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ON THE GEODETIC ROTATION OF THE MAJOR PLANETS, PLUTO, THE MOON AND THE SUN

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Abstract. The problem of the geodetic (relativistic) rotation of the major planets, the Moon and the Sun was studied in the paper (Eroshkin and Pashkevich, 2007) only for the components of the angular velocity vectors of the geodetic rotation, which are orthogonal to the plane of the fixed ecliptic J2000. This research represents an extension of the previous investigation to all the other components of the angular velocity vectors of the geodetic rotations with respect to the proper reference frames for these bodies (Seidelmann et al., 2005). The data provided by DE404/LE404 ephemeris (Standish et al., 1995) are applied. For each body the files with the data of the projections of the angular velocity vectors of the geodetic rotations on the axes of the proper reference frame are constructed over the time span from AD1000 to AD3000 with one day spacing. The most essential terms of the geodetic rotation are found by means of the least squares procedure and the spectral analysis methods.

1. INTRODUCTION

In every relativistic ephemeris of the Sun, major planets, and the Moon the principal gravitational interaction is modeled by considering these bodies as non-rotating point masses. Nevertheless an ephemeris of the major bodies of the solar system, based on the relativistic equations of the orbital motion, contains data necessary for the calculation of the secular and periodic components of the angular velocity of the geodetic rotation of these bodies. In accordance with the research (Landau and Lifshitz, 1975), a geodetic rotation arises when a body, having non-zero moments of inertia, is orbiting in the Riemannian space of general relativity. The vector of the angular velocity of the geodetic rotation, which is the most essential relativistic component of the body rotational motion around the proper center of mass, has the following expression:

$$\bar{\sigma}_i = \frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{\left|\bar{R}_i - \bar{R}_j\right|^3}\left(\bar{R}_i - \bar{R}_j\right) \times \left(\frac{3}{2}\dot{\bar{R}}_i - 2\dot{\bar{R}}_j\right).$$

Here c is the velocity of light; G is the gravitational constant; m_j is the mass of a body j; $\bar{R}_i$, $\bar{R}_j$, $\dot{\bar{R}}_i$, $\dot{\bar{R}}_j$ are the vectors of the barycentric position and velocity of bodies i

and j. The symbol × means a vector product; the subscripts i and j correspond to the major planets, the Moon, and the Sun.

2. RAW DATA

For each body the files with the data of the projections of the angular velocity vectors of the geodetic rotation on the axes of the proper reference frames are constructed over the time span from AD1000 to AD3000 with one day spacing by using DE404/LE404 ephemeris. The behavior of the components of these angular velocity vectors for the Earth, the Moon, the Sun, Mercury, Pluto and Uranus are depicted in Figs. 1 – 6, respectively. In these Figures σ_ψ represents the geodetic motion of the equator of a body on the fixed ecliptic J2000, σ_θ describes the geodetic variation of the obliquity of the equator of a body to the fixed ecliptic J2000, σ_φ represents the behavior of the projection of the angular velocity vector of the geodetic rotation on the axis of the angular velocity vector of the rotation.

Since the mass of the Sun is dominant in the solar system then the main part of the angular velocity vector of the geodetic rotation $\bar{\sigma}$ for each major planet and the Moon is a result of the heliocentric orbital motion of these bodies.

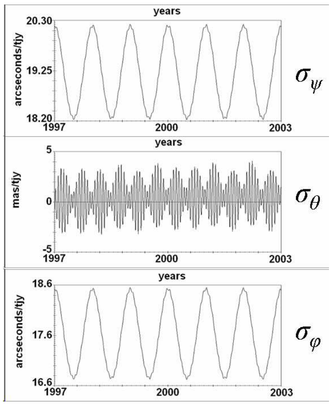

Figure 1: For the Earth.

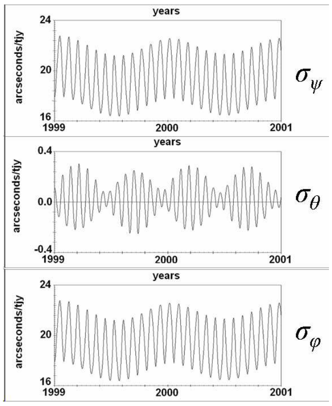

Figure 2: For the Moon.

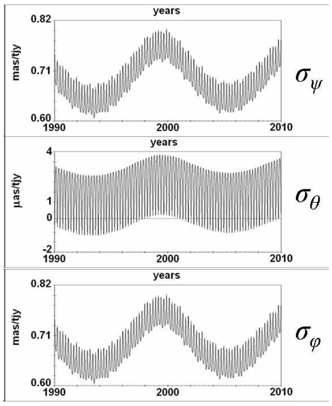

Figure 3: For the Sun.

The geodetic rotation of the Moon is determined not only by the Sun but also by the Earth. Fig. 2 demonstrates it visually.

The vector of the geodetic rotation of the Sun is determined by the orbital motion of the planets. Since the masses of the planets are essentially less than the mass of the Sun then the geodetic rotation of the Sun is very small. Its main part (presented in Fig. 3) depends on the orbital motions of Jupiter and Mercury.

A similar character of the behavior of the components of the angular velocity vector of the geodetic rotation for Mercury (presented in Fig. 4) and for Pluto (presented in Fig. 5) is explained by the fact that the values of the eccentricities of their orbits (given in Table 1) are close to each other and relatively large as compared to the other planet orbits. Since Mercury is the nearest planet to the Sun then it is clear that its geodetic rotation has to be the most significant in the solar system.

