CHALLENGES OF MODERN STELLAR SPECTROSCOPY:
DOPPLER IMAGING AND DOPPLER TOMOGRAPHY

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Abstract. Basics of the two modern indirect imaging techniques Doppler imaging and
Doppler tomography are reviewed. They benefited the studies of stellar surface inhomogene-
ities and flattened disc-like structures during the last decades. The foundations of Doppler
imaging lay in the fact that spottedness is the main reason for stellar variability due to
rotation. Stellar rotation modulates both photometric and spectroscopic characteristics of
the objects we observe, giving the possibility to reconstruct the spatial distribution of the
temperature or the concentration of chemical elements over the stellar surface. Patchy sur-
faces are typical for many early and late type variable stars, and it is generally thought
that starspots have a close connection with the stellar magnetic field geometry. Polarimet-
ric measurements allow this geometry to be successfully reconstructed via Zeeman-Doppler
method.

The other modern technique, Doppler tomography, is based primarily on the strong atomic
line emission, e.g. in hydrogen Balmer lines, generated by accretion discs. It transforms line
profiles observed at different orbital phases of a binary star into characteristics of the emission
regions over the disc or over the surface of primary component.

Observational constraints of the Doppler imaging and Doppler tomography are discussed
in brief.

1. INTRODUCTION

Various indirect imaging techniques used often in modern astrophysics tender an
enormous diagnostic power for analysing spatially distributed stellar phenomena.
According to the commonly used definiton applying indirect imaging of stellar surfaces
and flattened structures means to restore spatial distribution of some physical parame-
ters using time-dependant spectroscopic and/or photometric observational data.

In regard to stars we have Light Curve Modeling method that is based mainly on
photometric data. Different numerical techniques are exploited to reveal and model
Kjurkchneva (1990), and Kallrath and Milone (1999). When spectroscopic data so
called Doppler Imaging technique is used. It has been developed in years by Deutsch
(1970), Khokhlova (1975), Vogt and Penrod (1983), Piskunov et al. (1990), and
Strassmeier (2002). Because Doppler Imaging produces maps of stellar surfaces it is
also known as Doppler Mapping.
The corresponding techniques to disclose structures with quite different geometry
and physical background like accretion disks formed during binary systems evolution
are Eclipse Mapping that relies on the information contained in the eclipse light curves
(e.g. Horne 1985), and Doppler Tomography which utilises information taken mainly
from emission spectral lines and radial velocity curves (e.g. Marsh and Horne 1988,
Marsh 2001).

The subject of this short review is limited to the both spectroscopic approaches,
thus leaving for the photometric data only the supporting role.

2. DOPPLER IMAGING

Our Sun is the only star we can look at its surface directly. This way solar spots
are observed. Their number and total surface vary in time following solar activity
cycles. No chances to observe spots at the same size like solar on any other star, but
fortunately, there are much larger stars and much larger spots. Fairly good example
is the K0 giant star HD 12545 (XX Tri). As it has been reported this star exhibits
one cool high-altitude spot of gigantic dimensions - its oval shape bounds a surface
that exceeds twelve by twenty solar radii (Strassmeier 1999).

The idea that other stars could also have spots is more than three centuries old.
Together with the stellar pulsations nowadays spottedness is regarded among few main
reasons for stellar variability, since stellar rotation modulates both photometric and
spectroscopic behaviour of the objects. Possibility to interpret observational data as
a simple result of almost endless combinations between regular rotation and irregular
spots of different size and shape makes this idea very efficient and attractive.
Moreover, starspots bear crucial information about the stellar magnetic fields at all.
This is because only magnetic fields can make any surface structures like starspots to
appear or to vanish.

Two main types of spots are observed according to their origin and physical nature.
In late-type active stars, like RS CVn, BY Dra, FK Com, T Tau, UV Cet, single G –
K giants and main sequence stars, W UMa, Algols, and secondaries of cataclismic
variable stars temperature spots are found. They are cooler than the surrounding
surface just because convection is extinguished. As surface structures they are highly
changeable - its lifetime does not exceed few weeks, or months. These starspots can
be produced by weak and complex magnetic fields which strength is usually not larger
than few tens of Gauusses, such fields are still rarely observed. This kind of magnetic
fields are generated by stellar dynamo mechanism.

