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BEHAVIOR OF GRAVITO-MHD WAVES AT SOLAR PHOTOSPHERE-CORONA BOUNDARY

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Abstract. We study the boundary-value problem for waves propagating through the solar corona that are generated by standardly observed gravito-acoustic modes at the photosphere-corona interface. The applied model for the photosphere-corona system assumes two isothermal plasma regions with constant Alfvén speeds and horizontal magnetic field, and separated by a horizontal boundary. The analysis of the obtained wave-propagation condition shows the domains of propagation for gravitationally modified slow and fast MHD modes in the applied model.

1. INTRODUCTION

The solar interior, the convective zone primarily, is a source of permanent perturbations resulting in wave modes that propagate in various directions (see e.g. Christensen-Dalsgaard et al. 1998a, 1998b). Some of them enter the corona where they play a significant role in nonthermal coronal heating mechanisms (see e.g. Tirry et al. 1998).

In this paper, we start from a typical helio-seismic wave pattern power spectrum observed by the GONG (The Global Oscillation Network Group) network and analyze its consequences in terms of linear MHD waves that can cross the photosphere-corona interface and propagate further into the corona. Our analytical approach is based on an idealized model of the solar plasma with relevant physical parameters typical of solar conditions. The model comprises two domains describing the photospheric and coronal conditions respectively assuming isothermal plasma and a horizontal magnetic field with constant Alfvén speed. The two domains are separated by a horizontal plane where proper boundary conditions imposed on the unperturbed basic state quantities as well as on the perturbation-wave parameters have to be met. As a result, the obtained dispersion relation indicates the domains where waves generated below the boundary can cross it and propagate further into the corona (see e.g. Cambell & Roberts, 1989; Miles & Roberts, 1992; Pintér et al. 1998, 1999).

2. STARTING MHD EQUATIONS

2. 1. THE BASIC STATE

The initial basic state is defined by the model itself which implies a stationary, static (i.e. the hydrostatic equilibrium), gravitationally stratified isothermal plasma with embedded horizontal magnetic field.

Thus, in Cartesian coordinates, we have:

$$\begin{aligned} &\vec{g} = -g\hat{e_z}, \quad \text{g=const}, \\ &\vec{B}_0 = B_0(z)\hat{e_x}, \quad \rho_0 = \rho_0(z), \quad p_0 = p_0(z). \end{aligned} \tag{1}$$

The assumptions regarding the model are:

$$\begin{aligned} &\text{constant sound speed:} \quad V_s^2 \equiv \gamma \frac{p_0(z)}{\rho_0(z)} = \text{const}, \\ &\text{constant Alfvén speed:} \quad V_A^2 \equiv \frac{B_0^2(z)}{\mu_0 \rho_0(z)} = \text{const}. \end{aligned} \tag{2}$$

The hydrostatic equilibrium is given by:

$$\frac{d}{dz}\left[p_0(z) + \frac{B_0(z)^2}{2\mu_0}\right] + \rho_0(z) g = 0$$

or

$$\frac{d}{dz}\log\rho_0(z) + \frac{1}{H} = 0 \quad \text{with} \quad H \equiv \frac{V_s^2}{\gamma g} + \frac{V_A^2}{2g} = \frac{1+\beta}{\beta} H_0, \tag{3}$$

where:

$$\gamma = \frac{c_p}{c_v} = \frac{5}{3}, \quad \beta = \frac{p_0(z)}{p_{0m}(z)} = \frac{2v_s^2}{\gamma v_A^2} = \text{const}, \quad H_0 = \frac{v_s^2}{\gamma g} = \text{const}. \tag{4}$$

The solutions for density and magnetic field profiles that follow from Eqs. (3) and (4) are readily obtained as:

$$\rho_0(z) = \rho_0(0)\exp\left(-\frac{z}{H}\right), \quad B_0(z) = B_0(0)\exp\left(-\frac{z}{2H}\right). \tag{5}$$

