

## COSMIC NAVIGATION AND INERTIAL NAVIGATION SYSTEM

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**Abstract.** In this paper, we expose general performance of navigation, compared and opposed to astrodynamics and marine navigation, observing methods for determining the orbit and operating in the Space and on the Earth and, also, pre-satellite and post-satellite era of cosmic navigation..

### 1. INTRODUCTION

Navigation has been present for thousands of years in some form or another. The birds, the bees, and almost everything else in nature must be able to navigate from one point in space to another. For people, navigation had originally included using the sun and stars. Over the years we have been able to develop better and more accurate sensors to compensate for our limited range of senses. It is very useful to expose general presentation on navigation, to compare and to oppose astrodynamics and marine navigation, observative methods and methods for defining trajectory and managing into the outer space and on Earth as pre-satellite and post-satellite cosmic navigation.

### 2. COSMIC NAVIGATION

The problem of defining trajectory or orbit we can illustrate with a simple navigation task.

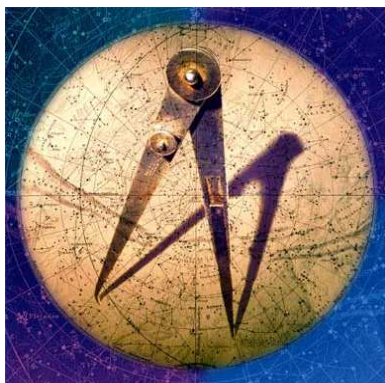
**Task 1.** *Two points A and B are given and the task is to define - calculate the course C and the distance D.*

In marine navigation the solution where the course C and the distance D are accepted as an arc of a large circle through the points A and B, with a center of the Earth related to it. Here we had introduced the term of optimization which was applied into the aviation later on. In the cosmic navigation such example represents the calculation of the flight of trajectory from the Earth over to the Mars. Firstly, you find the simplest solution, say Keplers orbit can be the demanded trajectory, so the task gradually became more complexed -we take into consideration the perturbation, the active segments of the artificial vehicle the convenient conditions, instrument errors... In the pre-satellite cosmic navigation even if it is well -enough solved task of the defining the orbit by watching it, the calculation of the orbit wasn't anything more

than theoretical and abstract exercise or mathematical abstraction for the ordinary people.

**Task 2.** *In the condition of given starting position  $A$ , course  $C$  and preliminary (for the given velocity and time) the distance  $D$ , should be found trajectory - orbit (by integration), i.e. to define the position  $B$ .*

In the cosmic navigation simple analytical integrals of Kepler's orbits are used or pretty more complexed effects are taken into consideration - perturbation, head resistance, relativistic effects etc. by numeric integration or through the order and integration into quadrature. The difference between the cosmic and the ground-based navigation, can be most obviously seen from the difference in the observing practice. The ground-based navigation means the immediate improvement of the instruments (from the sextant of one arcminute accuracy through a microscope-micrometer of tenths of arc second to the electronic accessories of hundredth of an arc second). Where it stops or reaches the limits of the accuracy of the navigation on the surface of the Earth, the observing accuracy of the cosmic navigation begins, and the further improvement can be reached both through the working out of new methods (interferometry, laser measuring of the distance and so on) and also through the working out of the special methods of the observation processing and removing the errors, say their optimization. Finally, the task of orbit correction is to remove the systematic errors from the observation and minimization of coincidental errors. To define the integration constants the time which passed from the beginning of the motion is important, and as the time passes, by the rule the accuracy of the defining different observing values rapidly grows. Even if statistical and recurrent methods are used, the results of the integration with the better accuracy can be achieved.



(a) Navigation is the science



(b) Last shuttle on pad, Sept.2010

Figure 1: The problem of defining trajectory... (a) and (b).

The procedure of more accurate derivation of approximate orbit, with the relevant supply of observing data, can be widened into three dimensional case of the navigation on the surface of the Earth (and if we have the data on velocity it could be six dimensional) based on the strong point. However, in the accurate cosmic navigation in

the first phase of the correcting orbit such procedure can cause more serious errors and shouldn't be applied. So-called differential navigation, which is the most favourable method of navigators in the aviations, in the cosmic navigation couldn't be successfully applied.

