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ATTRIBUTION OF SURVEY OBSERVATIONS TO KNOWN SOLAR SYSTEM BODIES

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Abstract. Already known solar system bodies, mostly asteroids, are unavoidably observed when a survey collects data from a large portion of the sky. It is thus essential to separate the observations belonging to these bodies from the rest. The attribution problem is a special case of the more general class of identification problems, pertaining to the situation in which a set of observations have been assigned to an object resulting in a least squares orbit, while the others are not enough for this purpose, but still need to be assigned if possible. The present paper deals with a strongly asymmetric case when an asteroid has a well constrained orbit, while the additional data to be attributed are just a few, typically a single tracklet, that is a very short arc of astrometric observations assembled by the observer.

1. THE PROBLEM

Surveys unavoidably observe known solar system objects, mostly asteroids. Separating observations of moving objects from the rest is thus essential for a number of obvious reasons, like to avoid claiming as a new discovery some well known object or to reduce the dataset to be analysed, but also to improve known asteroid orbits and to use residuals of the observations for statistical quality control of both, observations and orbits.

The procedure briefly presented in the following can be used in two cases: for new observations, just obtained by some telescope, to be identified with the objects with orbit computed with previous data, and for old observations stored in some archive, which had not been previously used to compute orbits, to be identified with objects discovered later.

In general, any procedure of assigning observations to objects belongs to a class of *i*dentification problems. An important subgroup of such procedures is assigning observations to known objects, and this represents the so-called *t*he attribution problems. More specifically, is the asteroid case, the attribution problem pertains to the situation in which a large enough set of observations have been assigned to one object to compute a least squares orbit, while the others are not enough for the purpose but still need to be assigned if possible. In practice this includes also the cases when the second object already has an orbit but weakly constrained, or the case when even the first object still does not have a well constrained orbit (Milani 1999; Milani and Gronchi 2010, Chap. 7).

In this paper, following Milani et al. (2011), we are dealing with even more particular case when previously known asteroids have well constrained orbits, while the additional data to be attributed are very few, typically a very short arcs of observations, assembled by the observer without using an orbit to fit the data, which is called a *tracklet*. Challenge in this case is the asymmetry of the data - few observations per object, but much more accurate, from the state-of-the-art surveys vs. many observations per well known object, but of lower accuracy, from the historic data.

In reality the data are often severely biased and a reliable error model (including RMS and correlations) for the astrometry is not yet available. Practical consequences of this situation are twofold:

- the better the orbit is constrained by the previous data, the worse is the effect of the biases on the ephemerides predicted from the orbit and its covariance matrix.
- a few observatories and next generation surveys produce astrometry with 0.10 - 0.15 arcsec accuracy. If such high quality data were fitted to orbits computed with historic biased data, the two data sets would appear statistically incompatible.

2. THE SOLUTION

To the above problem we propose a solution which consists ion two steps: we first devise a new statistical quality control and apply it *asymmetrically* to the old and new data; then we apply a debiasing procedure to observations stored in the MPC archives removing the most dangerous form of bias due to systematic errors in the star catalogs (Chesley et al. 2010).

2. 1. THE ATTRIBUTION ALGORITHMS

An *attribution problem* requires first an already determined orbit, with a vector of orbital elements at epoch t_1, and a corresponding *covariance matrix* describing their uncertainty in the linearised approximation. The second element of the problem is a vector of *observables*, measured at another epoch t_2. The third element is the prediction at time t_2, in the same space of observables, computed from the known orbit. The orbital elements uncertainty can be propagated to the space of the observables and the resulting covariance used to assess the likelyhood of the prediction being compatible with the hypothesis that the observation belongs to the same object.

For reasons of efficiency in handling large datasets (of both orbits and tracklets), this has to be tested by a sequence of filters, providing an increasing likelyhood of identification at the price of an increasing computational effort.

2. 2. FILTER 1

The first step is to compare the observations with prediction based on the known orbit in terms of the angular coordinates (α, δ), e.g., right ascension and declination.

The difference of computed and observed position is projected on the *tangent space to the celestial sphere*. From the covariance matrix of predicted position we compute the *normal matrix* and the corresponding *confidence ellipse*. The observation also has its own uncertainty, expressed by the covariance matrix and by the normal matrix. Thus there is another confidence ellipse for the observed angular position. The attribution requires that, for a reasonable value of the confidence parameters, the two ellipses intersect.

2. 3. FILTER 2

In the second step, in addition to positions that passed filter 1, we take into account also the velocities. Here we take advantage of the fact that the tracklet can be compressed into an *attributable* vector $(\alpha, \delta, \dot{\alpha}, \dot{\delta}) \in \mathbf{R}^4$ by a linear fit of the individual observations in the tracklet, which results in a best fit for the average time t_2 with a 4×4 normal matrix. The prediction from the state at time t_2 can also be performed in the attributables space, resulting in a nominal prediction with its 4×4 normal matrix. The compatibility between the two can be described by the *attribution penalty* K_4, which corresponds geometrically to testing the intersection of the confidence ellipsoids, with $\sigma_4 = \sqrt{K_4}$ playing the role of the confidence parameter.