2. 2. MHD EQUATIONS

Plasma dynamics of small amplitude waves is described by the standard set of non-linear MHD equations for ideal plasma:

$$\begin{aligned} &\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0, \qquad \frac{dp}{dt} - \gamma\frac{p}{\rho}\frac{d\rho}{dt} = 0, \\ &\rho\frac{\partial\vec{v}}{\partial t} + \nabla p - \rho\vec{g} - \frac{1}{\mu_0}\left(\nabla\times\vec{B}\right)\times\vec{B} = 0, \\ &\frac{\partial\vec{B}}{\partial t} - \nabla\times\left(\vec{v}\times\vec{B}\right) = 0, \qquad \nabla\cdot\vec{B} = 0, \end{aligned} \tag{6}$$

which are perturbed by taking any unknown physical quantity $f(x,y,z,t)$ in Eqs. (6) as a sum:

$$f(x,y,z,t) = f_0(z) + \delta f(x,y,z,t), \tag{7}$$

where perturbations $\delta f(x,y,z,t)$ have the following form:

$$\delta f(x,y,z,t) = f_1(z)\exp(-i\omega t + ik_x x + ik_y y), \tag{8}$$

while the unperturbed quantities $f_0(z)$ satisfy the hydrostatic balance equation Eq. (3) of the basic state.

2. 3. LINEARIZED EQUATIONS

Eqs. (6) linearized according to Eq. (7) can be cast in the following form (Pintér et al. 1998, 1999):

$$D\frac{d\xi_{1z}}{dz} = C_1\xi_{1z} - C_2P_1, \quad D\frac{dP_1}{dz} = C_3\xi_{1z} - C_1P_1, \tag{9}$$

where:

$$\begin{aligned} D(z) &= \rho_0(z)\left(v_s^2+v_A^2\right)\left(\omega^2-\omega_A^2\right)\left(\omega^2-\omega_c^2\right), \\ C_1(z) &= \rho_0(z)g\omega^2(\omega^2-\omega_A^2), \\ C_2(z) &= (\omega^2-\omega_A^2)(\omega^2-\omega_s^2) - k_y^2(v_s^2+v_A^2)(\omega^2-\omega_c^2), \\ C_3 &= \left[\rho_0(z)(\omega^2-\omega_A^2) + g\frac{d\rho_0(z)}{dz}\right]D(z) + g^2\rho_0^2(z)(\omega^2-\omega_A^2)^2, \end{aligned} \tag{10}$$

with:

$$\omega_A^2 = v_A^2k_x^2, \quad \omega_c^2 = v_c^2k_x^2 \quad \omega_s^2 = v_s^2k_x^2, \quad v_c^2 = \frac{v_A^2v_s^2}{v_A^2+v_s^2}k_x^2 \tag{11}$$

Here, ξ_{1z} is the $z-$component (i.e. the vertical component) of the Lagrangian displacement and P_1 is the total pressure perturbation, i.e.:

$$\xi_{1z} \equiv -iv_{1z}, \quad P_1 \equiv p_1 + p_{m1}.$$

2. 4. SOLUTIONS AND DISPERSION RELATION

Taking into account Eq. (5), the equations Eqs. (9) and (10) allow the following solutions for the vertical displacement ξ_{1z} and total pressure perturbation P_1:

$$\begin{aligned} \xi_{1z} &= \xi_{1z}(0)\exp\left(z/2H\right)\exp ik_zz, \\ P_1 &= P_1(0)\exp\left(-z/2H\right)\exp ik_zz. \end{aligned} \tag{12}$$

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Eqs. (9) with solutions Eq. (12) now yield the dispersion relation:

$$\begin{aligned} k_z^2 = &\frac{\omega^2(\omega^2-\omega_A^2) - \frac{g}{H}\omega_A^2}{(V_A^2+V_s^2)(\omega^2-\omega_A^2)(\omega-\omega_c^2)}\omega^2 - \frac{1}{4H^2} \\ &-\frac{(V_A^2+V_s^2)(\omega^2-\omega_A^2)(\omega-\omega_c^2) + g^2(\omega^2-\omega_A^2) - \frac{g}{H}(V_A^2+V_s^2)(\omega^2-\omega_c^2)}{(V_A^2+V_s^2)(\omega^2-\omega_A^2)(\omega-\omega_c^2)}k_\parallel^2, \end{aligned} \tag{13}$$

where $k_\parallel^2 = k_x^2 + k_y^2$ is the total horizontal component of the wave vector $\vec{k}$.

The criterion for wave propagation now follows from Eq. (13) as $k_z^2 > 0$ with initially prescribed ω, $k_\parallel$ and $k_x = k_\parallel \cos\phi$.