However, managing and physical correction of the orbit toward the goal or achievement of goal got its place in the applied cosmic navigation. Algorithms, by which the necessary data for the pilots of the artificial vehicles are needed, are similar to mathematical algorithms for correcting orbits. Intermedial and terminal managing are the terms formed in the process of developing the navigation and must be clearly defined, especially in the problems of rendez-vous, intercept and collision of the artificial vehicles. It doesn't have to emphasise that the presence of modern and specialized computer gyroscopic systems, both on the vehicle and on the dislocated control point, are the conditions of existence and survival of the cosmic navigation.

### 3. INERTIAL NAVIGATION SYSTEM

Studies of free and forced motions of spinning rigid bodies of various geometries have led to the development of important scientific instruments (gyroscopes, etc.) and to the concept of spin stabilization of modern spacecraft. Inertial guidance system is electronic system that continuously monitors the position, velocity, and acceleration of a vehicle, usually a submarine, missile, or airplane, and thus provides navigational data or control without need for communicating with a base station. The basic components of an inertial guidance system are gyroscopes, accelerometers, and a computer. The gyroscopes provide fixed reference directions or turning rate measurements, and accelerometers measure changes in the velocity of the system. The computer processes information on changes in direction and acceleration and feeds its results to the vehicle's navigation system. There are two fundamentally different types of inertial navigation systems: gimbaling systems (the Stable Platform) and

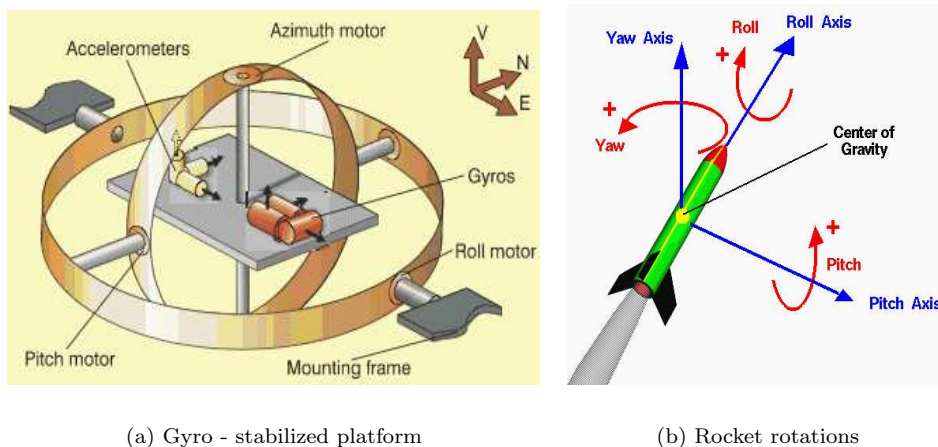


Figure 2: The basic components (a) and body axes (b).

strapdown systems. A typical gimbaling inertial navigation system, such as might be used on board a missile, uses three gyroscopes and three accelerometers. The three gimbal-mounted gyroscopes of the stable platform and the three accelerometers oriented in mutually perpendicular directions can supply all the information for establishing the motion of a rigid body, and the high degree of accuracy. The platform, in turn, is mounted on two gimbals which allow it three degrees of angular freedom. If the platform is perfectly balanced and the bearings are frictionless, no torque will be experienced by the platform, and its orientation will be maintained regardless of the motion of the carrier. However, due to unbalance and friction which cannot be eliminated entirely, disturbing torques will be felt by the platform. It is the function of the gyros to sense this disturbance and, through a servo system, counteract the disturbing torque to produce essentially a torque-free system.

For example, if a rotation around one axis is sensed by a gyro, the signal is measured by the pickoff and transmitted to the associated gimbal servo motor. This is called the *servo-loop*. The signal of the angle is measured by the resolver on the same gimbal. The platform with the gyros and accelerometers itself keeps its orientation to the inertial space and therefore it moves with reference to the ground one time in 24 hours in a direction opposite to the earth's rotation. These errors as well as the drift and acceleration errors have to be compensated by a computer program.