The other type of spots is characteristic for early-type magnetic chemically peculiar
stars, often called Bp and Ap stars. The abundance pattern of their atmospheres
differ significantly from what can be found in solar atmosphere, as for example. Since
the particle transport across the magnetic lines is strongly restricted large regions of
higher concentration of various chemical elements are created - and starspots are
abundance spots. As surface structures they are steady and stable, no changes are
observed for decades and even more. Magnetic fields are strong and ordered (poloidal,
with poles), their strength reaches few tens of kiloGausses. In oposite to the case of
late-type stars such magnetic fields have fossil origin.

Following the steps described in the original paper of Vogt and Penrod (1983) to
accomplish Doppler imaging means to gain information about stellar surface struc-
tures from spectral line profiles and their variations due to rotation. As a spot moves
across the star, line profile changes. An image of the stellar surface can be then reconstructed from the observed line profile. Extended and informative Doppler maps of different type of stars that illustrates well Doppler imaging process and its results can be found widely in the literature - distribution maps of some chemical elements on the surface of the Ap star prototype $\alpha^2$CVn (Kochukhov et al. 2002), connection with the magnetic field geometry in another Ap star - HR 3851 (Kochukhov et al. 2004), starspots evolution in RS CVn type star UZ Lib (Strassmeier 1997), and flip-flop activity cycle in another RS CVn star II Peg (Berdyugina et al. 1999).

When information about the circular and linear polarization is also included in calculations the technique is called Zeeman-Doppler imaging. It is an extension to the temperature and abundance mapping, and was developed by Semel (1989), Donati et al. (1989), and Brown et al. (1991). Zeeman-Doppler maps and examples also can be found - strong magnetic field are connected with a hot spot on the surface of II Peg (Carroll et al., 2007), strong and poloidal magnetic field combined with a solid body rotation of very cool and fully convective late-type star V374 Peg (Donati et al. 2006), rapidly oscillating magnetic Ap star HD 24712 (Lüftinger et al. 2009).

It looks very easy to describe the observational constraints of Doppler imaging technique. To get a successful Doppler map you need to study stars with rotational velocities $\text{vsin}i$ between 10 km/s and 100 km/s, with as higher as possible signal-to-noise ratio - between 200 and 700, with good phase coverage - best known data-sets comprise of measurements made in more than forty individual rotational phases, and with a spectral resolution that begins with 30 000 and finishes beyond 100 000. Data processing and computations behind are also very important, since spatial restoration using line profiles is almost perfect example of an ill-posed problem. As a rule it has no unique solution and usually needs some additional information to be taken into account as a prerequisite. Among the mathematical foundations of all the numerical codes developed so far one should mention first Maximum Entropy Method, developed by Vogt et al. (1987), Brown et al. (1991), Rice and Strassmeier (2000), Tikhonov regularisation method by Piskunov et al. (1990), a CLEAN clone by Küster (1993), and last but not least, an Occamian approach (to find the simplest opportunity) - Berdyugina (1998), Savanov and Strassmeier (2005). The encouraging conclusion made by Berdyugina (2005) that “the difference (in the final results) is diminished when the data used are of high quality” just shows the right way of avoiding a lot of troubles and many ambiguities.

3. DOPPLER TOMOGRAPHY

According to the modern dictionaries and encyclopedias word “tomography” means “imaging by sections, or sectioning”. In general tomography refers to the process of reconstructing N-dimensional object from its (N-1)-dimensional projections (or slices). Thus, the main goal of any data collection is to make slices trough the object or observe projections around it. Using projections to judge about the true shape of an object is very old idea. Discussing round contours of the Earth’s shadow visible during the Lunar eclipses Aristotle reached to the absolutely right conclusion that Earth should have a spherical form.

Computational roots of Doppler tomography originated from a fine mathematical tool called Radon transform. Johann Radon (1887–1956), an Austrian born mathematician described how a 3D-object can be presented numerically by its 2D-projections.
Inverse Radon transform is much more interesting for astronomers, actually. It can be used to build a 3D object having its 2D projections. Radon transform basics can be illustrated with the help of well-known transform between cartesian and polar coordinates. For every straight line we can write:

\[
\rho = x \cos \theta + y \sin \theta, \tag{1}
\]

where \(\theta\) is the polar angle, and \(\rho\) is the shortest distance between the pole and the given line. Therefore, in a \((\rho, \theta)\)-space this line would be presented with a single point. Squared object comprising four lines in \((x, y)\)-space will be transformed to four points only in \((\rho, \theta)\)-space. With this analogy in mind we can write:

\[
R(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) \, dx \, dy, \tag{2}
\]

where \(f(x, y)\) stands for the object in cartesian space, \(R(\rho, \theta)\) - for its image in \((\rho, \theta)\)-space, and \(\delta(0) = \infty\), otherwise \(\delta(a) = 0\).