2. 5. WAVES IN THE CORONA

We consider the following scenario for waves entering the corona through the boundary interface with the photosphere:

- Wave parameters ω and $k_{\|}$ are determined from observations of gravito-acoustic modes at the photosphere-corona boundary as seen in Fig.1-Top.
- These modes are refracted through the boundary if $k_z^2 > 0$ (Eq. (13)) in the corona.
- In what follows, we take the following values for physical parameters:
  - Physical parameters of the corona: $\rho_c = 10^{-11}$kg m^{-3}, $T_c = 10^6$K, $B_c = 3\times10^{-2}$T yielding $V_{sc}^2 = 2.7\times10^{10}$m^{2}s^{-2} and $V_{Ac}^2 = 7\times10^{13}$m^{2}s^{-2} implying a low β plasma: $\beta \equiv 2V_{sc}^2/(\gamma V_{Ac}) \approx 5\times10^{-4}$ (here $\gamma = 5/3$).
  - Physical parameters of the photosphere: $\rho_0 = 10^{-4}$kg m^{-3}, $T_0 = 6\times10^3$K, $B_0 \approx 0$ yielding $V_{s0}^2 = 1.6\times10^8$m^{2}s^{-2}, $V_{A0}^2 \approx 0$.
  - The gravitational acceleration and the solar radius at the interface are $g_\odot = 273$ m s^{-2} and $R_\odot = 7\times10^8$m respectively.

Our further analysis will be restricted to propagation of pure modes only. Eq. (13) for k_z^2 has namely two singularities:

$$\omega^2 = \omega_A^2 \equiv V_A^2 k_{\|}^2 \cos^2\phi \quad \text{and} \quad \omega^2 = \omega_c^2 \equiv V_c^2 k_{\|}^2 \cos^2\phi, \tag{14}$$

in whose neighborhood the condition $k_z^2 > 0$ for wave propagation is surely satisfied. These two domains are thus related to fast and slow MHD mode respectively i.e. in the coronal low plasma-β, they are:

$$\begin{aligned} &\text{fast mode waves:} && \omega^2 \approx V_{Ac}^2 k_{\|}^2 \cos^2\phi, \\ &\text{slow mode waves:} && \omega^2 \approx V_{sc}^2 k_{\|}^2 \cos^2\phi. \end{aligned} \tag{15}$$

In the domain of fast mode waves, Eq. (13) reduces to:

$$k_z^2 = \frac{2g^2 k_{\|} \sin^2\phi}{V_{Ac}^2(\omega^2 - V_{Ac}^2 k_{\|}^2 \cos^2\phi)}, \tag{16}$$

indicating the wave propagation condition:

$$V_{Ac}^2 k_{\|}^2 \geq \omega^2 > V_{Ac}^2 k_{\|}^2 \cos^2\phi \geq 0. \tag{17}$$

In the slow mode domain, Eq. (13) becomes:

$$k_z^2 = \frac{V_{sc}^4 k_{\|}^2 \cos^4\phi - g^2}{V_{Ac}^2(\omega^2 - V_{sc}^2 k_{\|}^2 \cos^2\phi)}, \tag{18}$$

which gives the wave propagation condition:

$$V_{sc}^2 k_{\|}^2 \geq \omega^2 \geq 0. \tag{19}$$

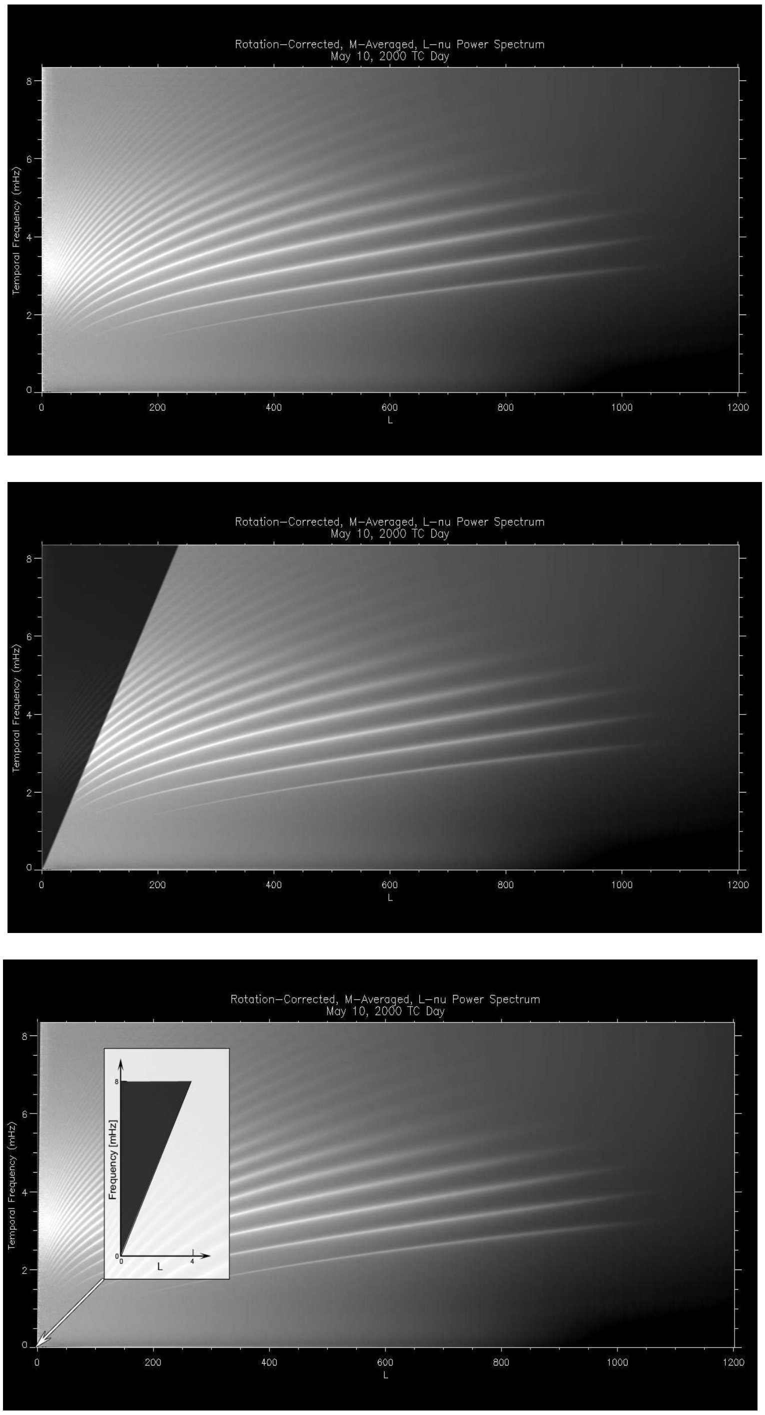

Figure 1: Top: the recorded helio-seismic p-mode spectrum (http://gong.nso.edu). Middle: the domain of induced fast mode waves in the corona. Bottom: the domain of induced slow mode waves in the corona.

Finally, we apply the two wave propagation criteria Eqs. (17) and Eq. (19) to a particular solar global oscillation spectrum registered by the GONG (Global Oscillation Network Group, http://gong.nso.edu/) shown in Fig.1-Top. The vertical coordinate axis reads the wave frequency $\nu \equiv \omega/2\pi$ in [mHz] while the horizontal axis gives the wave-order L, a dimensionless number related to the horizontal wave-number $k_{\parallel}$ as $L \equiv k_{\parallel} R_{\odot}$.

The boundary condition for harmonic waves at the photosphere-corona interface requires continuity of the horizontal component of the wave-number $k_{\parallel}$ and continuity of the wave frequency ω; the only wave parameter that changes across the interface is the normal component k_z of the wave vector $\vec{k}$. Thus, the domain of wave frequencies ν and wave orders L for helio-seismic modes in Fig.1-Top applies also to the induced waves in the corona i.e. it can be taken as the boundary condition for driving waves into the region above the interface.

The wave propagation domains according to criteria Eqs. (17) and Eq. (19) now take the following form in terms of ν and L:

$$\begin{aligned} &2L \geq \nu[\text{mHz}] \geq 0 && \text{for the fast mode and} \\ &3.6 \times 10^{-2} L \geq \nu[\text{mHz}] \geq 0 && \text{for the slow mode.} \end{aligned} \tag{20}$$

Fig.1-middle and Fig.1-bottom show the domains of propagation for the fast and slow mode respectively where the conditions given by Eq. (20) are satisfied. The darkened sections in Fig.1-middle and Fig.1-bottom correspond to domains which cannot generate the indicated wave-modes propagating in the corona.

3. CONCLUDING REMARKS

The considered model of the coronal medium, albeit simplified and idealized, offers some realistic insight in effects of solar global oscillations on wave generation and their propagation in the corona. Results of our analysis given in Fig.1 show that the fast and slow mode MHD waves are easily excited at the photosphere-corona interface by the helio-seismic p-mode oscillations. These waves are therefore a mechanism of energy transport from the solar interior into the corona and they are considered as the primary source for the coronal heating.

Comparing Fig.1-middle and Fig.1-bottom, we see that the fast mode covers a broader domain of frequencies and wave orders L than the slow MHD mode.

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