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External disturbing torque  $\Rightarrow$  Platform rotates  $\Rightarrow$  Gyro rotates (opposite)  $\Rightarrow$  Signal voltage  $\Rightarrow$  Motor operates  $\Rightarrow$  Platform rotates to original leveled position

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To obtain an understanding of how the inertial navigator works, we assume that the vehicle starts at the equator and that the plane of the stable platform is horizontal with the arrow pointing in the  $N$  polar direction. If the vehicle moves towards the north along a longitude, and the accelerometer table is always kept normal to the geocentric radius  $r$ , the  $N$ - $S$  accelerometer will measure the acceleration  $a_x$ . The proper rate of rotation of the table about the  $y$  axis is then  $\omega_y = v_x/r$ , where  $v_x$  is determined from the first integral of  $a_x$ . The latitude motor then rotates the table at a rate  $\omega_y$  to keep the  $N$ - $S$  line on the table normal to  $r$ . Due to rotation of the Earth towards the east, the  $E$ - $W$  line of the table must be rotated by the longitude motor to unwind the earth's rotation. Since during the motion of the vehicle the orientation of the stable platform remains fixed in inertial space (towards the N star) the required rotation of the accelerometer table about the  $x$  axis of the stable platform at any latitude is  $\Omega$  or  $15^\circ/\text{hr}$ . To this rotation must be added the rotation about the platform  $x$  axis due to the  $E$ - $W$  motion of the vehicle relative to the original longitude. By integrating the output of the  $E$ - $W$  accelerometer and dividing by  $r \cos \lambda$ , the additional rotation to maintain the  $E$ - $W$  line of the table normal to  $r$  is  $\omega_x = v_y/r \cos \lambda$ . These computations are performed by a computer which must be an integral part of the inertial system. Thus the inertial navigator must consist of the stable platform, accelerometers with integrators, a computer to compute the proper angular rates of the table due to vehicle motion, a clock to unwind the earth's rotation, and the servomotors to actually carry out these functions.

The original applications of Inertial Navigation System (INS) technology used stable platform techniques. Platform systems are still in use, particularly for those applications requiring very accurate estimates of navigation data, such as ships and submarines. However, modern systems have removed most of the mechanical complexity

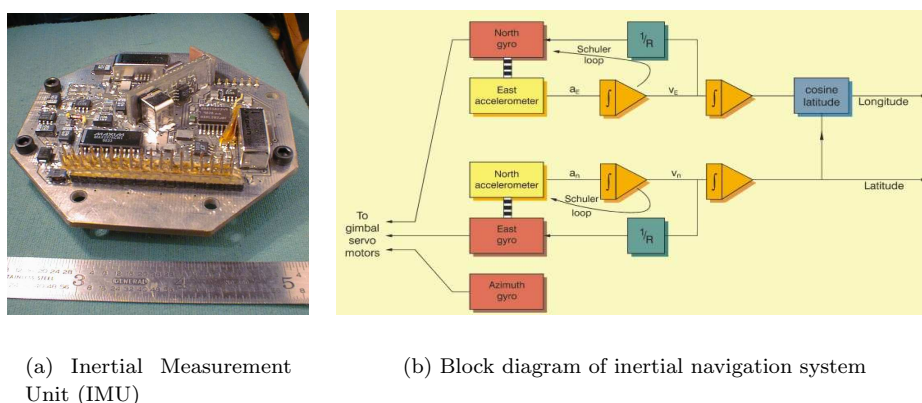


Figure 3: IMU is the main component of inertial navigation systems (a) and Calculation of the orientation and position (b).