In this case the inverse Radon transform will be given by:

\[
f(x, y) = \int_{0}^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\rho, \theta) |\omega| e^{2\pi i \omega (x \cos \theta + y \sin \theta - \rho)} \, d\rho \, d\omega \, d\theta \tag{3}
\]

Using first derivatives (velocities) instead of cartesian coordinates leads us to the basic equation used in Doppler tomography:

\[
f(v_x, v_y) = \int_{0}^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(v_r, \phi) |\omega| e^{2\pi i \omega (v_x \cos \phi + v_y \sin \phi - v_r)} \, dv_r \, d\omega \, d\phi, \tag{4}
\]

where \(v_r = v_x \cos \phi + v_y \sin \phi\) is the radial velocity, \(\phi\) is the orbital phase, and \(\omega\) is the angular frequency.

Results of Doppler tomography are widely used to obtain visualisation of the line-forming regions in different kind of binary systems, like Algols (gas streams, atmospheric structures) or cataclismic variable stars (accretion disks, accretion disk structures, hot spots, etc.). The observational data set consists of emission line profiles taken at different rotational phases. Since the line profiles are broadened by rotation, they present projections of density and velocity distribution along the line of sight. As the binary system orbits it offers sequence of such velocity projections. The set of Doppler broadened emission line-profiles observed at different orbital phases gives information enough to build Doppler maps, or tomograms. The tomogram itself is a map of velocities and fluxes, it presents the intensity-distribution of light-emitting regions in the two-dimensional velocity space.

Due to the common mathematical backgrounds the implementation of Doppler tomography looks similar to that of Doppler imaging. There are two main approaches again. The first one is Fourier-filtered back-projections. As described above it employs emission line-profiles in velocity coordinates. Problems with the noise are solved by using Fourier filtration. This approach is fast and easier from computing point of view, but it should be stressed that some problems usually arise when working with saturated spectral lines or complex line blends. Second approach is concerned with the Maximum Entropy Method. A rich and extended set of model maps (~1000 and even more) is calculated and then compared backward map by map with the
observations by using $\chi^2$ statistic to measure the goodness of fit. Because of the noise $\chi^2$ optimization procedures are often used to make model data and observational data consistent.

Observational constraints of the Doppler tomography rely on the ways used to resolve the main problem of all spectroscopic observations - how to raise time resolution keeping signal-to-noise ratio and spectral resolution as high as possible. Practice has clearly shown that the solution is not easy, not unique, and not always successful. Let’s finish the lecture featuring Marsh (2001) paper, and using an observational example. Let the ratio $\frac{t}{P}$ is the phase resolution. Here $t$ is the exposure time and $P$ is the orbital period of the binary system. If $\Delta v$ is our spectral resolution in km/s, and the speed of an accretion disk feature we want to study is $K$, then, by geometrical reasons we can write that:

$$\frac{t}{P} \sim \frac{\pi \Delta v}{2 K}.$$  \hspace{1cm} (5)

Modest spectral resolution of about 50000 means $\Delta v \sim 6$ km/s, and the typical value of $K$ for many accretion disks in cataclysmic variable stars is about 400 km/s. Phase resolution with these numbers will be $\frac{t}{P} \sim 0.003$. Finally, with an orbital period of about 16 hours we would get for a single exposure $t$, the value of nearly 2.5 minutes. This is a real challenge for high resolution spectroscopy because cataclysmic variable systems are typically faint, and building good and reasonable Doppler tomograms requires high signal-to-noise spectroscopic observations in more than forty orbital phases. Doppler tomography is not only the observational challenge, but the game is worth all the efforts, because several hundreds papers have been published so far, revealing many intriguing and important details of the mass transfer processes in binary systems.

Acknowledgments

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References