2. 4. FILTER 3

The proposed attribution which have passed the test of filter 2 need to be confirmed by a least squares fit with all the observational data, both the ones already used to compute the previously available orbit and the ones of the additional tracklet. The newly obtained solution is then passed to the rigorous *statistical quality control* based on the following 10 metrics:

- RMS normalized root mean square of the astrometric residuals
- $BIAS_\alpha, BIAS_\delta$ bias, i.e. average, of the residuals,
- $SPAN_\alpha, SPAN_\delta$ first derivatives of the residuals,
- $CURV_\alpha, CURV_\delta$ second derivatives of the residuals,
- $ZSIGN_\alpha, ZSIGN_\delta$ third derivatives of the residuals,
- $RMSH$ RMS of photometric residuals in magnitudes.

2. 5. ATTRIBUTION QUALITY CONTROL

This statistical quality control has been shown to be very effective in recovering the true identifications and removing most false ones (Milani et al. 2008). However, tuning of the control parameters is case dependent. When using simulations one a priori knows which identifications are the right ones and which are false. By using this information it is possible to adjust the control parameters for an optimal performance with good efficiency and accuracy.

On the other hand, when attributing new observations to real objects with strongly overdetermined orbits, such as numbered asteroids, we cannot use tight controls, otherwise we would get the paradoxical result that, due to the lower accuracy of the historic data, the already computed orbit should be discarded even before adding new data. If we use high values of the controls we may not reject false attributions.

Our solution is to use not just the values of the metrics, but also the amount by which they change as a result of the proposed attribution. E.g., for the RMS metrics, we accept an attribution only if the increase resulting from the attribution is small (we currently require an increase by < 0.15).

Since tracklets contain few observations, typically $2 \div 8$ and the previous data set is large, typically with tens or even hundreds of observations, the new data may have little effect on the statistical properties of the complete set of residuals. Thus, we need also to separately consider the residuals of the attributed observations. Metrics, RMS, BIAS and SPAN (typical tracklets have no significant curvature, even less Z-sign). Currently used control values for these quantities are in the range $2 \div 3$. The procedure of attribution is recursive, that is tracklets are added one by one, proceeding in order from the most likely (as measured by the penalty K_4). Once the attribution has passed all the quality controls, the orbit fitted to all the data, including the new tracklet, becomes the reference one for the asteroid.

The above procedure is quite robust, but, because of the stochastic nature of the observational errors, the identifications are nevertheless probabilistic.

3. DEBIASING THE HISTORIC DATA

Systematic errors in observations of solar system objects are for the most part due to the presence of systematic errors in the catalogs of stars used for the astrometric reduction. The normalized biases of astrometric residuals exhibit strong asymmetries even for well determined orbits. The worst case is with declination, for which the mean value of the normalized biases is 2.17, the standard deviation is 1.86, and the distribution of biases differs from a Gaussian; for right ascension the mean is 0.12 and the standard deviation is 0.77, while the shape of the distribution of biases is not too different from a normal distribution. For the other controls, such as span, curvature and Z-sign, there are similar signatures.

A method has been recently proposed to debias the astrometric asteroid data by using the measured regional biases of the catalogs computed as differences, averaged over a given portion of the sky, with respect to a catalog considered to be the most accurate (Chesley et al. 2010). The 2MASS star catalog has been used as reference, because it is of good accuracy, dense enough and covers the entire sky. The biases are given as average differences between a given star catalog and the corresponding entries in 2MASS over patches of a tessellation of the celestial sphere.

Having these regional biases of catalogs, we have built the error model for debiased asteroid observations. A weight inversely proportional to the RMS of the debiased residuals (for the same observatory and the same catalog), as given in Table 6 of Chesley et al. (2010), was assigned to the observations for which we knew the catalog used in data reduction,; as a matter of fact, the weight was actually $1/(2 \cdot RMS)$. For the observations performed by photographic and CCD techniques with no catalog information, we assigned the weight of $1/1.5$ arcsec^{-1} for data after 1950.

After applying such a debiasing procedure we obtained much less asymmetric distribution of biases. The mean of the normalized biases for declination is now 1.00 (with standard deviation 1.26), and 0.05 for right ascension (standard deviation 0.80). As one can easily infer, the declination is still biased, but we reduced the effects by a factor 2; the right ascension has biases at the level of the quality of the best catalogs, hence, for now, it cannot be better. In conclusion, the debiasing significantly improves the results obtained by the asymmetric attribution procedure, but what we hitherto achieved still needs to be improved.

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