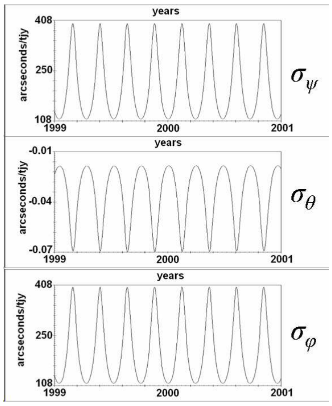

Figure 4: For Mercury.

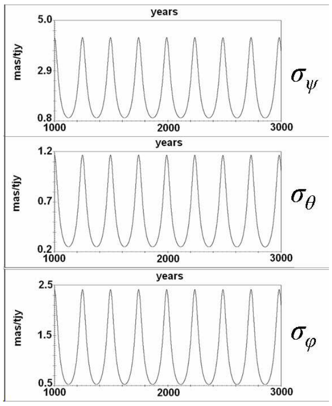

Figure 5: For Pluto.

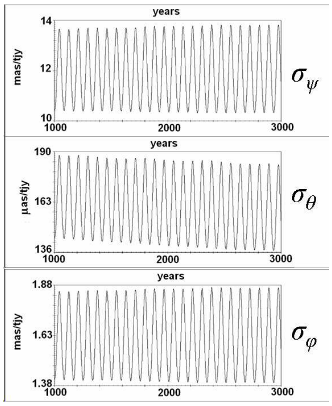

Figure 6: For Uranus.

The position of the rotation axis of Uranus is close to the ecliptic plane. As a consequence, the difference between the values of the components σ_ψ and σ_ϕ on Fig. 6 is very significant as compared to the similar components for the other bodies.

3. RESULTS

Table 1: The main secular and periodic terms of the geodetic rotation in the longitude of the body equator

Object	Secular part	Periodic part	Eccentricity of the orbit
Mercury	$213''.3T$	$(1078''.175\sin\lambda_1 - 4845''.763\cos\lambda_1)10^{-6}$	0.206
Venus	$43''.048T$	$(-56''.807\sin\lambda_2 - 64''.066\cos\lambda_2)10^{-6}$	0.007
The Earth	$19''.199T$	$(-34''.283\sin\lambda_3 - 149''.221\cos\lambda_3)10^{-6}$	0.017
The Moon	$19''.494T$	$(-34''.278\sin\lambda_3 - 149''.2\cos\lambda_3 + 30''.212\sin D)10^{-6}$	
Mars	$6''.752T$	$(515''.779\sin\lambda_4 + 229''.128\cos\lambda_4)10^{-6}$	0.093
Jupiter	$0''.312T$	$(82''.801\sin\lambda_5 - 21''.288\cos\lambda_5)10^{-6}$	0.048
Saturn	$0''.069T$	$(-2''.691\sin\lambda_6 - 52''.94\cos\lambda_6)10^{-6}$	0.056
Uranus	$0''.012T$	$(-22''.266\sin\lambda_7 - 3''.466\cos\lambda_7)10^{-6}$	0.046
Neptune	$0''.004T$	$(1''.839\sin\lambda_8 - 1''.78\cos\lambda_8)10^{-6}$	0.009
Pluto	$0''.002T$	$(59''.423\sin\lambda_9 + 0''.273\cos\lambda_9)10^{-6}$	0.249
The Sun	$7''10^{-4}T$	$(0''.105\sin\lambda_5 - 0''.027\cos\lambda_5 - 0''.001\cos\lambda_1)10^{-6}$	

The most essential terms of the geodetic rotation are found by means of the least squares procedure and the spectral analysis methods. The mean longitudes of the planets, the Moon and Pluto, adjusted to the DE404/LE404 ephemeris, are taken from the previous investigation (Eroshkin and Pashkevich, 2007). Table 1 represents the main secular and periodic terms of the geodetic rotation in the longitude of the body equator, which are found after the integration of the corresponding components σ_{ψ}. Here λ_j $(j = 1, ...9)$ are the mean longitudes of the planets; λ_{10} is the mean geocentric longitude of the Moon; $D = \lambda_{10} - \lambda_3 + 180^o$; T is means the Dynamical Barycentric Time (TDB), measured in thousand Julian years (tjy) from J2000. It is easy to see that the values of the secular and periodic parts of the geodetic rotation of each planet depends on its distance from the Sun.

4. CONCLUSIONS AND PERSPECTIVES

1. For the Sun, giant planets and Pluto the geodetic rotation is insignificant.
2. For the terrestrial planets and the Moon the geodetic rotation is significant and has to be taken into account for the construction of the high-precision theories of the rotational motion of these bodies.
3. Geodetic rotation has to be taken into account if the influence of the dynamical figure of a body on its orbital-rotational motion is studied in the post-Newtonian approximation.
4. The lunar laser ranging data processing has to use the relativistic theory of the rotation of the Moon, as well as that of the Earth.

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References

Eroshkin, G. I., Pashkevich, V. V.: 2007, Geodetic rotation of the Solar system bodies, Artificial Satellites, Vol. 42, No. 1, pp. 5970.

Landau, L. D. and Lifshitz, E. M.: 1975, The Classical Theory of Fields, Oxford: Pergamon Press.

Seidelmann, P. K., Archinal, B. A., A'Hearn, M. F., Cruikshank, D. P., Hilton, J. L., Keller, H. U., Oberst, J., Simon, J. L., Stooke, P., Tholen, D. J. and Thomas, P. C.: 2005, Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements: 2003, *Celestial Mechanics and Dynamical Astronomy*, **91**, pp. 203-215.

Standish, E. M., Newhall, X X, Williams, J. G. and Folkner, W. M.: 1995, JPL Planetary and stellar Ephemerides, DE403/LE403, JPL IOM 314.10-127.