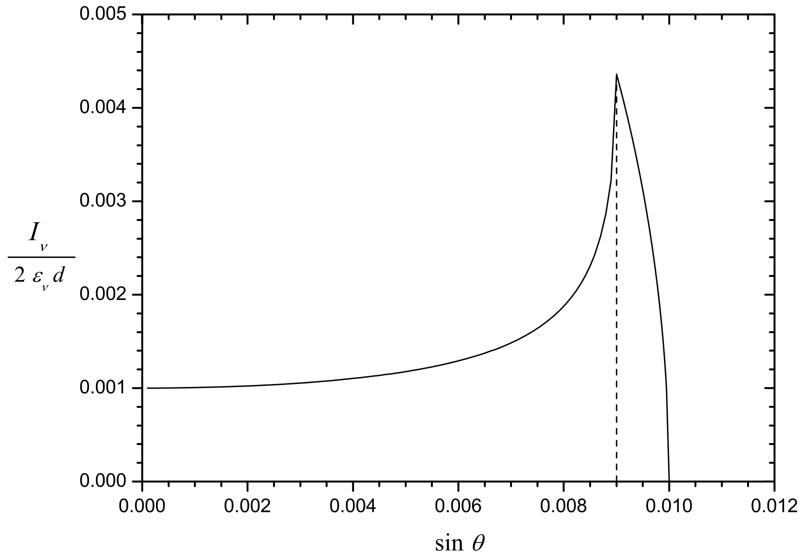

Figure 2: Brightness distribution for an optically thin homogenous shell-like source with $\delta = \Delta/R = 0.1$ and $R/d = 0.01$.

Sine theorem (see Fig. 1) gives us

$$\frac{r'}{d} = \frac{\sin\Theta}{\sin\theta'}, \quad \Theta = \theta' - \theta, \tag{10}$$

i.e.

$$r' = d(\cos\theta - \sin\theta \ \cot\theta') \tag{11}$$

and

$$\mathrm{d}s = \mathrm{d}r' = d\sin\theta\frac{\mathrm{d}\theta'}{\sin^2\theta'} \tag{12}$$

Intensity is then

$$I_\nu = \int \varepsilon_\nu \mathrm{d}s = \tilde{\varepsilon}_\nu d\sin\theta \int (\sin\theta')^{\alpha-1}\mathrm{d}\theta' \tag{13}$$

i.e.

$$I_\nu = \begin{cases} 2C_\nu \sin\theta \int_{\mu_{1-}}^{\mu_{2-}} \left(1-\mu^2\right)^{\frac{\alpha-2}{2}} d\mu, & 0 < \theta < \theta_1 \\ C_\nu \sin\theta \int_{\mu_{2+}}^{\mu_{2-}} \left(1-\mu^2\right)^{\frac{\alpha-2}{2}} d\mu, & \theta_1 \le \theta \le \theta_2, \end{cases} \tag{14}$$

where $\mu = \cos\theta'$, $\mu_{1,2\pm} = \mp\frac{\sqrt{\sin^2\theta_{1,2} - \sin^2\theta}}{\sin\theta_{1,2}}$ and $C_\nu = \tilde{\varepsilon_\nu} d$.

Rather than direct integration we will find flux density through $S_\nu = \frac{L_\nu}{4\pi d^2} = \frac{\mathcal{E}_\nu V}{4\pi d^2}$, where

$$\mathcal{E}_\nu = \int_{4\pi} \varepsilon_\nu \mathrm{d}\omega' = \int_0^{2\pi}\int_0^{\pi} \tilde{\varepsilon}_\nu (\sin\theta')^{\alpha+1} \sin\theta' \mathrm{d}\theta' \mathrm{d}\varphi = 2\pi\tilde{\varepsilon}_\nu \int_0^{\pi} (\sin\theta')^{\alpha+2}\mathrm{d}\theta' \tag{15}$$

