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INSTABILITIES IN STRATIFIED MAGNETIZED STELLAR ATMOSPHERES

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Abstract. Gravity causes a stratification of plasma which introduces a possibility for instabilities to set in. In what follows, we assume a stratified plasma with horizontal magnetic field, prescribed gravity and temperature profile. Several different possible thermodynamic properties of plasma are taken into consideration in discussions of their influence on distributions of mass density, and consequently of potential energy, with height, in order to estimate the free energy of the unperturbed system that might be released through induced plasma motions if the system is unstable to linear perturbations. Finally, the linear approximation and normal mode analysis are applied to derive analytical dispersion relation and instability criterion for a special and commonly used case when perturbations are taken adiabatic while the magnetized plasma is assumed ideal and acting as a perfect gas.

1. INTRODUCTION

Many features in astrophysical plasmas can be studied by methods of fluid dynamics when the ionized gas is treated as a continuous medium in which individual particles lose their identity. Such an approach is physically and mathematically justified whenever the considered structure has length and time scales largely exceeding the mean free path, and time between collisions of plasma particles, respectively. Moreover, the fluid approximation can often be applied also to collisionless plasmas in magnetic fields provided the length scale of the observed phenomenon is much bigger than the Larmour radius of charged particles, and its typical frequency much smaller than the related cyclotron frequency. Under these conditions, the standard equations of magneto-hydrodynamics (MHD) are allowed in analytical and numerical treatments of plasma phenomena.

The problem of stability of magnetic fields in stratified astrophysical plasmas has been studied for a long time since early papers by Tserkovnikov 1960 and Newcomb 1961 till recent works by Ferrière et al. 1999 and Pinter et al. 1999, and references therein. Different applications of linear waves and stability can be found among others in Lemaire 1999, 2001 (the terrestrial magnetosphere), Čadež et al. 1996 (magnetic arcades in the solar corona), Parker 1979 and Fan et al. 1999 (sunspots).

2. EQUILIBRIUM BASIC STATE

We shall consider general properties of linear MHD waves in gravitationally stratified stationary plasmas permeated by a nonuniform and horizontal, azimuthal, magnetic field $\vec{B}_0$ as usually taken in typical models of stellar/solar coronae and planetary magnetospheres in equatorial regions. The gravity vector $\vec{g}$ and unperturbed basic state quantities are assumed functions of the radial coordinate r only. Our basic state is thus uniform on surfaces of constant gravity or, in other words, it does not vary neither along the magnetic field lines nor in the perpendicular direction in the horizontal surface.

The initial equilibrium of forces is now given by:

$$\nabla p_0 = \rho_0 \vec{g} + \vec{j}_0 \times \vec{B}_0 - \rho_0 (\vec{v}_0 \cdot \nabla) \vec{v}_0 \tag{1}$$

where the form of pressure relation $p_0 = p_0(\rho_0, T_0)$ depends on the thermodynamic properties of the plasma assumed in the model. For example, one may use one or a combination of the following expressions:

$$\begin{array}{ll} p_0 = \mathcal{R}\rho_0 T_0 & \text{- perfect gas pressure,} \\ p_0 = a_1 T_0^4 & \text{- radiation pressure,} \\ p_0 = a_2 \rho_0^n & \text{- degenerate gas, polytropic gas pressure,} \\ p_0 = \dfrac{\mathcal{R}\rho_0 T_0}{1 - b_1\rho_0} - b_2\rho_0^2 & \text{- Van der Waals gas pressure.} \end{array} \tag{2}$$

As for the other physical quantities in Eq(1), we consider a radial gravitational acceleration $\vec{g} = g(r)\hat{e}_r$ with $g(r) = r_0^2 g(r_0)/r^2$, the magnetic field is toroidal with a prescribed r-dependent intensity $\vec{B}_0 = B_0(r)\hat{e}_\phi$ while the macroscopic fluid velocities if taken into an account, are assumed parallel to the magnetic field $\vec{B}_0(r)$. This allows for the following two interesting possibilities: $\vec{v}_0 = \Omega r \hat{e}_\phi$ if effects of solid rotation of the system are included and $\vec{v}_0 = v_0(r, \theta)\hat{e}_\phi$ if a horizontal shear flow exists.

As an example, let us assume a rotating system with plasma obeying the perfect gas law in which case Eq (1) yields the following initial density distribution:

$$\mathcal{R} T_0 \frac{d\rho_0}{dr} = -\rho_0 g^* - \mathcal{R}\rho_0 \frac{dT_0}{dr} - \frac{B_0^2}{\mu_0 r} - \frac{B_0}{\mu_0}\frac{dB_0}{dr}. \tag{3}$$

where the gravity g^* is given by $g^* \equiv g(r) - \Omega^2 r$, and the temperature profile $T_0(r)$ is known i.e. assumed initially prescribed.