of platform systems by having the sensors attached rigidly, or "strapped down", to the body of the host vehicle. The potential benefits of this approach are lower cost, reduced size, and greater reliability compared with equivalent platform systems. The major disadvantage is a substantial increase in computing complexity. In a strap-down inertial system, the three accelerometers and three gyros are hard-mounted to the vehicle. As the vehicle moves, the gyro and accelerometer outputs are read approximately every 1/100 of a second, the new direction is established, and the velocity change and position change are calculated for the past 1/100 s. These changes are then summed with previous data to give total velocity and position changes. The computation is very complicated, especially for the coordinate transformations necessary to maintain the mathematical equivalent of the gimbal position in inertial space. Strap-down systems also need to be Schuler-tuned, where this operation is accomplished mathematically in the navigation computer. Two major contributors to the practicality of the modern strap-down system were the solid-state high-speed computer and the development of gyros that do not require torquers and have outputs that are insensitive to vehicle accelerations, such as the ring laser gyro. Nearly all applications now use strap-down systems. However, at present the most precise navigators are still gimballed systems. In a strap-down system, the "vertical accelerometers" cannot be deleted since the accelerometers are tied to the vehicle and may assume any attitude.

### 3. 1. CALCULATION OF ORIENTATIONS AND POSITIONS

#### Calculating orientation:

The vehicle's orientation in the global coordinate system is simply represented by the rotation angles  $(\phi_x, \phi_y, \phi_z)$ , which is detected and generated by the three gyroscopes.

#### Calculating position:

Given the acceleration  $\vec{a} = [a_x, a_y, a_z]$  in the global coordinate system generated by the

three gyroscopes, we can find the velocity and position information by integrations:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}, \quad \vec{v} = \vec{v}_0 + \int_0^t \vec{a} dt, \quad \vec{r} = \vec{r}_0 + \int_0^t \vec{v} dt = \vec{r}_0 + \vec{v}_0 t + \int_0^t \int_0^t \vec{a} dt^2$$

Let  $v_{NS}(t)$  and  $v_{EW}(t)$  be the velocity components in the north and east directions, respectively. Then the latitude  $\phi(t)$  and longitude  $\lambda(t)$  (in radians) at time  $t$  can be obtained as

$$\phi(t) = \phi_0 + \frac{1}{R} \int_0^t v_{NS}(\tau) d\tau, \quad \lambda(t) = \phi_0 + \frac{1}{R} \int_0^t \frac{v_{EW}(\tau)}{\cos \phi(\tau)} d\tau$$

where  $R$  is the radius of the Earth. In order to maintain the stable platform leveled, it needs to be adjusted according to its location so that it is always tangential to the Earth surface. This is realized by rotating the gimbals at an angular velocity determined by the linear velocities in  $NS$  and  $EW$  directions:

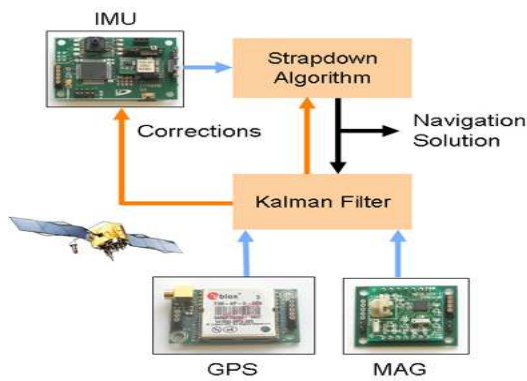
$$\dot{\theta}_{NS} = \frac{v_{NS}}{R}, \quad \dot{\theta}_{EW} = \frac{v_{EW}}{R}$$

In many modern inertial navigation systems, such as those used on commercial jetliners, booster rockets, and orbiting satellites, the turning rates are measured by ring laser gyroscopes or by fibre-optic gyroscopes. Minute errors in the measuring capabilities of the accelerometers or in the balance of the gyroscopes can introduce large errors into the information that the inertial guidance system provides. These instruments must, therefore, be constructed and maintained to strict tolerances, carefully aligned, and reinitialized at frequent intervals using an independent navigation system such as the global positioning system (GPS). Recent developments in the miniaturization of inertial instruments and GPS receiver hardware have led to the introduction of small, low cost integrated navigation systems under circumstances where GPS remains available. Under situations where GPS is unavailable or intermittent such as urban, indoor or subterranean environments, navigation performance is limited by inertial sensor performance. Given the size, power and cost constraints of miniature systems, currently only tactical grade MEMS<sup>1</sup> gyros and accelerometers (performing at around 1 deg/h and 1 milli-g bias stabilities, respectively) are suitable for use in these applications. Consequently position accuracy rapidly degrades in a tactical grade inertial/GPS system when GPS is denied.