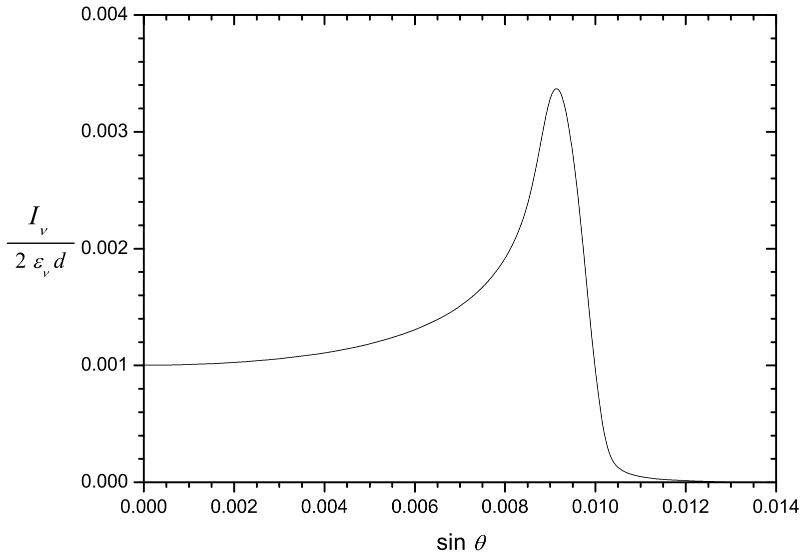

Figure 3: Radial profile with $\delta = \Delta/R = 0.1$ and $R/d = 0.01$ convolved with a Bessel function with HPBW=5×10^{-4} rad.

i.e. $\mathcal{E}_\nu = 2\pi\sqrt{\pi}\frac{\Gamma(\frac{\alpha+3}{2})}{\Gamma(\frac{\alpha+4}{2})}\tilde{\varepsilon}_\nu$ and the shell volume is as before $V = \frac{4\pi}{3} f R^3$, $f = 1-(1-\delta)^3$.

2. IDL CODES

2. 1. DIRECT PROBLEM: SIMULATION OF OBSERVATIONS OF SHELL SUPERNOVA REMNANTS WITH RADIO TELESCOPE

If we observe a shell SNR with a radio telescope, picture we get is a convolution of real intensity of radiation of the SNR and power pattern of the telescope, so we get "convolved" intensity (Fig 3). When simulating this convolution numerically, one must choose an expression for real intensity and an expression for power pattern of a radio telescope. In our case, expressions for intensities are (6) and (14), according to the models. For power pattern $P_n(\theta)$, two possible cases have been chosen: Gaussian approximation and approximation with Bessel function of the first kind. Each pattern has defined half power beam width (HPBW). Usually one takes HPBW=$1.02\frac{\lambda}{D}$ from Bessel function approximation (D is diameter of radio telescope and λ is wavelength, see Rohlfs and Wilson 1996, Urošević and Milogradov-Turin 2007). Because of technical limitations, we must consider that power pattern takes zero value for angles greater than some critical angle θ_c. In the case of Gaussian approximation of power pattern, θ_c of 5 sigmas (σ = HPBW$/(2\sqrt{\ln 2})$), while for the approximation with Bessel function θ_c of 8 HPBW has been chosen.

Expression that is used for numerical simulation of convolution of intensity of radio emission from a SNR and power pattern of a radio telescope is:

$$I_\nu^{\rm conv}(\theta_0) = \frac{\int\int_{\rm intersection} I_\nu(\theta)\, P_n(\theta^{'}) \sin\theta d\theta d\varphi}{2\pi \int_0^{\theta_c} P_n(\theta) \sin\theta d\theta}. \quad (16)$$

Angle $\theta^{'}$ is related to other angular parameters through following relation of spherical trigonometry:

$$\cos\theta^{'} = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos\varphi. \quad (17)$$

Region of double integration in numerator of the expression (16) is the intersection between regions where two convolving functions $I_\nu(\theta)$ and $P_n(\theta)$ are defined. That double integration is performed by the IDL function `INT_2D`. Integration in denominator of the expression (16) is performed by the IDL function `QROMB`. Integrations in the expression (14) are performed by the IDL functions `QROMB` and `INT_TABULATED`, as well as by "handwritten" function that calculates definite integrals using rectangular method.

In the case of first model (with constant emissivity), user of the program enters parameters C_ν, θ_1, θ_2, as well as parameter of antenna HPBW, and the program performs a convolution. In the case of second model (with radial magnetic field), user also enters an additional parameter of object - spectral index α.

2. 2. INDIRECT PROBLEM: FITTING MODEL TO OBSERVED PROFILE OF A SUPERNOVA REMNANT

Indirect problem is the following: user enters observed radial emission profile of a shell SNR in the form of a table, as well as the parameter HPBW, and the program should find the best values for parameters C_ν, θ_1 and θ_2 in the case of first model, or C_ν, θ_1, θ_2 and α in the case of second model, by fitting the chosen model to the entered data. This is performed by the iterative IDL procedure `CURVEFIT`. To perform this procedure, user has to estimate initial values for the parameters. That can be done using observed radial profile and the expression (6) or (14). Initial value for spectral index α can be taken to be 0.5. This parameter is, however, better to be held fixed (assuming that it is known from spectra). In addition to finding the best values of the parameters, the program also calculates their errors (i.e. standard deviations). Finally, the program calculates the flux density of an SNR.

The program has been tested with artificially generated data. Results of the application of the program to the real data will be given elsewhere.

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