A simple analysis of Eq(3) now offers a preliminary insight into possible physical effects of each term on its right-hand side and their contributions to the radial distribution of plasma density. Thus, the gravity term g^* introduces the intrinsic density stratification of a compressible fluid in an external gravitational field. If temperature is constant and magnetic field is absent, the density simply decreases with r according to the standard barometric formula. Introduction of a temperature gradient alters the density profile so that, for example, if $dT_0/dr < 0$ the density decrease with r is reduced meaning that more mass is now laying along the height r i.e. the potential energy energy of the whole system is bigger than in the case with $T_0 =$ const. Consequently, one can expect instabilities in such a system as a part of the excess potential energy can be converted into kinetic energy, which indeed happens through the known convective instability at super-adiabatic temperature gradients. As to the magnetic field effects, they are twofold. The negative term $-B_0^2/(\mu_0 r)$ arising from the magnetic field line curvature increases the density fall-off with r and therefore stabilizes the system i.e. reduces the total potential energy. The second term is proportional to the derivative $-dB_0/dr$ meaning that it may have either sign depending on the existing magnetic field distribution. Clearly, $dB_0/dr > 0$ stabilizes the system as the density fall-off with r is additionally increased i.e. the potential energy is reduced. However, if the magnetic field decreases with r i.e. if $dB_0/dr < 0$, this term has a destabilizing action on the system and competes with the stabilizing effect of the magnetic curvature term. The overall effect of a nonuniform toroidal magnetic field is therefore a resultant of the two actions. For example, if $dB_0/dr < -B_0/r$ which is the same as $d/dr(\log(B_0 r) < 0$, the negative density gradient is reduced and extra potential energy is available as a possible energy source for instability. The opposite effect is achieved if $d/dr(\log(B_0 r) > 0$ in which case the magnetic field tends to stabilize the system.

The described ad hoc analysis of stability properties can be performed in an analogous way also for other types of pressure relations and physical conditions listed in Eq(2).

The full treatment of instabilities and their onset criteria requires a proper analytical handling of the total set of linearized MHD equations. In a typical example what follows, we shall restrict the analysis to adiabatic perturbations in a stratified magnetized plasma having properties of a perfect gas.

3. NORMAL MODE ANALYSIS

Linearized set of MHD equations for small adiabatic perturbations of the assumed basic state is now given by:

$$\begin{aligned}
&\rho_0 \frac{\partial \vec{v}_1}{\partial t} + 2\rho_0 \vec{\Omega} \times \vec{v}_1 \\
&\quad = -\nabla p_1 + [g(r) - \Omega^2 r]\rho_1 \hat{e}_r + \frac{1}{\mu_0}(\nabla \times \vec{B}_0) \times \vec{B}_1 + \frac{1}{\mu_0}(\nabla \times \vec{B}_1) \times \vec{B}_0, \\
&\frac{\partial \vec{B}_1}{\partial t} = \nabla \times (\vec{v}_1 \times \vec{B}_0), \qquad \frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \vec{v}_1) = 0, \\
&\nabla \cdot \vec{B}_1 = 0, \qquad \frac{\partial p_1}{\partial t} + \vec{v}_1 \cdot \nabla p_0 = \gamma \mathcal{R} T_0 \left[\frac{\partial \rho_1}{\partial t} + \vec{v}_1 \cdot \nabla \rho_0\right]
\end{aligned} \tag{4}$$

These equations are treated by a specific semi-local normal mode analysis that is described in details in Cadez 2005. The obtained dispersion relation has either real solutions for perturbation frequencies ω or purely imaginary in which case the system is unstable with amplitudes exponentially growing in time. The instability criterion, i.e. the condition for the dispersion relation to yield imaginary solutions for ω is given by the inequality (Cadez, 2005):

$$0 \quad < V_A^2 k_\parallel^2 \quad < \quad -\left(g - \Omega^2 r - \frac{2V_s^2}{r}\right) \frac{V_A^2}{V_s^2} \frac{d}{dr} \ln(B_0 r) - N_0^2 \tag{5}$$

where V_A^2 and V_s^2 are the Alfvén speed and sound speed squared respectively, $k_\parallel$ is the wavenumber parallel to the magnetic field, and:

$$N_0^2 \equiv (g - \Omega^2 r)\left[\frac{1}{\gamma}\frac{d}{dr}\ln T_0 - \left(1-\frac{1}{\gamma}\right)\frac{d}{dr}\ln\rho_0\right]$$

is the square of Brunt-Väisäläa frequency in a non-magnetized rotating atmosphere representing adiabatic oscillations of fluid particles about equilibrium levels if $N_0^2 > 0$, or the standard convective instability growth-rate squared if $N_0^2 < 0$.

As we see, the instability criterion (5) indicates that perturbation with sufficiently large wavenumber $k_\parallel$, i.e. whose wavelength $\lambda_\parallel \equiv 2\pi/k_\parallel$ does not exceed a given value, are stable. Unstable perturbations are therefore those with large wavelength $\lambda_\parallel$ having a form of rising and sinking undulated magnetic tubes with the magnetic buoyancy force being the main driving mechanism and the opposing stabilizing force due to the magnetic field line curvature related to the wavelength $\lambda_\parallel$.

The condition (5) also shows that the presence of nonuniform magnetic field can stabilize/destabilize the convective instability in an initially non magnetized atmosphere with a super-adiabatic temperature gradient when $N_0^2 < 0$. This fact is important in determining domains of convective zones in stellar interiors among other things.

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