To recover navigation accuracy in miniature systems then, it is necessary to use additional sensors (e.g., velocity meters, magnetometers, barometers) and algorithms to augment the inertial system. The size, cost and weight are roughly about 2 – 3 times better than those of the 'latest' gimballed INS, with about the same level of performance. A new feature is that many INS today contain an embedded GPS *receiver module*.

GPS and INS are ideal synergistic partners, as their error dynamics are totally different and uncorrelated. Aiding the navigation system by using information from the GPS allows improved navigation accuracy when using lower-performing gyros and

<sup>1</sup>Micro-electromechanical systems (MEMS) - the combination of mechanical functions (sensing, moving, heating) and electrical functions (switching, deciding) on the same chip using microfabrication technology

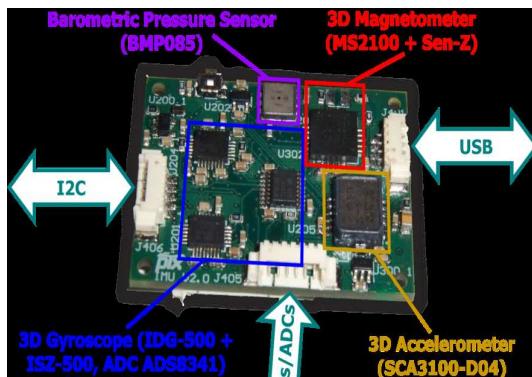


(a) INS and GPS can complement each other to enhance positioning performance

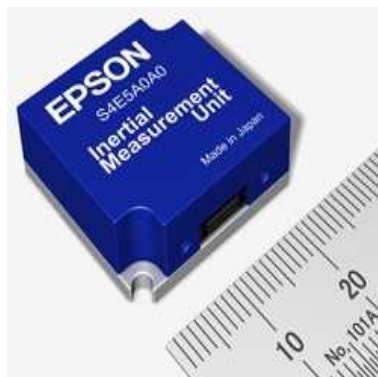


(b) The system of a mass about 600 g combines a small GPS receiver with only 1 to 2cm<sup>2</sup> micro gyros and accelerometers

Figure 4: INS and GPS are ideal synergistic partners (a) and Micro GPS/INS integrated navigation system (b).



(a) IMU - Dimensions: 30.2 x 40 mm, Weight: 15g



(b) Quartz-microelectromechanical-system sensor

Figure 5: Integration (a) and Miniaturization (b).

accelerometers, and has been responsible for the introduction of inertial navigation to many new applications, such as automobiles and guided bombs. Each technology counteracts the weakness of the other. GPS errors are bounded, but updates are available at a lower rate, and there is the possibility of jamming or of signals becoming unavailable (for example, in parts of city centers or in buildings). Inertial navigation system errors are unbounded (grow over time), but they provide higher rate updates,

and cannot be jammed. Also, the inertial navigation system can perform the autopilot function (stabilization and control) for the vehicle. Most navigation systems are no longer purely inertial but are now INS/GPS systems, and would not be able to achieve the desired performance with the inertial navigation system alone.

### CONCLUSION:

Developments in the field of variable mass systems have led to the development of important scientific instruments (gyroscopes, etc.) and to the concept of spin stabilization of modern spacecraft. The finite limits of electrical energy, thrust, and propellant on a vehicle are the driving forces that demand efficient and accurate equipment to perform the functions of steering and navigation while keeping the vehicle attitude stabilized. Added to these restrictions are the needs for minimum volume and mass. An intricate system results that is extremely difficult to design, construct, and test and usually is one of the most expensive on the vehicle. Modern inertial sensors and systems cover more than five decades of continuous research and development. Integrated navigation system with the emergence of lower cost MEMS-based inertial sensors replacing the earlier ones for reasons such as performance, cost, size, power requirement and reliability. Continued miniaturization of hardware resulted in increased complexity for nanosatellite formation missions. Satellite formation flying has the potential to greatly enhance space based capability for observation, monitoring and experimentation.

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